Graph Theory
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Lecture No. # 27
More Special Classes of Graphs

In this lecture, we will consider some special classes of graph. So, in the last two classes, we were looking at perfect graphs and we in fact studied several special classes of graphs before that, because to introduce the perfect graph, we studied chordal graphs, comparability graphs, several such classes we studied.

So, this is in some sense, a class which will just introduce some more special classes, which are not yet covered and because I think that good idea to have some awareness of these graph classes. So when you do graph theory, if you are interested in doing research in graph theory because, so this just a preliminary introduction of some of these things.

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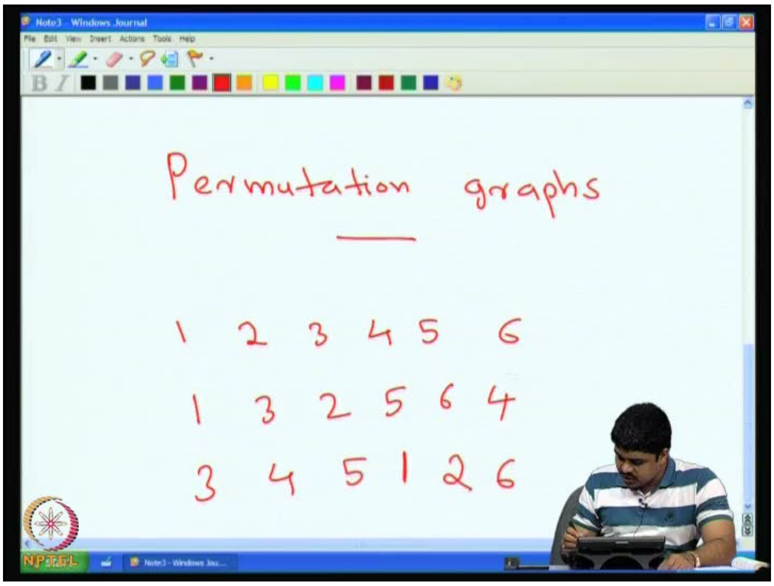

So here is one graph class which I want to introduced, is the permutation graphs. What are permutation graphs? So consider a permutation, say, suppose 1 2 3 4 5 6, this, a permutation or you can say 1 3 2 5 6 4, say another permutation or 3 4 5 1 2 and 6, this, another permutation.

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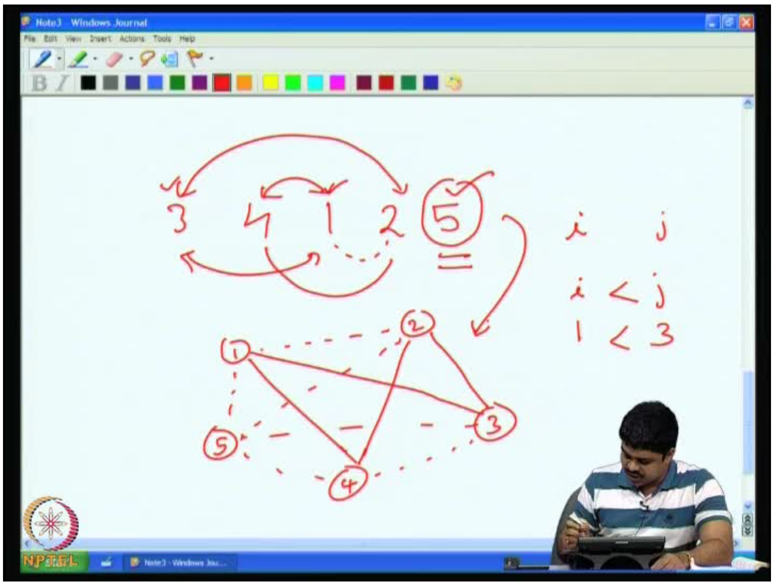

So for each of this thing corresponding to each of this permutation, we can introduce a graph like this, suppose you consider this permutation, namely 3 4 1 2 5, so often to 5, so introduce some vertices, 1 corresponding to each of this numbers 1 2 3 4 5.

Now what we do is, whenever you have the number, say 2 pairs, if you take i and j, so without loss, of then generally i is less than j.

Now in the permutation, see they are ordering, supposes if i is equal to 1 and j is equal to 3, now this 1 and 3 comes in the reverse order in the permutation, it is in the wrong order, so then we put an edge.

So for instance 1 and 2 they are in the correct order in the permutation, see here, therefore, so we want put an edge, now 2 and 3, 2 and 3 they are in the reverse order in the permutation because they come as 3 2, so the we put an edge here.

So now 1 and 4, 1 and 4 is in the reverse order therefore you put an edge here and what about 2 and 4, 2 and 4 is also on the reverse order, we will put an edge here.

And then what about 1 and 5, 1 and 5 is in the correct order, so we, you do not put an edge here and what about 4 and 4, 3 and 4 is in the correct order, therefore you would not put in the edge, 4 and 5 correct order, therefore here, this is a graph.

So the, you can see that 5 did not get any neighbor here because it was in the correct order with every vertex because 5 is a biggest number, it came in the last, therefore its non adjacent.

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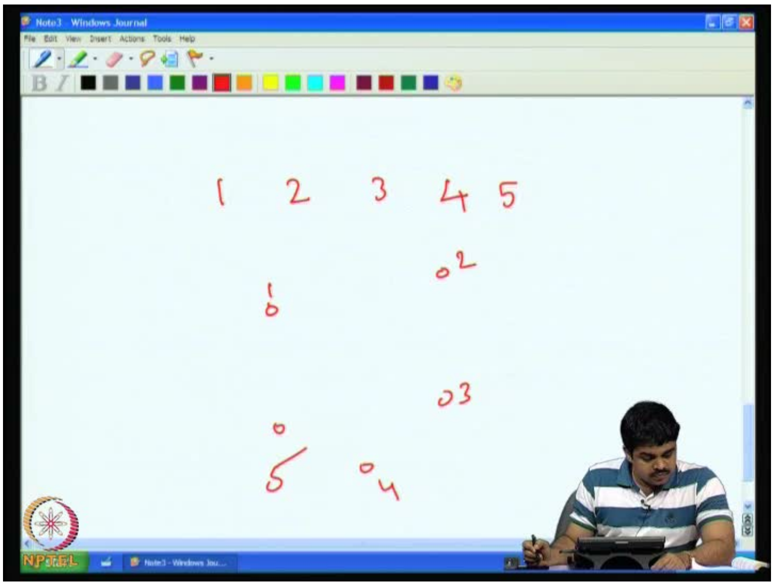

So this is the graph corresponding to this permutation. So you can ask, suppose this permutation was 1 2 3 4 5, like this, then what will happen, then everything will be in the, any pair if you take they are appearing in the correct order therefore our graph will be a collection of independence, so an independence set 1 2 3 4 5 isn't it.

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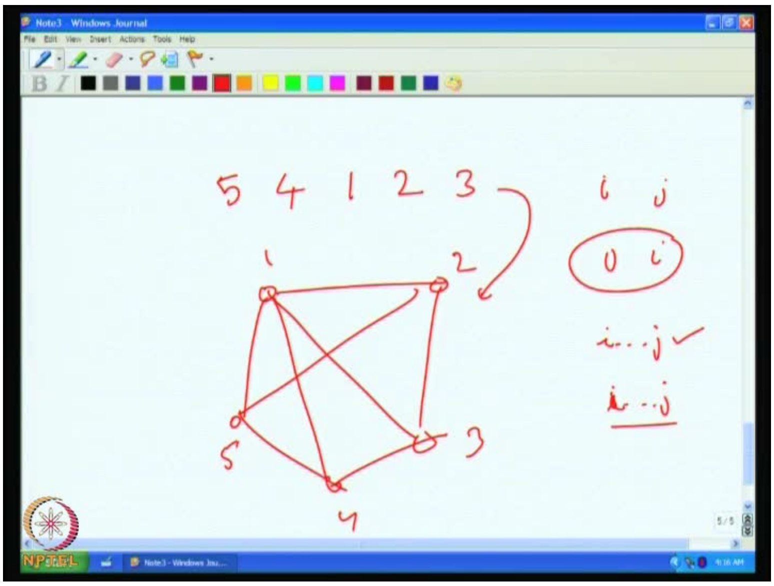

Now you can consider this reverse permutation, for instance, 5 4 1 2 3, how will the graph corresponding to that will come, so it will come like this, 1 2 3 4 5, here any pair, if we take the come in reverse order, therefore you will get the entire clique, all the adjust will be there in the graph.

Now you can see that. So then, if you get a permutation and if you draw the graph corresponding to that, now if you reverse the permutation and draw the graph corresponding to that, then what will happen?

So then naturally if i j was in the correct order, in this original permutation when you reveres it, it will become in the wrong order, therefore then the edge will appear or for instance if i j was in the wrong order in the original permutation, when you reveres they will become in the right order and therefore the edge will disappear.

So it will correspond to those. So when you take a permutation, draw its graph and then you take the reverse of the permutation and draw another graph, so these two graphs will complements of each other.

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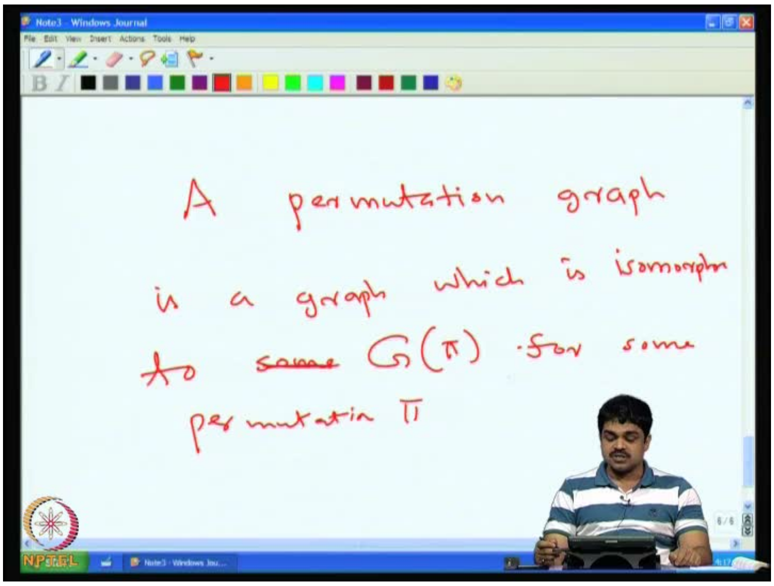

So what is, then we have this notion of permutation graphs, so the question is, what the permutation graph is? So a permutation graph is a graph which is isomorphic to some G of pi, for some permutation pi.

What is G of pi? G of pi is the graph corresponding to that permutation, so in other words the issue is to get, so we have to demonstrate some permutation, if you want to show that this graph is a permutation graph, if you want to demonstrate a permutation and then show the numbering on the vertices of the graph, say that and then if this is the numbering and then consider is permutation and draw the corresponding to that, then we have this graph, should be exactly that graph, (( )) so we are only saying that isomorphic to G of phi, for some permutation phi, because we have the option of renumbering the vertices in a such way that it becomes isomorphic to G of pi, for some permutation, so this is what is called a permutation graph.

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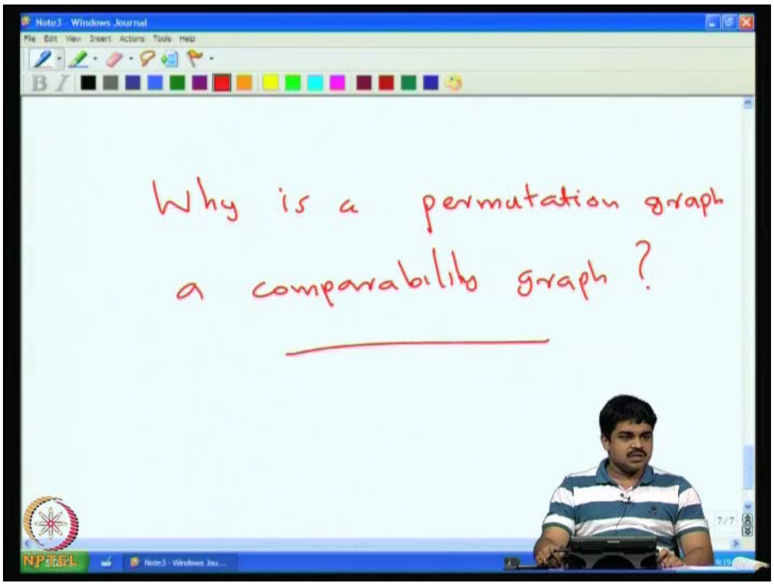

Now this is also known to be a perfect graph because it is known to be a sub class of comparability graph. Is it easy to see that any permutation graph is a comparability graph? So why is a permutation graph a comparability graph?

First let us remind what a comparability graph. So we had discussed it in the 3 classes before, just before introducing the perfect graphs. We had discuss the comparability graph class, essentially, comparability graphs corresponds to a partial order and in other words they are the ones who can be obtained from a partial order by discarding the self the directions on the edges and so this loops and the directions on the edges in other words, it captures which pairs of sector comparable and in other words we can say that if a graph is comparability, if we can transitively orient the edges of the graph.

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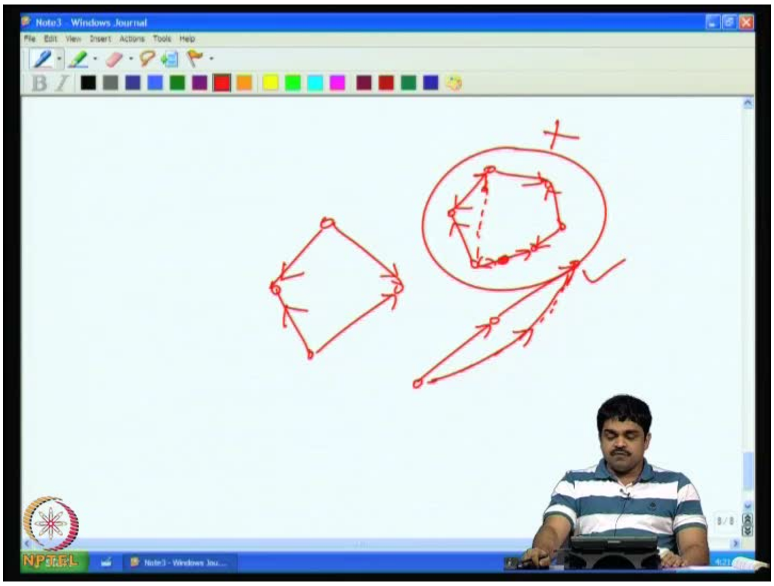

In other words, for instance if we had taken this cycle you can orient the edges of this graph like this, it is a transitive orientation and sense that whenever you see 2 edges in the same direction and then this edges should be present this, third edge should be present and the direction should be like this, that is, such an orientation is called transitive orientation, As we have seen some graphs can be transitively orientated, for instance like this, but if it was an odd cycle could not have transitively oriented the graph 1 2 3 4 5, for instance this graph cannot be transitively oriented, why is it so?

For instances if we put like this, then this cannot be, this edges not there, so we have to keep putting like this, so we have to put like this, we have to put like this, we have to put like this, then here this edge, so this is that we have to put it across this, but then this says that should so this cannot be, odd cycle cannot be transitive.

So all these things we had discussed, and now the question is, why is a permutation graph a comparability graph? can we show that a permutation graph is a comparability graph?

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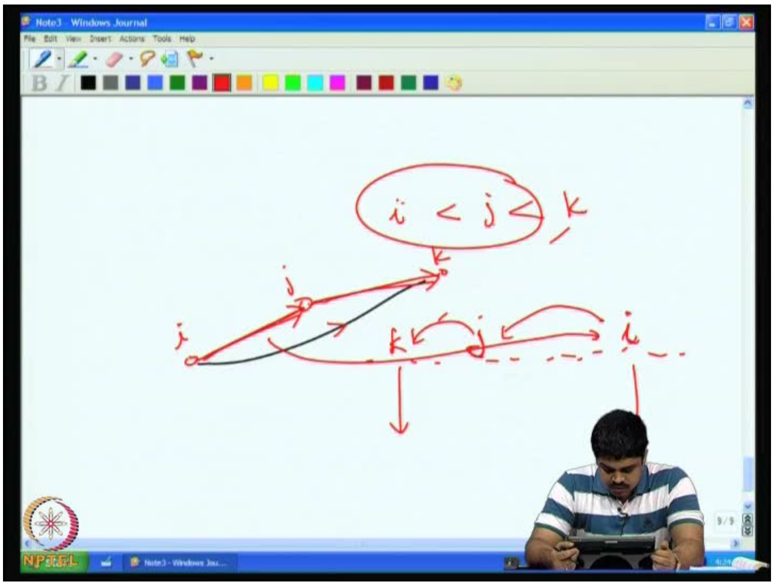

So to show that, we only have to demonstrate, how we can transitively orient the edges of a graph. Consider some permutation, now suppose it so happens that in the permutation- i, so let us consider i is less than j and so there is edge, between these are the 2 vertices, i is less than j, i j k, suppose you have 3 edges and suppose you decide to orient these two things, then we should show that there is this edge and also direction on the edge is like this, so this two things are important.

Now the question is to somehow orient the edges, so we will decide this rule whenever j is greater than i, let as put the arrow from i to j, therefore, so if there are 2, i j k, i j is an edge, and j is a, so the i less than j, less than k should be correct if you group follow this rule.

Now the point is the fact that i less than j and there is an edge between i and j means in the permutations, suppose if you look at the permutation, j occurred in an earlier position than i, that is why because that is a wrong order in the permutation, that is why this edge is present, here this means this j occurred earlier than i.

Now what about j k edge? k is a bigger quantity than j but the edge occurred means, so the k should be earlier than j. Now what can I tell about k and i, k is bigger than i and also there is an edge, so the order of k and i will definitely be reveres because j itself is earlier than i, k is even earlier than j, so k is definitely earlier than i, therefore k and i are in the wrong order, therefore they will be an edge between them.

Now the direction is always from i to k because we are always directing the edges from i to k, therefore we have demonstrate a transitive orientation for the permutation graphs.

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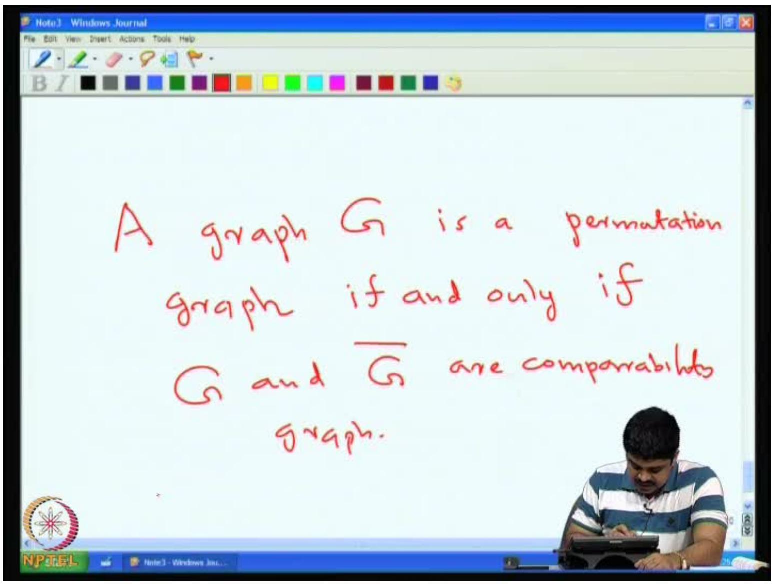

Therefore the permutation graphs are transitively orientable and then we therefore, we can infer that they can be the comparable graphs. Now an interesting theorem about permutation graphs is that, this permutation graphs are exactly the intersection of comparability graph and co-comparability graphs, that means a graph is permutation if and only if it is a comparability graph and complements is also a co-comparability graph, G and G complement are co-comparability graphs or in other words, the other side is that, if a graph is and complement and its complement both of them are comparability graph, it has to be a permutation graph.

So we write like this, a graph G is a permutation graph if and only if G and G bar are comparability graphs, see from all these things, so what we are saying, that the suppose if I write the setup permutation graphs as P, then P equal to the class of comparability graphs, intersection the complements of that co-comparability graphs.

So maybe this is not a correct way of replacing the essentially co-comparability graphs; is what should write co-comparability graphs (( )) intersection.

So then, now we will try to prove this theorem. So to prove this thing we have to prove two sides; first thing is to show that if it is a permutation graph, then it is a comparability graph and its complement is also comparability graph, this is already we have seen a permutation graph is indeed comparability graph.

What about the complement of a permutation graph? As we have seen, a permutation graph means, it has a certain permutation associate with it, that means you can identify a permutation on 1 to n, such that when you construct the G pi- G of pi, the graph of that permutation we will indeed get this particular graph.

And now what about reversing the permutation and taking the graph? So of course we will get its complement, therefore the complement of our graph is also a permutation graph and thus it is a comparability graph.

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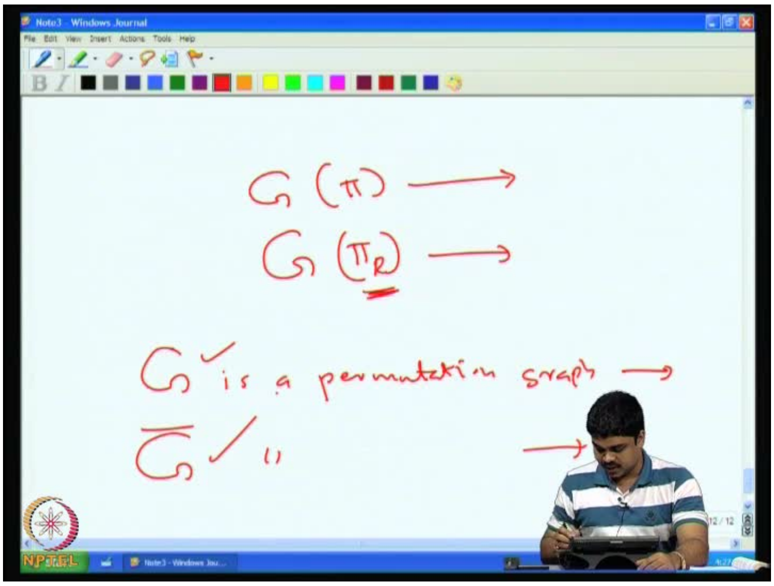

So what we wanted to prove is that, if G is a permutation graph, this is the sequence, then G complement is also a permutation graph and because it correspond to the reverse permutation and any permutation graph is comparability graph, therefore both G and G comparability graph, so that is the easy part.

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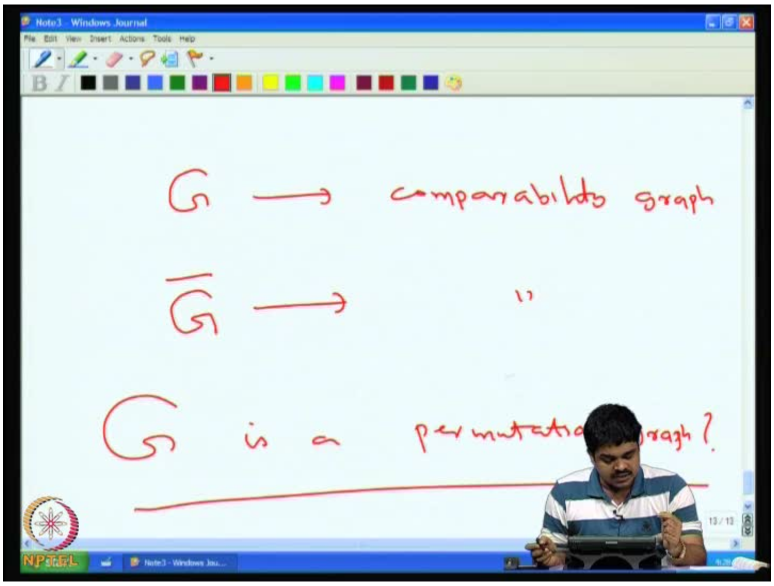

Now the more difficult part, suppose a graph is given- G, we know that it is a comparability graph, not only we know that, its complement is also a comparability graph. Now we want to show is that, G is a permutation graph, then we want to show that it is a permutation graph, in other words we want to demonstrate a permutation associated with G, such that G is the graph of that permutation, how do you do it?

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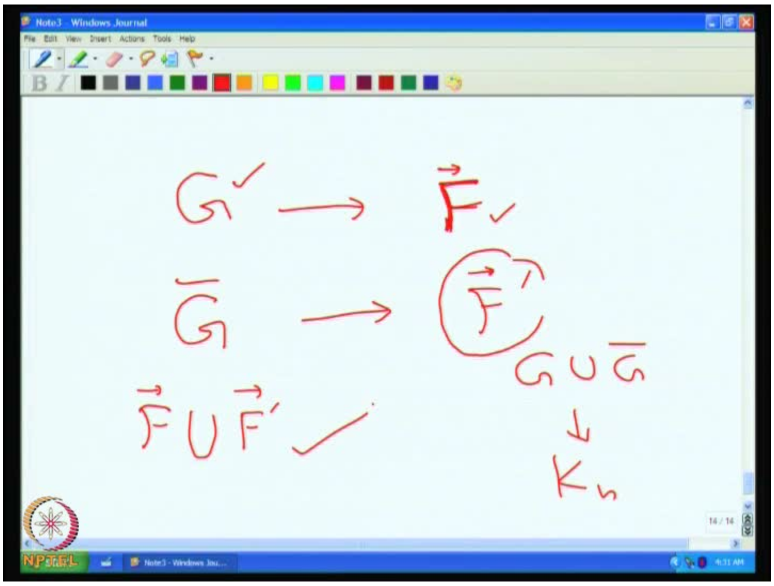

So to do this thing we first consider a transitive orientation of G, so G is a comparability graph, therefore it has a transitive orientation. Let us call this transitive orientation is F.

What do you mean by F, it is the directed edges, because every edge in G gets a direction, so the collection of directed edges will be called F because this is not a new entity because essentially an edge can be given two different direction, so we cannot identify it with just the edges, so we will call it F transitive orientation.

Now the G complement is also a comparability graph and its edges also can get a transitive orientation, let us call it F dash, so we can even put an arrow, if you want, this is the thing.

Now what will happen if I take F union F dash, so of course now if this the edge set of G union G complement, essentially correspond to the edge setup of a complete graph on n vertices, all the edges will come then.

Now what we see is, F union F dash is indeed orientation for the edges of a complete graph; K n, is not it because if we have collected the edges of G and given orientation for each of them, it is a indeed a transitive orientation.

Now G bar you took and then you gave a transitive orientation for that, this G bar contains exactly the remaining edges, then they are all the edges of the complete graphs, gets an orientation, now that is what has happened when you consider the F union F dash.

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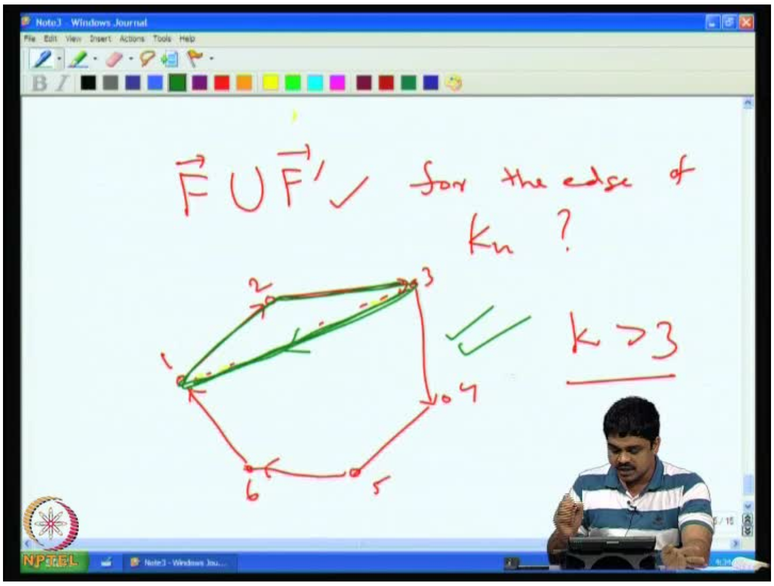

Now the next point is that, what can I tell about this orientation F given by F union F dash for the edges of K n, the complete graph on n vertices? we claim that it is acyclic, what do you mean by acyclic, that means they would not be any directed cycles with respect to these orientation of the edges in F union F dash, we would not have any directed cycles, this is indeed easy to prove, in fact, suppose there is a directed cycle, then what will happen, suppose there is a directed cycle, so then we can say that this is the directed cycle. We consider 2 cases; this directed cycle contains 1 2 3 4 5 6, so we consider the case, suppose the directed cycle contains k notes, k strictly greater than 3, here I drew 6 notes, so it can be 4 5 6 anything, suppose many edges are contained in the directed cycle.

Now you can see that, lets consider this thing, this possible 1 3 because it is a complete graph of edges, there there are 2 possibilities, see suppose there is a cycle, what we can do is, we can always ask for, there is a cycle, we can always ask for the smallest such directed cycle, the big smallest such directed cycle.

The smallest such directed cycle, suppose is of length more than 3, that is the case we are considering, we will contradict the fact that, it is smallest, where is the contradiction, you know this edge because it is complete graph, we have this edge the direction can be either like this. If it is like this, then what will happen? then you see we can follow this edge and then like this and we get a smaller directed cycle, so the flag that we took, the assumption that we took, the smallest directed cycle is violated here is not correct but then you may say that, what if the direction was not like that? So for instance it can be, the direction can be other way because it can be this way, then we consider this directed cycle, here this is a smaller directed cycle, then, this entire thing k.

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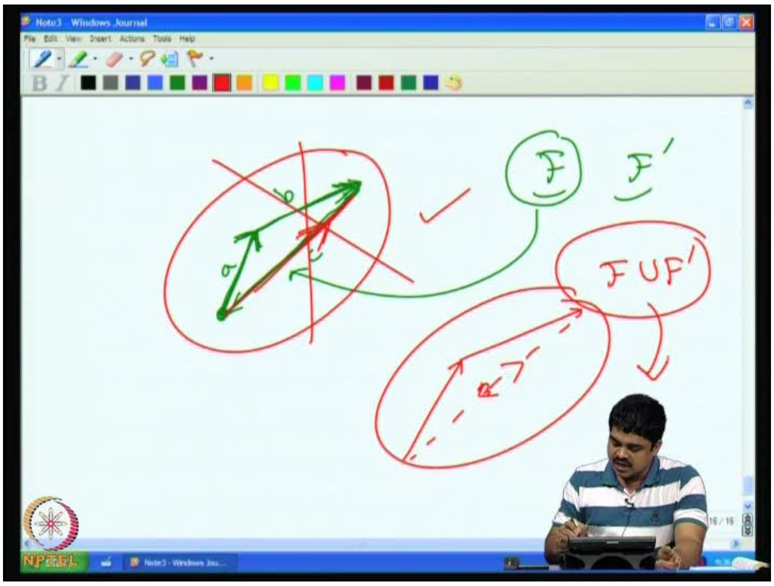

So in either case, whether the direction in this way or in this way in both ways you get a smaller directed cycle. So what can infer, the smallest directed cycle has to be of length 3, if at all there is a cycle also then you consider the smallest directed cycle of length 3, it can be like this

Now you can ask, there are this 3 edges; let say a b c, now that two of them should belong F or F dash, one of them it is not necessary, that should be in F or it can be F dash also at least two of them should belong to 1 group, is without loss of generality this a and b because it is same, only a and b belongs to this group because this structure is same because a and b, b and c, c and a both are 2 edges, one after the other, suppose it is a and b they belong to say F or F dash whichever 1 group.

Then what happens is, because it is a transitive orientation, this F is a transitive orientation and a and b belongs to F, then a, this one, this edge starting from this- to- this, this edge should be there in F itself because it is not possible to give it to the complement the other side, because otherwise how can this be transitive orientation, we gave directions like this, then this edge also should be present there and moreover if it is present, the direction should be this way, how can you give the direction the other way?

So therefore it is a total contradiction, here it is not possible, so if you have out of the 3, 2 edges will come to the same group, which is a transitive orientation, the third one also should belong to them and we have given with the wrong direction because we have given the circular direction, transitive orientation says it should be like this.

So therefore we got a contradiction and what was our assumption? What it we contradict? A contradict, the assumption that, there was a cycle, therefore the resulting orientation F union F dash, for the complete graph is indeed acyclic orientation, so we got acyclic orientation for complete graph.

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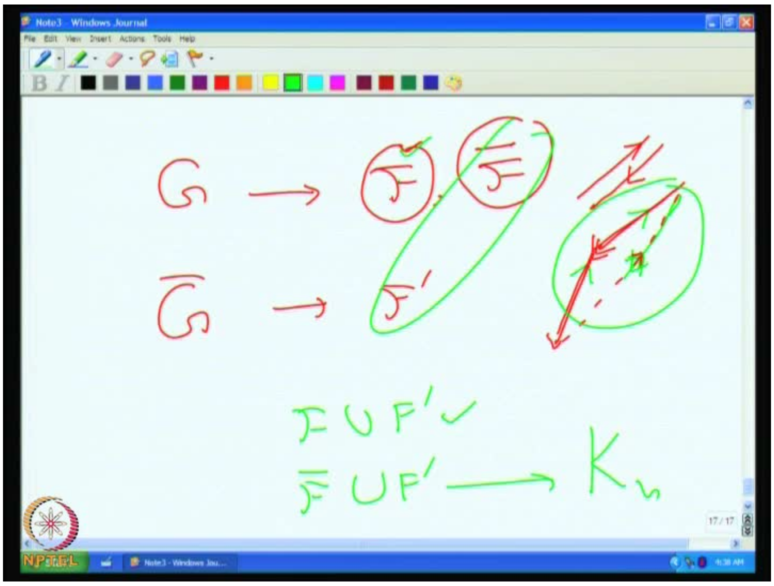

Now what we do is, you recall this F was the transitive orientation corresponding to G and F dash was the transitive orientation corresponding to d dash. Now suppose we have considered F bar, F bar means, the same edges, edge set is same but then whenever we directed like an edge, like this, an F we will direct it in the opposite way in the F dash, so I have reverse the directions of all the edges in F, then there we get a F bar.

So this is also a transitive orientation of G because it is very clear because this is suppose F bar is not transitive, that means that, we have something like this, so, some 2 edges like this and we do not have this or it is orientated wrongly.

But then if you had reversed, then in F we will see this, this, and this should be present, so if this present in F 5 in F dash also it should be present and also the F, the direction has to be like in F it should reverse and give it the correct form.

So it is very easy to check that if F is transitive oriented, and then F bar is also a transitive orientation, in other words we just reversing all, the directions cannot make it non transitive, so instead of F we could have taken F bar and then instead of making F union F dash, we could have considered this union, that means F bar union F dash.

So the same arguments will halt and this will be also a transitive, and orientation of K n and also acyclic orientation of K n, as we have proved, because if, you, again we can consider a shortest cycle show that a shorter cycle will eventually lead contradiction.

So our inference is that, F bar union F dash also an acyclic orientation of K. So what is good about this acyclic orientations, so then we can order them in such way that, now we get a total order corresponding to K n, because if it is acyclic, so then if you do a topological sort, because anyway it is a total order, because all the pairs, because the complete graph, we have a direction between every pair, so we will get, because its acyclic also we can also order them in such way that is total it becomes a total order.

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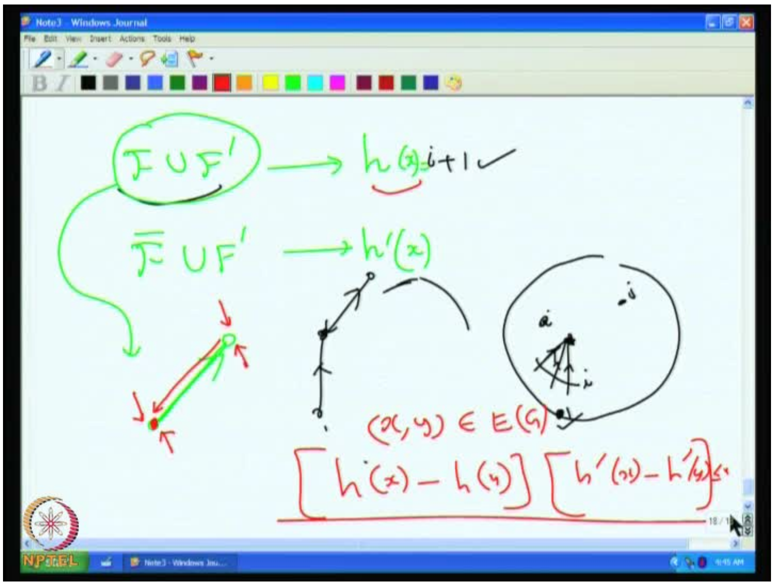

Now what we can do is because it is a total order, what we can do is, with respect to the first one, the corresponding to F union F dash, we can create an ordering- a permutation, so such that we can call it, say the ordering n, so maybe we can use another notation for that.

So h of, so corresponding to this thing. So what we do is, we go to the graph, the complete graph, and we look at the vertex numbered i, so the initially, we have a number i, we are going to renumber the vertices now in such way that, we will look at the incoming edges of it with respect to this orientation because this incoming edges of it.

How many edges are coming inside? Suppose there are i edges coming, then we will, see make for instance, initially we will see a zero edge, because see, we claim that, what we can do is, we claim that there is a vertex with a incoming edges equal to 0 0- number of edges coming inside that vertex.

Why is it so? Because if every vertex is with an incoming edge, the, now if I track back, for instance if I go backward through the incoming edges, where will I reach, because every time I can go back through an incoming edge and then because it is a finite graph you have to stop some work, so the only way to stop, is to stop is by forming a cycle, you will enter in to some one of the this thing.

So therefore, because it acyclic you will have to find out 1 vertex with a no incoming edges to it. So we can start with that vertex, we will, we will numbered it 1 and then once you remove it, all the edges of it are going out ward only, therefore now we can see that in the remaining graph there is a again, a vertex with incoming degree 0, but then this vertex will be providing exactly 1 edge in to it, therefore its incoming degree in the entire thing is 1, therefore that can be called the second vertex and so on, that means there will be a vertex with i incoming edges in it, then that vertex will be called i plus 1, 0 incoming edge we called 1- first vertex, the one incoming edges can exactly will be called 1, 2 incoming edge will be called exactly 2. So i incoming edge will be called the vertex i plus 1.

So we number the vertices like that, based on that, similarly will consider the other orientation, namely F bar union F dash and based on that we will define numbering, we can call this numbering as h, the vertex h will x will given the number of incoming edges plus 1 will be the name for that.

Similarly with respect to this orientation we again make in other, this thing, so for h dash of x. Now the interesting thing about this h of x and h dash of x is that, if you take any edge of the original graph- G, if you consider any edge of original graph- G, so what is interesting is, in this first one, if the direction is like this, in this second one the direction will be reversed, it will be like this.

So therefore if, with respect h; this got a lower number and this got a higher number, with respect to h dash; this will get a lower number and this will get a higher number, so the essentially they will reverse their roles.

In other words, what we can say is that, if x y is an edge of G, now h of x minus h of y into h dash of x minus h dash of y, will be less than 0, so that is what, so that product will be negative, in other words, we do not have to worry too much about it. So essentially what we are trying to tell is, whenever x and y, if x is greater than y, in one of them the values corresponding to x and y is such that, h of x is greater and than h of y in the first one, then the order will reveres in other one.

So therefore when you put h of x minus h of y into h dash of minus h of y, h dash of y, we will get a negative value.

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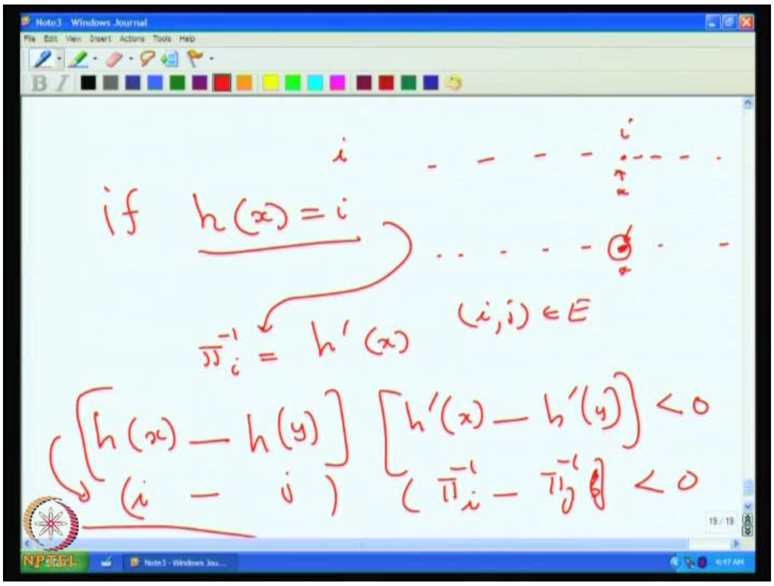

So now we will construct the permutation like this. So we will take a vertex i and this vertex in the first with respect to h of x may be some, we will find some x, so that h of x is i, somewhere in the, so that permutation i will appear somewhere in the permutation i.

So this is the position x, x position i, have some appeared, now what you do is, so we will consider in the correct corresponding position, what is there here, now that position will be made, so we will take this value in the corresponding position, we will place i, that means, pi inverse of i will be made h dash of x, if h of x is equal to i then we will make pi i inverse equal to x dash of x.

The position number corresponding to i will be made h dash of x, now we know that h of x, so whenever there is an edge, h of x minus h of y into h dash of x minus h dash of y is negative, so is less than 0.

So, but then this is essentially, or say suppose i j is an edge, then this is essentiality some x and y corresponding to, so this I, this is j and then this is what, this is pi inverse of i minus pi inverse of j, because this is by dissection, so this is will be less than 0, this is we are making use of this property.

What is this last, latter inequality mean? This latter inequality means, if i is smaller than j, then their positions, corresponding positions, will be reversed. i will be in a if i is a smaller number than j, then pi i should be greater number, then pi i inverse should be greater number than pi j inverse.

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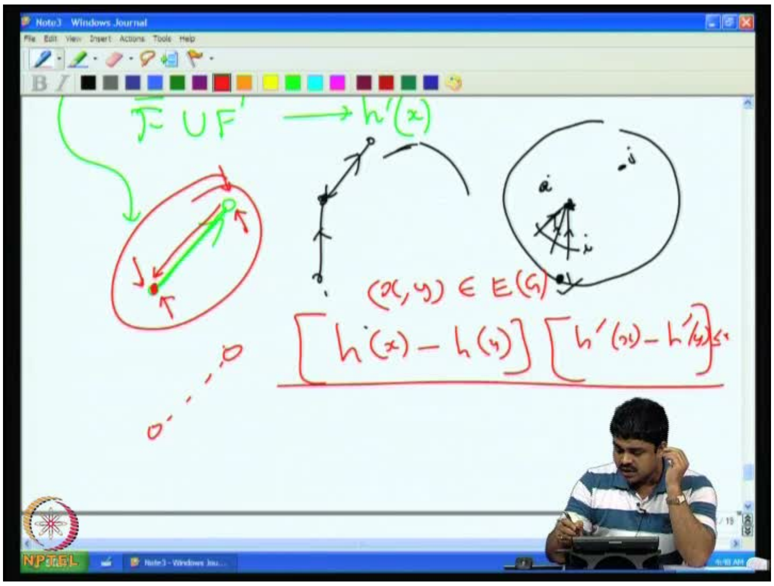

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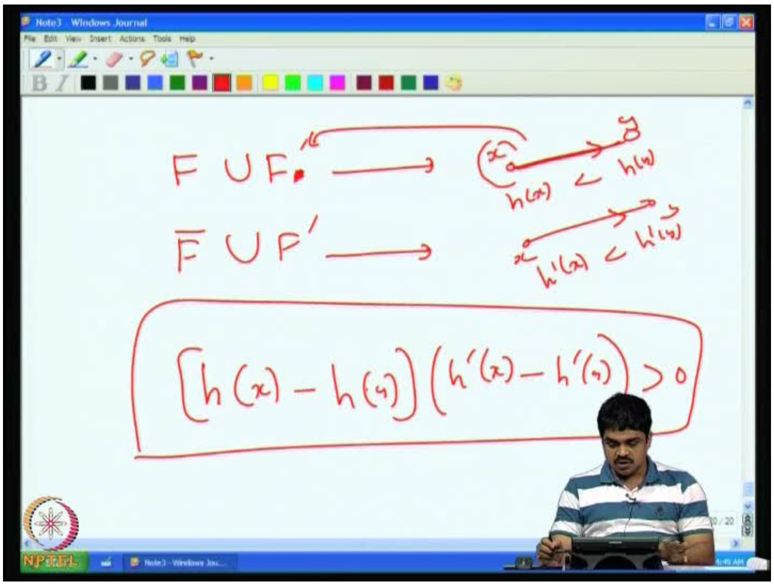

So therefore they will appear in the permutation in the wrong order, the corresponding permutation, we also have to consider the edge set, suppose there is no edge between, so we consider, when there is an edge here, suppose there was no edge here, if there was no edge here, then we know in both cases F, this also this also in both the orientations we will get the same directions, for that, the corresponding pair, the edge, because we are not changing, we are only changing, so we had once F union F dash, the other thing was F bar union F dash.

In this thing, suppose the pair was like this, in this thing, the pair for instances x pi is a non edge, so then that belongs to F dash only, so we gave this direction and then here also x y will belong to this F dash, we are not changing it, so it will be the same orientation.

So if the value is assign to this h of x, if it is less than h of y, here so h dash of x will be less than h dash of y, also here because the orientation, the direction are same. Therefore whenever there is non-edge, this h of x, so then h of x minus h of y into h dash of x minus h dash of y will be greater than 0, because they are will be positive, either they are both positive or both negative, therefore they are positive.

Therefore we can say that this property of being negative is will happen only when this x y is an edge, if x y not an edge that will not reverse, so we have described now how a permutation can be constructed from the graph verses.

So we will summaries the idea of the proof, so we wanted to show that if G and G complement both have transitive orientations, then G is a permutation graph. So what we did is, we considered the transitive orientations F and F dash of G and G complement, if F is a transitive orientation, the F bar which is obtain by reversing all the directions is also a transitive orientation of G.

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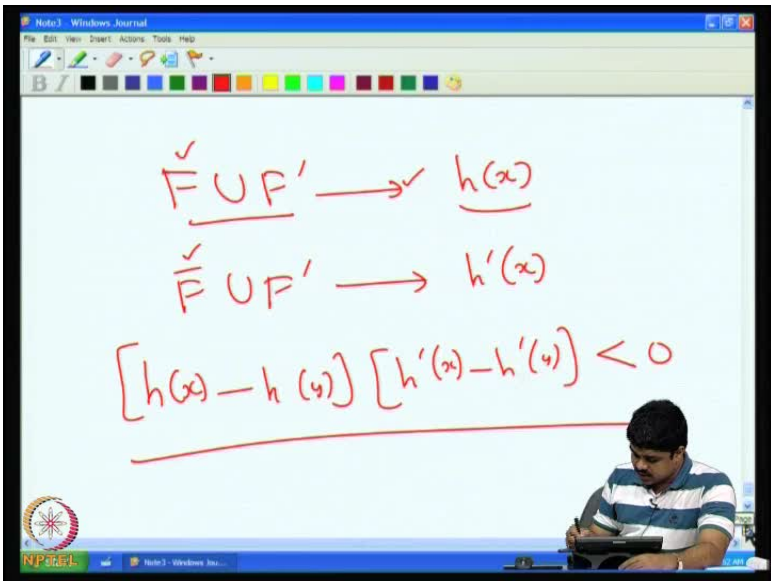

So then we consider two kinds of orientations of the complete graph, nearly F F dash and then F bar F dash, this was the crucial thing and both are acyclic, this is what we proved both are acyclic and acyclic means, we have a total ordering there and then we considered a numbering h of x from based on this and considered a numbering h dash of x based on this.

This h of x it is a numbering, just you, that counted incoming edges for a vertex and plus one was given, as it number, that vertex was labeled with that number, that is a h of x, that is a h dash of x and this h of x and h dash of x will have the property that if we consider this product h of x minus h of y into for 2, vertices x and y, suppose if you had consider this product, hence when you minus this h of x minus h of y and h dash of x minus h dash of y this is less than 0, if and only if x and y are adjacent. Once if there adjacent, this has to be two different because we reverse the edge directions in both the cases, so one will be if this is positive, this has to become negative and vice versa.

Similarly if there is no edge then both will have the same parity. If this is negative, this also will be negative, this is positive this also will be positive, therefore that will be positive.

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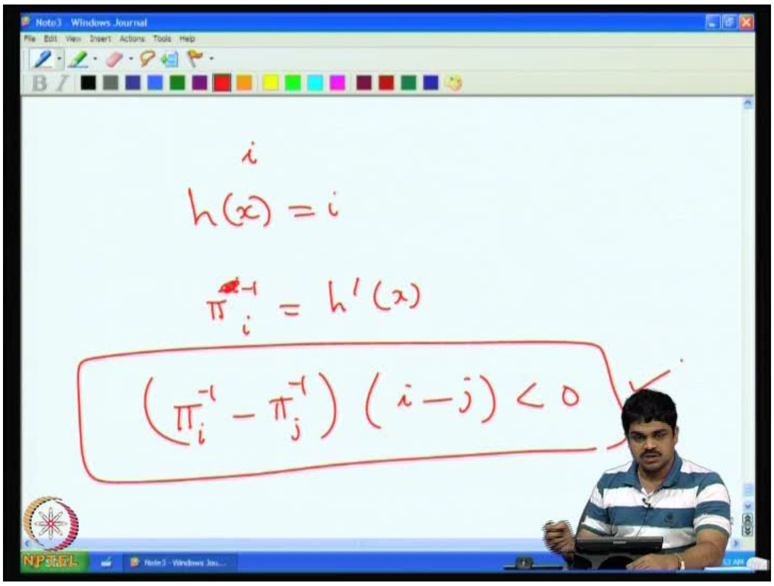

So then, based on this h of x and h of, we constructed a permutation, how did we do that? We just considered for every vertex i, for every i, we looked at, for which, x is h of x equal to i.

And then we placed i in the permutation, in which, position by inverse i is equal to h dash of x. We looked at the permutation h dash of x, in that x position whatever you see in that was the position number corresponding to y and then you see because of we immediately get this, this thing that pi i inverse minus pi j inverse into i minus i j is less than 0, if and only if i j is an edge.

So therefore this means that whenever i j is an edge, i and j come in the reverse ordering in the corresponding permutation, therefore it is a permutation graph, so this is, so we conclude the permutation graph of this thing.

Now we will consider some other graph classes also this is maybe, I will just give a quick over view of things. So one graph class we want to consider is, in the some, see, we will say that generalization of interval- interval graphs are very important class, therefore people have try to generalized in various ways.

So in this fifteen minutes what I will do is to introduced a concept, which tries to generalize the interval graphs, so because again this is a see as we know interval graphs are the intersection graphs of intervals on the real line because each vertex is association interval on the real line, whenever 2 vertices are adjacent we put an edge between them otherwise we do not we put an edge between them, that is the meaning of intersection graph of intervals of on the real line.

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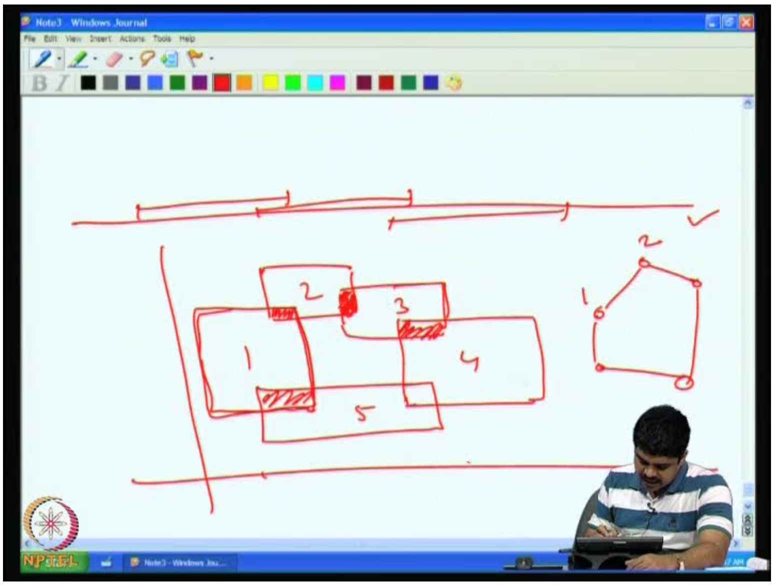

Now suppose you want to, instead of considering intervals on the real line, what if I want to considering rectangles on the plane, for instance, i want to associate with each vertex, a rectangle access parallel rectangle on the plane, such that 2 access parallel rectangles intersect if and only if those two vertices are adjacent, this is called a rectangle graph, in fact if a graph can be represented like this, it is called a rectangle graph, interesting for instance for instance, if it is, take cycle we can represent it like this, 5 cycle c so 5 cycle.

So what you do, this is 1, this is 2, this is 3, this is 4, this is 5, we correspond to each vertex, we are placing an access parallel rectangle and then we are allowing them to intersect only when there is an edge between this, for instance, here 1 and 2, here is an intersection between these thing because 1 and 2 are adjacent which with each other.

And similarly 2 and 3 are made to adjacent, made to intersect, because they have an intersection between them. Now you can see that, this is a kind of generalization of interval graph in the following sense.

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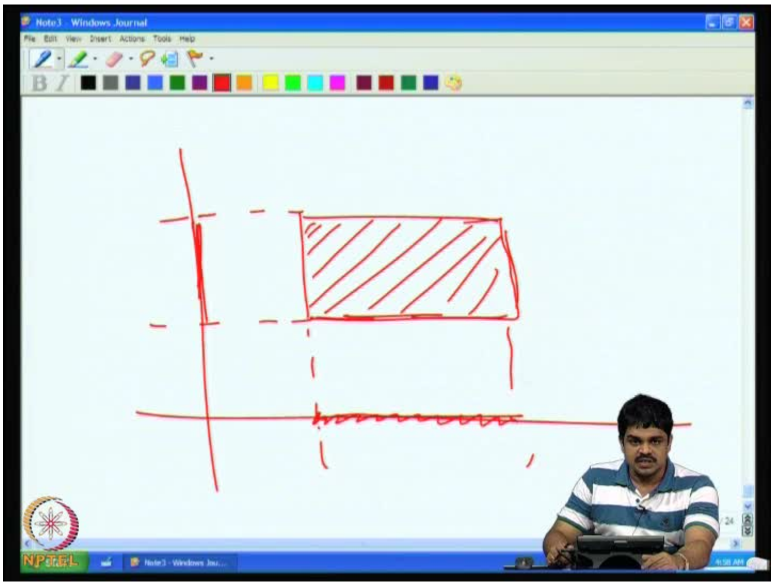

So because this interval graph are intervals on 1 dimension because this this rectangle, for instance, if we take a rectangle, it can be seen that an interval is growing, for instance, it is a cortication product of 2 interval, for instance, this is 1 interval if you consider this interval, so it is a cortication product of interval, so this can be consider as a 2 dimensional interval.

Because, so if you give this interval and this interval, their cortication product, so this is the corresponding interval, then just like interval graph (( )) intersection graphs of one dimension interval, these are the interval intersection graphs of 2 dimensional intervals.

Now you can ask this question, so we know that there are interval graphs, which cannot be represented as the intersection graph 1 dimensional interval.

Similarly there are graphs which cannot be represented as the intersection graphs of 2 dimensional intervals can you quickly find out 1 graph, which cannot be represented as the intersection graph of 2 dimensional intervals?

So this is one interesting question, so you can see like this, to study this thing and we need some kind of model, so suppose you consider a rectangle graph this are rectangle graph.

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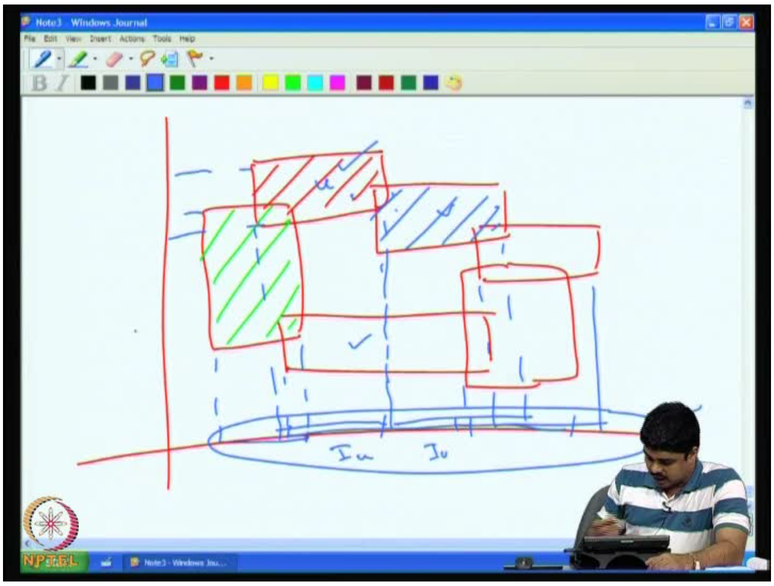

Now if you are given a rectangle representation of the given graph, that means a vertex is associated with some rectangle, so we draw a rectangle corresponding to any vertex and then whenever they are adjacent we have to make the rectangles intersect.

Now we can create 2 interval graphs from this thing, so what we do is, we project this rectangles to the corresponding accesses, so this is one, this is one, the corresponding to this this is third interval.

So if we take the projection for each rectangle in to the accesses, then what will happen is, we will get an interval graph corresponding to that, suppose this is vertex u, then you take the projection, you will get an interval corresponding to I u.

Similarly when I take the projection of this thing, you will get interval corresponding to I v, we can say now, you can see that, whenever this 2 boxes here, rectangle here intersect, their corresponding projections also should intersect.

So therefore whenever there is edge in rectangle graph in the interval graph, obtained by taking the projections, also should have the edge.

So, but when there are no edges between the for instance, between this box and this box there is no intersection, therefore the corresponding vertices will not have edge between them. Now if we take the projection, then also it is not guaranteed that they are again non intersecting because these two projections are intersecting.

So what we can say is that, the interval graph we obtain by taking the projection of this rectangles, is indeed a super graph of the given rectangle graph, it is not actually the exact graph, they can be some extra edges but all the edges which have present should be always present but some extra edges may come, therefore it is only a super graph.

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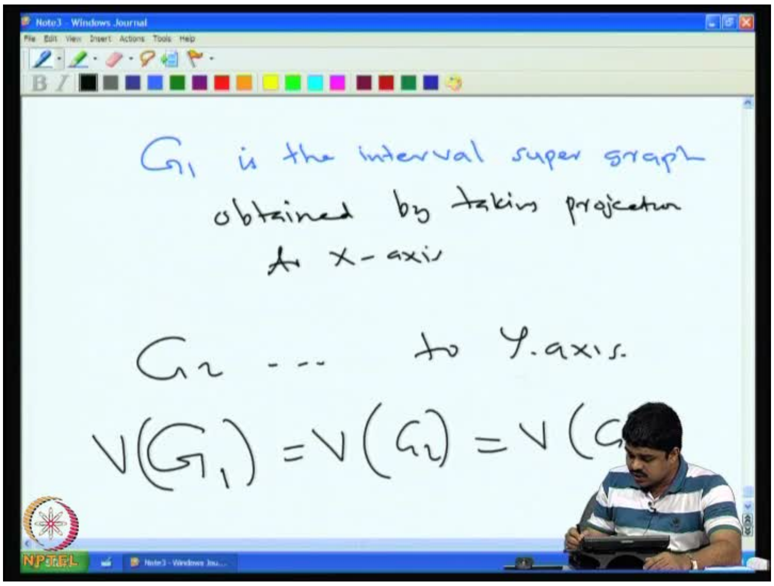

Similarly if we had taken the projection to the y axis, you would have got another super graph. So the interesting thing, for instance, suppose G 1 is the interval, super graph is obtained by taking projections to x axis and G 2 is the interval, super graph obtained by taking projections to y axis, then we can easily see that, if I had taken G 1 and G 2, both are the same vertex because the corresponding to each box we took projection to x, as well as we took a projection to y.

So therefore in both cases we see that there is the projection corresponding to 1 vertex, there is a box corresponding to 1 vertex and therefore when you project, projection corresponding to, so it is on the same vertex set G 1 vertex set of G 1 equal to vertex set of G 2 equal to vertex set of G.

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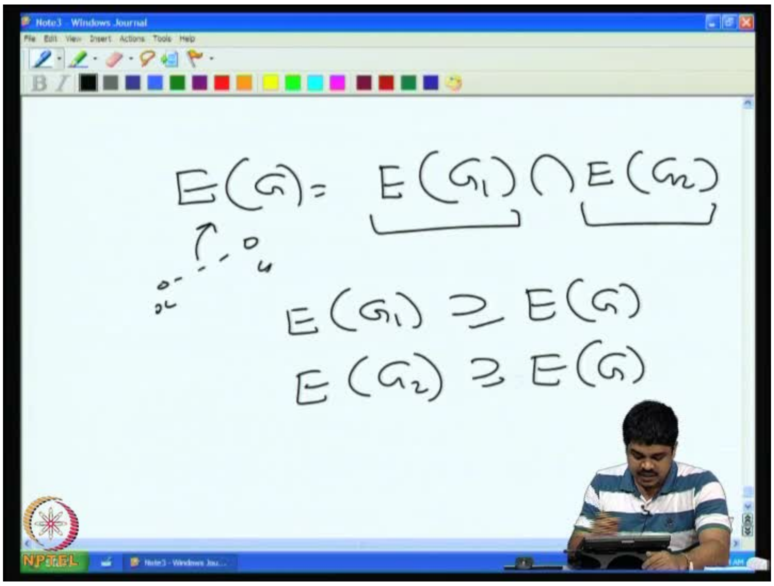

But the on the other hand if you consider the edge sets, we can easily see that, the edge set of G is equal to the intersection of the edge set of G 1 and the edge set of G 2, see this thing, we have already observed that E G 1 and E G 2 both are super sets of E G, super set of E G is not it.

So because you know, whenever there is an intersection, we all we anyway get that, now if some edge x y is missing in G, suppose there is this x y which is not there in E G, then that means 2 rectangles corresponding to x and y do not intersect it, will like this.

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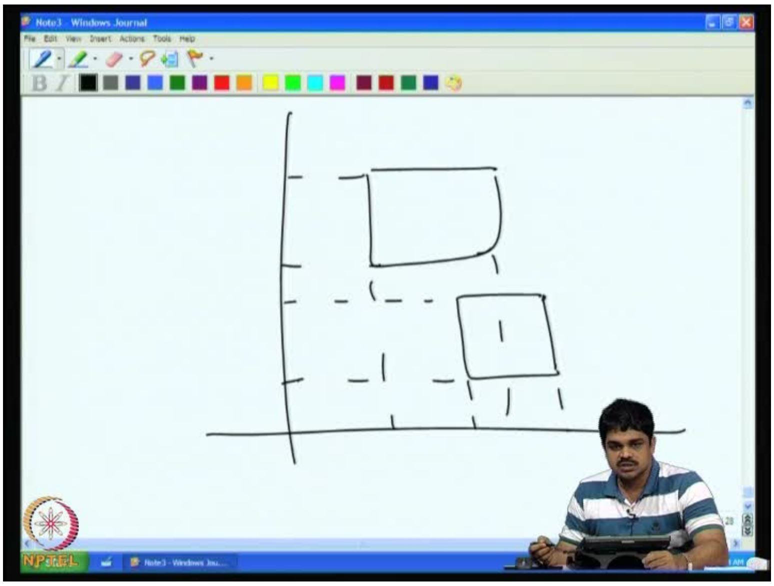

So it can be some x and it can be some y, it can be something like this. Now it is possible that in one of the accesses, it they are projections intersect, but it is not possible that in both axis they are in projection, intersect both access, if their projection intersect, then the rectangle themselves have to intersect because the rectangles are the Cartesian product of this project. It is very easy to see that if there is a common point in both the projections, then their Cartesian product have to intersect.

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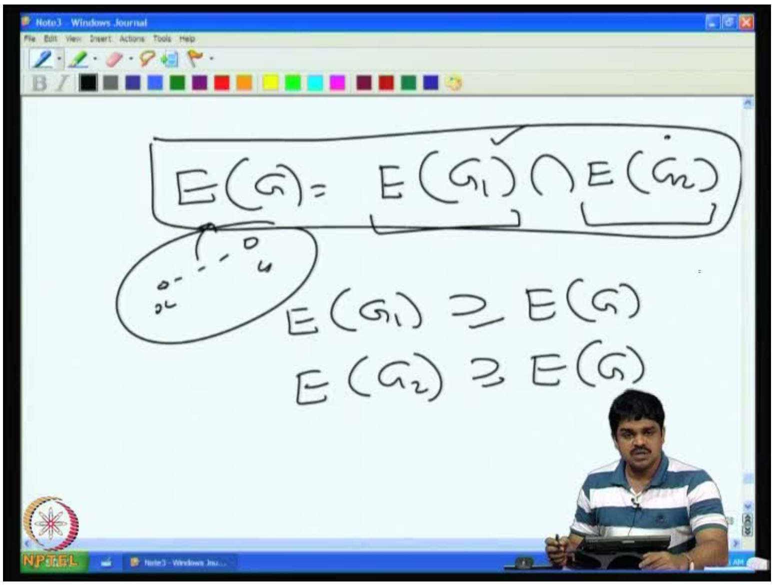

Therefore we can easily see that, in either G 1 or G 2 will be such that, this edge, which is not there in E G, will not be there in E G 1 or in one of those gaps, that is why this E of G becomes the intersection of E of G 1 and E of G 2.

Now why did we tell this thing? Suppose you have, if a graph can be drawn on the 2 dimensional plane, as the axis parallel intersection graph of axis parallel rectangles.

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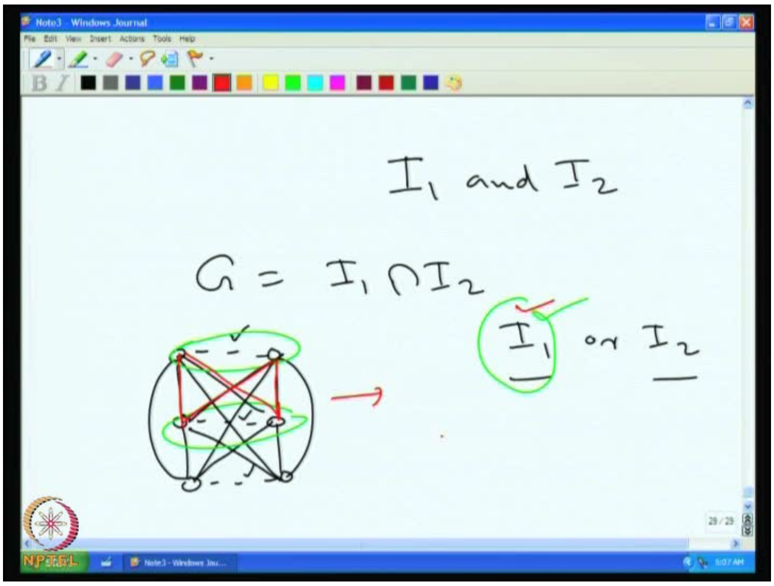

Then we can find out 2 interval graphs I 1 and I 2, such that G is equal to I 1 intersection I 2, when I say 1 intersection I 2, the vertex that is same for all the graph, the edge sets intersection edge set of G will be the intersection of edge set of G.

Now we claim that if we consider this small graph, so it takes this graph, now this is missing, so this connection is there and this is missing, this connection is there, this is connection, this is missing

Now we claim that, if we consider this graph, this cannot have a representation in 2 dimensional planes, so this is because, so this 3 edges should be missing in I 1 or I 2, each one of them should be missing one of them.

Now (( )) principle says that, there are three of them, there are 2 interval graphs, now two of them should be missing in one of the interval graph, suppose without loss of generality this and this edge and this edge becomes missing in I 1, then what will happen, then you see that there is a 4 cycle, like this, which will because this, all the edges which are there, here, should be in I 1, also because their I 1 is a super graph and then this 2 edges are missing, that means, there is a induced 4 cycle which is contribution, therefore 2 interval graphs cannot give G as an intersection. So thank you, we will stop the class with this.