Graph Theory

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Module No. # 06

Lecture No. # 35

Random Graphs and Probabilistic Method: Preliminaries

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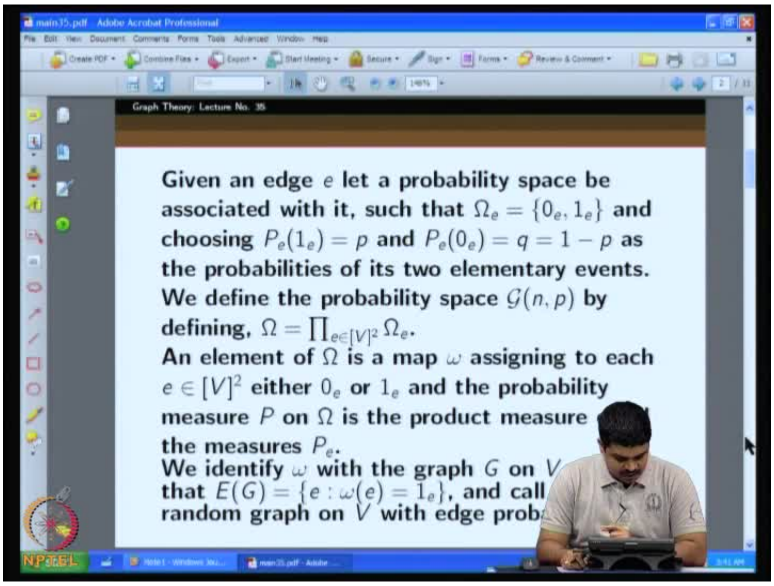

Welcome to lecture number thirty-five of graph theory. Today, we start with a new topic so, about random graphs; so, for these things, a small amount of a probability is knowledge about probability theory is required, but only a little amount of knowledge is enough so, but I want getting to so much of it; so, I am assuming that the students are familiar with the required amount of material.

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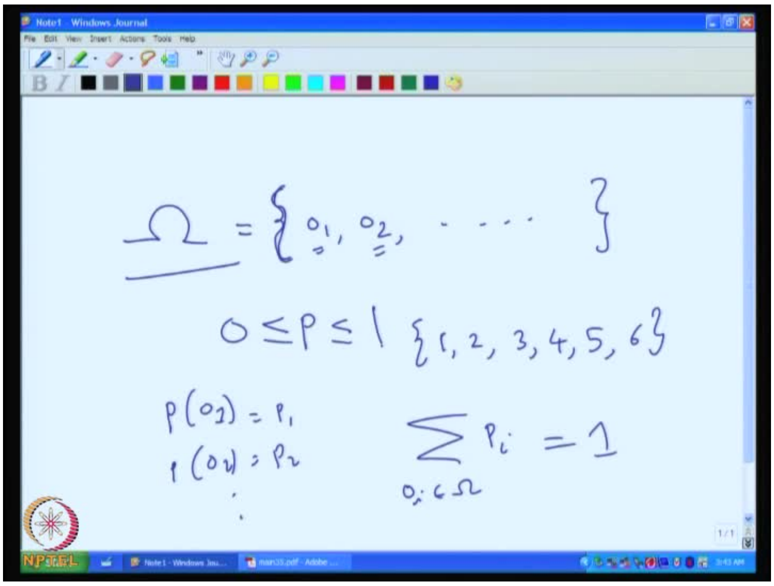

So, but still is a just to remind the things, the students so find it difficult to remember, so see we have an experiment, so like tossing a coin, then the samples space omega is the set of outcomes, say it can be the outcome outcome, one outcome, two like set outcomes.

So, in the, in our case, we are always dealing with discrete probability and also the number of the sample spaces are finite and discrete; therefore, so thus we do not have to get into complicated details, we just have to say that, there are this set of outcomes; so, if for instance, if I conduct an experiment like tossing a coin, these are the only possible outcomes that can happen; and for instance, this can be outcome that I had turned up; this can be the outcome that tail turn up; or if it is the tossing of a dice, it can be that number on the face that turn up like, 1, 2, 3, 4, 5, 6 outcomes there.

Now, in a probability space each outcome is associated with the probability value, sets p of o 1 we will write certain value is a p 1, so p of 2 is p 2 and so on; the idea is, when you sum up this probability values over all the outcomes in this sample space, all the possible outcomes will turn out to the 1, and the probability values themselves are always in between 0 and 1; these are the things we have to remember.

So, there are the set of outcomes and then each of outcomes is given by setting probability value, which is always between 0 and 1; and then the sum of the values are assigned to them is equal to 1, the total probability is equal to 1; and another thing we have to remember, see remember, we are only talking about the case when the sample space is omega is finite and also discrete.

So, the now, see now, all other example only such situations will comes, so the students who is not very much comfortable with the probability theory also can understand what is going on so; and also even if you have not studied the probability theory in a formal course, we can just read up a little bit, may be the first chapter on a probability theory, introductory book, and that would be enough.

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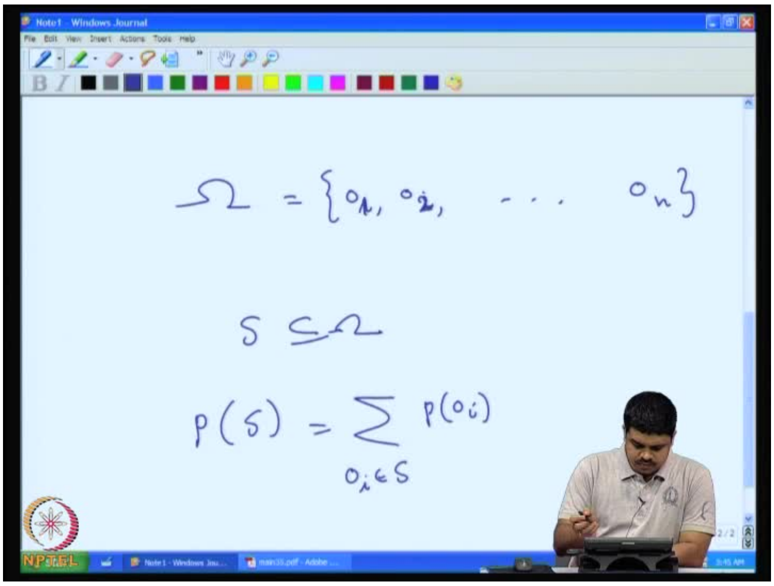

So, then another thing we need is the idea that, any subset of this omega will be considered as an event; so, this for instance, if the set of outcomes is o 1 o 2 say upto o n sum n, then sum s comes subset of omega is called an event, so that is the way we call a sub set with sample space in the probability theory.

Now, what will be a probability of this event probability of this event? That will be the sum of see this outcomes which constitutes that event, this is what the probability of that event, this is yeah , so this this notion is enough.

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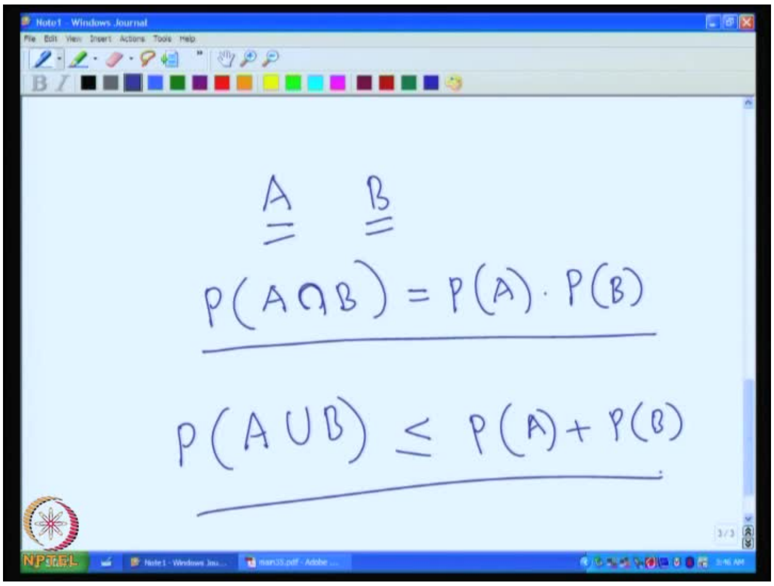

Now, the only notion we need is the notion of independence; that means, suppose we have two events say a and b and then what is the probability that it together happen, that means, probability of the event a intersection b, so we say that a and b are independent, the two event a and b are independent, if probability of a intersection b equal to probability of a into probability of b.

So, this is without notion we need; and previous things we have to notice is, suppose a and b are two events and then what is what can I tell about the probability of a union b, so this is always less than equal to probability of a plus probability of b, this also we will be using; now, but I leave to the student to study a little bit of basic probability theory, if is very uncomfortable with the relevant concepts on probability theory.

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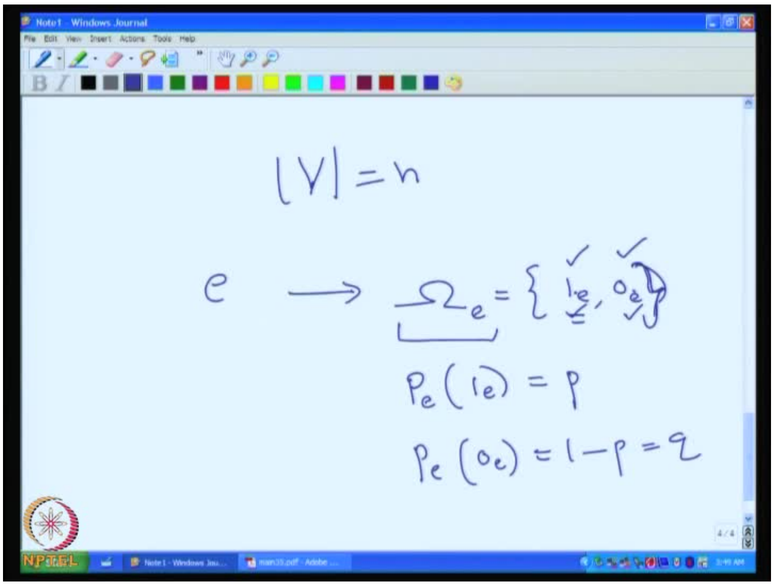

Now, here in for our purpose, we are dealing with graphs; so, consider a graph where the vertex set is v, so let us say v is equal to n - there are n vertices; now, so there are n choose to pairs here associated with each of these edges, so associated with each of these pairs we can have an edge or not, so we will suppose edge, e is an edge and then we will associate as small probability space for that edge itself.

So, we will call omega e to be the set of outcomes namely 1 e and 0 e; what does it meant? So, it means that, 1 e is the outcome which tells that the edge is selected edge, because we are going to create a finally a graph by selecting some edges and not selecting some edges; so, we are defining a probability space with a given edge, where the outcomes are 1 e and 0 e, 1 e denotes that, that edge is selected; for instance, if you are it is like thinking that there is a coin associated with an edge a given edge e and then when we are tossing that coin if the head terms are we will allow that edge we present in the final graph, otherwise we would not allow that.

So, that head, the event that the head occurs corresponds to only the even that tail occurs corresponds to 0 e; and we also have to give the probability values, see the probability of 1 e, when 1 e happening is say we say it is a probability, it is a value these are assigning in between 0 and 1, and let it be p fix value p; and then what will be the probability of 0 e? Because the probability assign for this and this should be adds up to 1, this should be 1 minus p, typically it is called as q.

Now, this is very simple probability space, which we usually associated with the coin tossing the same same thing; now, the probability space that we are interested in this, this so we have suppose so this vertex set is v, corresponding to each pair in it we have this say we can enumerate it as e 1, e 2, e 3, e n - choose two, these are the possible edges

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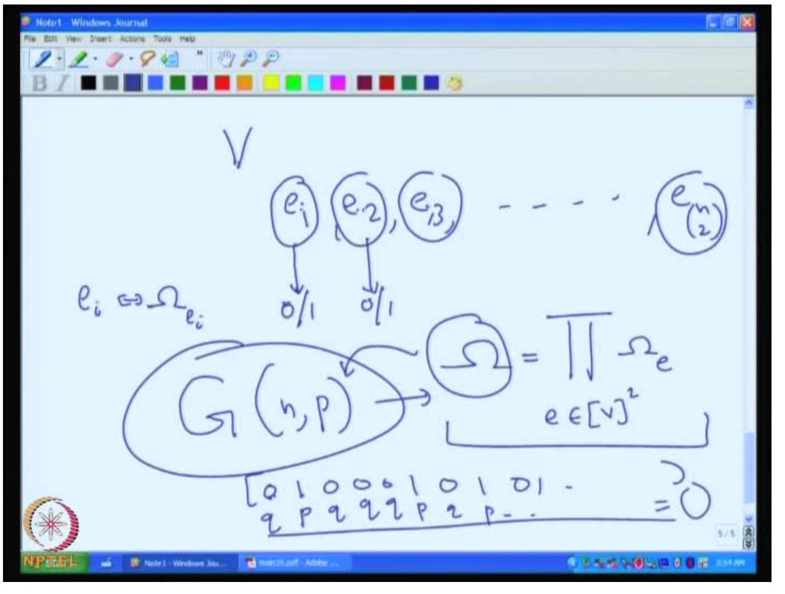

Now, corresponding to each of these I have this corresponding probability space; so, for instance, with e i I have associated the sample space omega e i and the corresponding probability values namely 1 e i will happen with probability p and say e i will happen with probability 1 minus p.

Now, we will define the g and p, new probability space called g n p, we will define a new probability space called g n p and it is the its given the samples, its space it is samples space is essentially the cartesian product of the samples space associated with each of this edges; you know, the sample space corresponding to this edge the corresponding probability space has two events in it, the two outcomes for it here to outcomes here to outcomes.

So, when I take Cartesian product of all this small sample spaces e element of v square we get the sample spaces; what is this? This is the essentially like you know either in the position of this we will get four e 1, we will get a 0 or 1, here also a 0 or 1, 0 or 1; so, we can think that, this sample space is essentially is extreme is an extreme of length and choose, where at each then ith big position 1 or 0 corresponds to the ith edge and we are deciding whether that edge should belong or should not belong to the this thing.

So, essentially, we can say it is a mapping from the edge set to 0 e or 1 e, so, 0 or 1 whether it is selected or not about that; so, in other words, if you think with it means that when we have assigned, when we are decided whether 0 should be here or 1 should be here, 0 should be here, or 1 should be here, so we are selecting some edges and d selecting some other edges; whenever 1 is selected, that means, that edges is there in the graph and series given for that means that edge is not there.

So, essentially, we can interpret this string as a graph itself with the corresponding edges, the edges corresponding to those with 1, a value 1; therefore, the sample space here this omega the sample space corresponding to g n p is essentially the strings corresponding to the set of edges, so n choose to possible edges; and then essentially that string can be interrupted as the graph depending on whether the whether each edge is selected or not.

So, then the problem how do you assign probability values to each of the outcomes in this things; for instance, if a string is a like 0 1 0 0 0 1 0 1 0 1 like that, then we can assign this means that e 1 has not been selected, that means, this corresponds to the probability q it is selected, so this correspond to probability p e 3 is not selected, so it corresponds to probability q.

So, this is again q, so this is again q, so this is p, this is q, this is p, and so on so; finally, if we multiply this values, we will get a certain probability value that can be associated as to this particular outcome as its probability value.

So, this probability distribution is g n p; the we then we want to verify, so they student may want to verify that, this is indeed the probability space in the design that if you sum up the probability values that we are assigning to each of this events we will indeed and end up with 1; if you sum up them it will indeed end up with 1, so it is induce a probability space.

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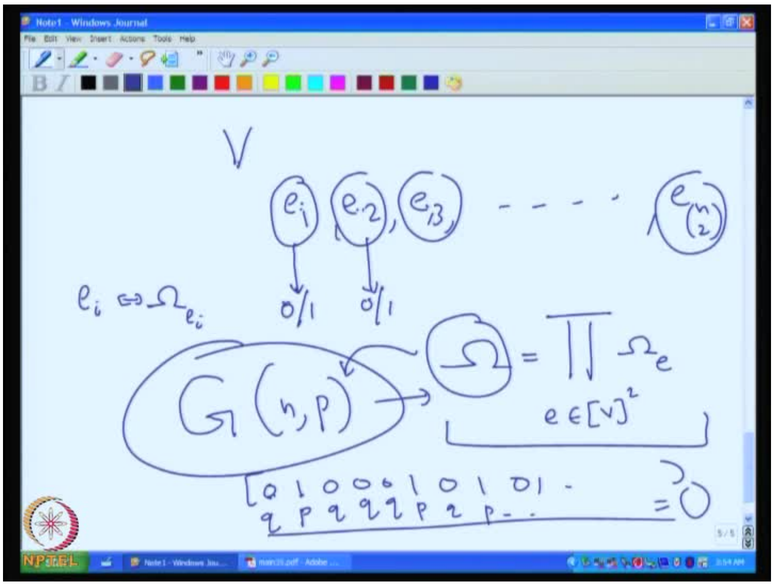

So, for instance, you see if you take a particular outcome has I have told, so it corresponds to a certain graph, so the omega that bit stream corresponds to certain graph depending whenever 0 comes that particular edge is not there and whenever it is 1 then that particular edge is there. So, this particular graph we can ask, what is the probability of this graph appearing? So, this probability of this graph appearing corresponds to the probability of that particular bit string in omega.

So, we say that this g is a random graph and it is probability is are the corresponding probability; so, it is essentially, if you know the number of edges it is very easy to calculate, because for each of the edges which are present we will be giving p, so p raise to m into q raise to n choose to minus m will be the probability of a given graph with respect to with respect to this probability space g n p.

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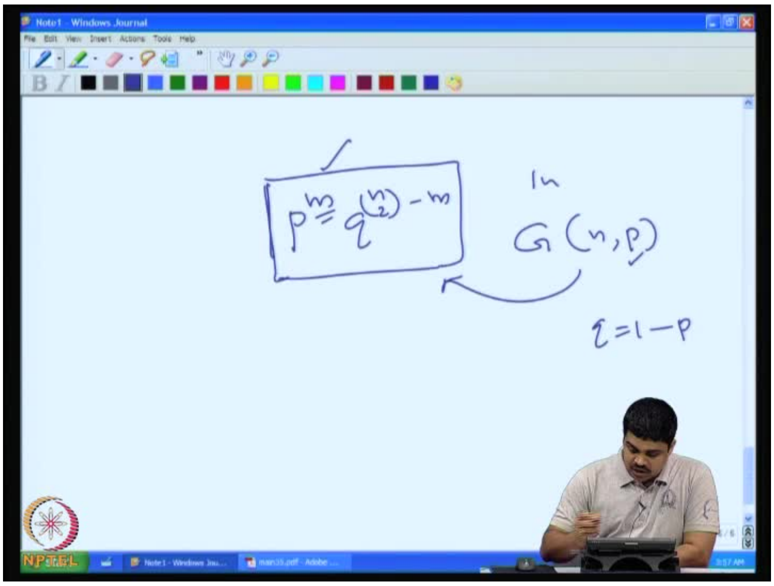

In g n p, this will the value of this will be the value of the probabilities; so, again to summaries what the students has to understand is that, we have created a probabilities space g n p, the outcomes in the sample space, the elements of the sample space of g n p is essentially the graphs on n vertices - all possible graphs on n vertices; and the probability for a particular graph theory when the outcome is equal to a certain graph; the probability value assign to it is essentially this much p raise to m into q raise to choose to minus m, where p is some fix number; so, g n p is p q d 1 minus p and m is the number of edges in that graph.

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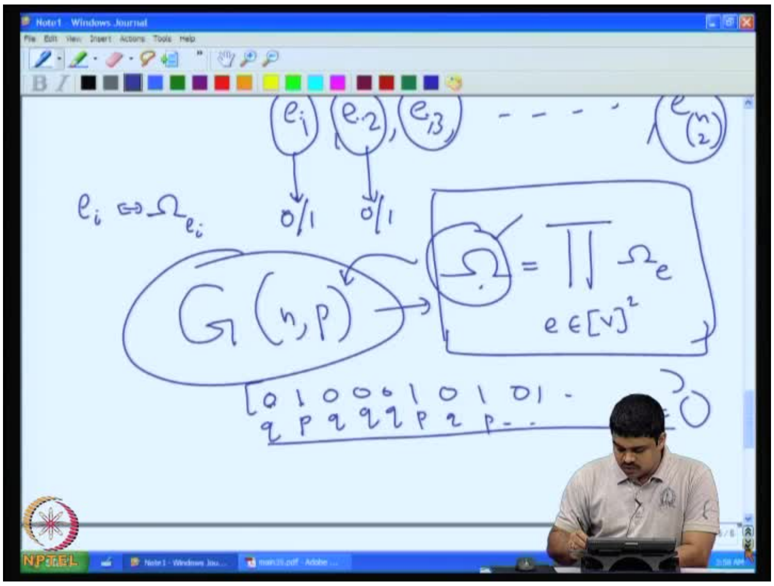

Now, the wave we have introduced it is as the products space of many small spaces, because the small probability space was associated with the each edge with just two outcomes 0 or 1, which means that edge is selected or not selected with probability p or 1 minus p; and then we could interpret each graph as the cartesian product of these set of outcome, that is set edge is selected or not selected over of all the edges, that is what we had drawn it.

So, the elements of this omega is essentially the cartesian product of the elements of each of this small things and therefore there all they can all be interpreted as binary strings, but then those binary string can be map to corresponding graph, so that is that is why we say that the elements of the omega are essentially the graphs on n vertices and the probabilities are clear, so this is g n p.

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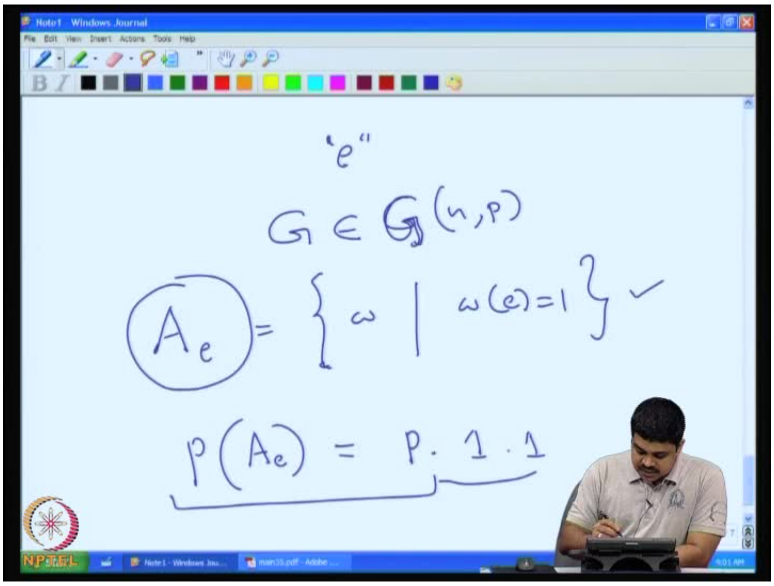

The next the only the next concept you want is that, so consider the event with just consider the event with a given edge e being present in the random graph; so, suppose if we select g element of g n p, in the element of g n p, so what is the probability that a given edge e is present in g, this is the question.

So, how will you find it, so because if this e is present, so this event can be written as a e; so, the present, what is the event? The probability of the event we are interested in the probability of the event that e is present in the graph, essentially, we see that this is the all those graphs with e as in it or all those strings omega, such that, omega e is equal to 1;

all those functions, such that, omega of e is equal to 1, if we if we are taking it has a function from the edge set to 1 or 0.

And the probability, now the probability of a e probability of a e can be calculated as because this e has to get the value 1, that happens to probability p all other edges each other any other edge if you can take 0 over 1, it therefore, that is the probability is 1, there so 1 into 1 into 1 like that.

So, this probability will be, so before that what we should understand is, this I think we should do it to from, so what is it meant by the probability of a e, that is, essentially the sum of the probability values associated with the graphs with that edge present in it.

So, now, how can you find it out? Suppose, this edge is present and what about the next edge; the next edge in some graphs it is present, in some graphs it is not present, therefore, we can see that when you sum up over all both the categories, we will get all, so the 1 for that, because that is the probability 1 is also there, 0 is also there, so both the events are present; therefore, we can get 1 there; and therefore, this probability will be equal to 1 if you sum up over this things.

So, the student can convince by summing this up and then we can see that is that this probability indeed p; and now, the only thing other thing we have to understand is that, because each of the suppose that probability that for a particular edge to be present in the graph is p, and now suppose there are two edges e 1 and e 2, so what is the probability? That both of them are present p e 1 e 2 is also present, e 2 is also present, that corresponds to the some of the probabilities over all the graphs with containing both e 1 and e 2 together. So, that can be seen as p into p, p square, therefore, probability of a e 1 and probability a e 2 is equal to probability of a e 1 into probability of a e 2; therefore, these events are independent also.

So, so the anyway to summaries, what we are telling is, this in this g n p the calculations become easy because the event that a particular edge is present in that g n p is p and two probability that a particular event e 1 is present and sorry the event that a particular edge e 1 is present and even the particular edge e 2 is present, they are independent events.

So, because their probabilities the probability that both of them are together present actually equal to the product of the probabilities that e 1 is present in the probability that e 2 is present; therefore, this is what we should keep in mind this two things make it very easy to do calculations in these probability space instead of we do not have to think about what is the each outcomes are graphs complete graphs, but when you are dealing with we can thing that, we are thinking of each individual edge and we can manipulate based on that.

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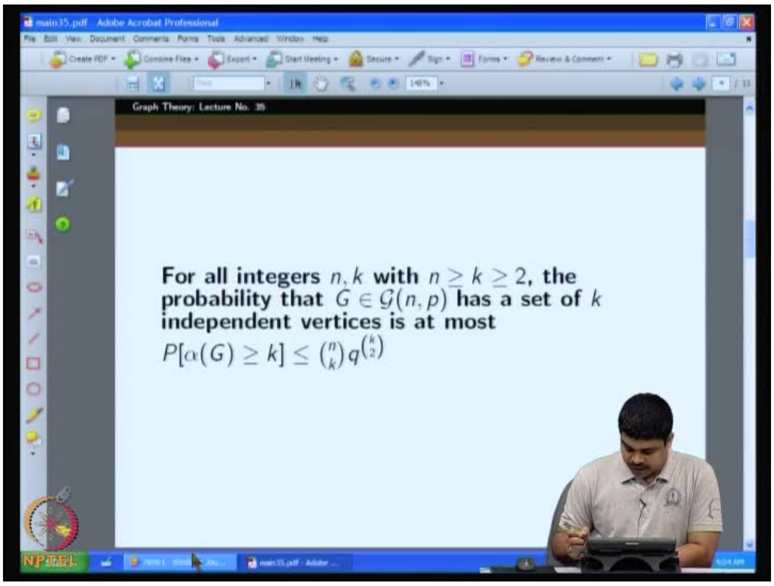

So, these are just background; therefore, if the as we go on it will it will become much easier to understand; now, the next point is to do some examples, for instance, we can think of this question, what is the probability that the given graph contains an independent the random graph contains g element of g n p has a set of k independent vertices, but what is the probability? How can i calculate?

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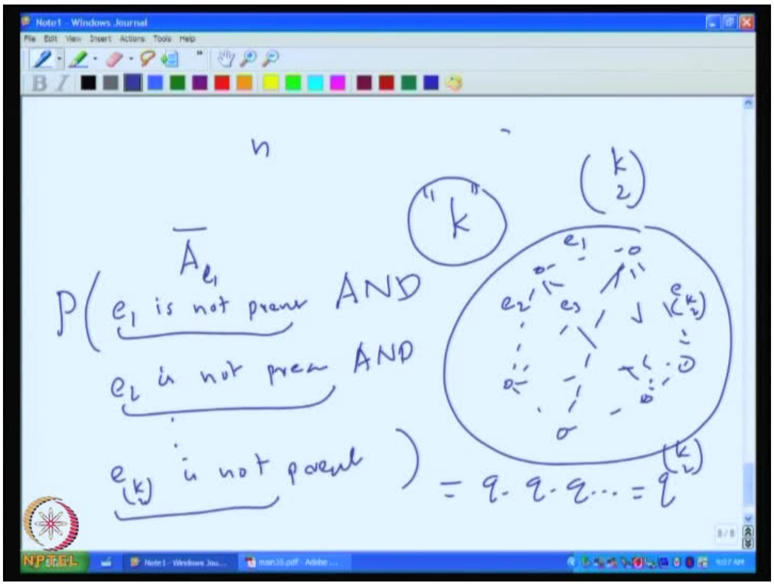

So, the in fact, if I want a good upper bound flow for it, this is the way we do this, so g so if we can out of n vertices, we can select k vertices, so what is we can ask we can consider this event that, these on these k vertices we have an independent set.

So, what is the probability that, this is indeed and independent set; now, the probability that we have an independent set on a this given k vertices is essentially correspond to the combined occurrences of simultaneous occurrence of this following events namely, for instance, if this are the k vertices, so this edge should not be present, this edge should not be present, this edge should not be present, this should not be present, this should not be present, this should not be present, this should not be present, this should not be present, this should not be present, none of them should be present, that means, there are k choose to edges none of the cases chose to edges should be present to present.

So, let us say, this is e 1, this is e 2, without edges using this names e 3, say e n, e k chose to; so, the probability that on this set of k vertices we have an independent set correspond to the probability that we have e 1 is not present, that is, this will correspond to a e 1 bar a e 1 bar, and e 2 is not present, and e k chose to is not present; so, this k chose to even together should happen, but then we know that each of these things are independent; therefore, we can multiply them together, this is the essentially q into q into q into how many times? That is k chose 2 times k q raise to k chose 2, this is the probability that, this is an independent set.

Now, if you ask can I get an upper bound that, there is a k sized independent set in the graph; independent set can occur in any of the possible k vertex sets, so there are n chose k such possible vertex sets; in any of the k subsets among this n chose k possible subsets, the probability that we have an independent set is q raise to k chose 2.

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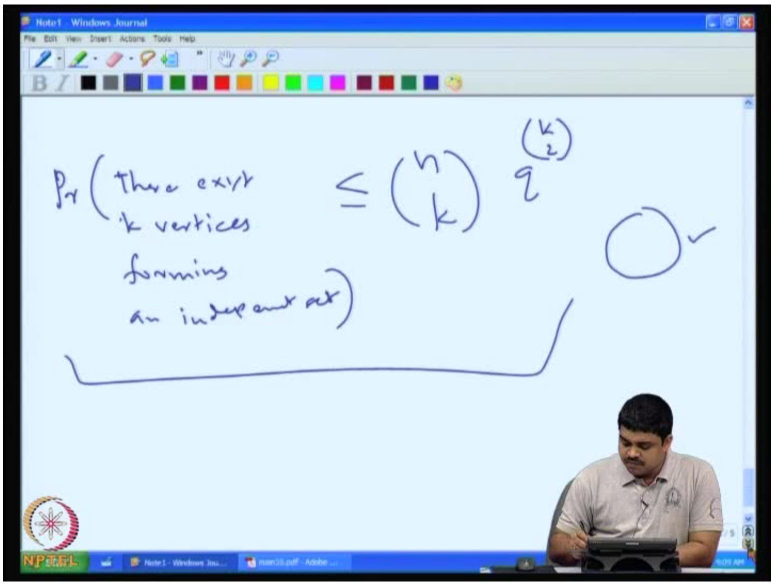

Now, the probability that there exists an independent set in any of them will be at most n chose k into q raise to q raise to k chose 2, this will be that is this is the probability that there is a there exist k vertices forming an independent set.

So, why did you say less than equal to greater than summing up, because so they are for instance in this k selections some of these events may occur, see it is not that they are all mutually exclusive; in fact, it is possible that when one of them happen the other may also happen; so, therefore, we can only say that the total is at most in chose k if as if they were all mutually exclusive; we had summed up, we would have got this, so it can be smaller that is the reason why we are taking n chose k into q resistance by 2.

A similar question, suppose, if I asked instead of independent sets can I find an upper bound that there exist a clique on k vertices in g, how will we find? The same method, we will use the same method namely, we can fix a particular k vertices, and then ask what is the probability that there is a clique in it.

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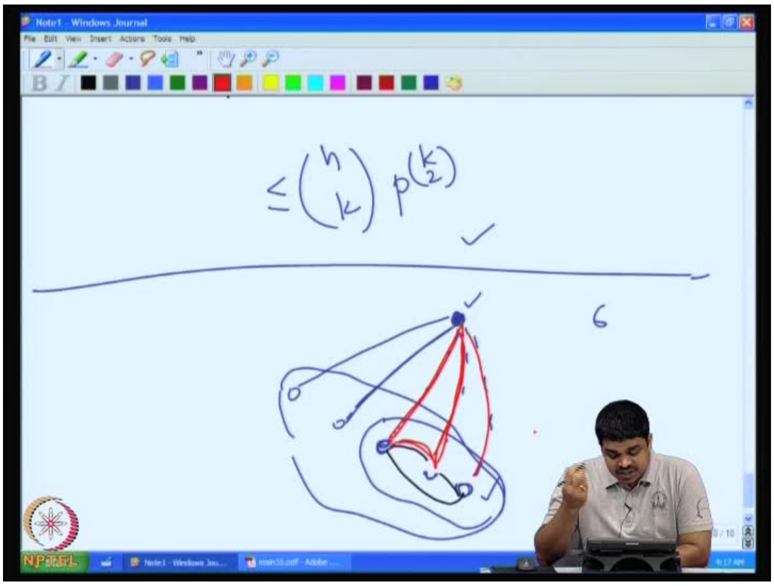

Clearly, the probabilities p raise to k chose to why is p raise to k chose to, because for that k vertices to form a clique, we need each pair of vertices to be adjacent, there are k chose to adjust within that k vertices, so for each of these edges to be present the probability is p and all of them to be present together, it is p raise to k chose 2.

Now, what is the probability there exist at least one such clique in it, so the clique may happen in any of the k vertices; so, we can sum up the probabilities on each of the possible case sub sets of the g, so n vertices, there are n chose k possibilities, therefore, n chose k into n chose k into p raise to k by 2 will be an upper bound for the probability that there exists a clique of sizes k in g.

So, these are just examples; I just wanted to illustrate some other way the way in which we calculate this probabilities in g n p; so, what we have done is, we have we have introduced a probabilities space called g n p, where we can visualize this g n p as like each edge is selected with probability p and not selected with probability 1 minus p and so if that is the situation we can access, we can at least get an some bounds for the probability of some events which are interested in, like we considered couple of events here, one event be consider was what may be a good upper bound for or what may be a upper bound for the probability that there exist k sized independent set in the randomly taken graph.

So, and another thing we consider will be an upper bound of the probability that there exists a k sized clique in the randomly selected graph; so, this calculations were done with some simple observations that for a particular given vertex k to be a clique, then we should have all the edges present, therefore, if things they all should together happen using the independence of these events; that means, an edge occurring and another edge occurring is independent events; therefore, we can some we can multiply the probabilities together; therefore, p raise to k chose 2 is the probability that a clique occurs in a given k set of k vertices, but then there are such n chose k possibilities.

So, we can sum them over all of the subsets, but then it is only an upper bound, because some of them may happen together, so then what we mutually exclusive so this is, it will illustrate the calculations in.

Now, see how do we make use of this g n p to do some graph theoretic to solve some graph theoretic crystals; so, in probabilistic in we use probabilistic methods many times; we prove certain statements which are completely combinatorial, that means, there is no real probability content in the statement of the theorem, but in the proof we will use the probabilistic, so this a way, so it is a very nice example will be explained now.

So, we many of we might have heard about the following problem; see this problem that, suppose you go to a party with at least six people in the party and then in this party there is at least three people, always we can find three people, who are either mutual friends or mutually are not to each other how do you show this thing?

So, this is like - when I say mutual - when two people then either know each other or they are unknown to each other, now we can think of it a graph, if this is the first person when consider others other five people so and then out of this five people so he knows some of them, so if you knows some of them we can put an edge, otherwise if you do not put on edge between them. So, this is the graph; so, here also will some edges and non-edges; now, we will see, out of this five people, so we are grouping the five people into two categories - the friends of this vertex and here the unknown to this person.

Now, among these six people five people, so at least three should come in the same category; so, let us say here edges we at shown that three of them are unknown to him mark, so I will mark with red color here, three are unknown to him; now, here we see that, among three, if all of them are known to each other, we already got a mutual friends here three, but on the other hand if even one of them is unknown to each other then together with this present we have a three mutually unknown present.

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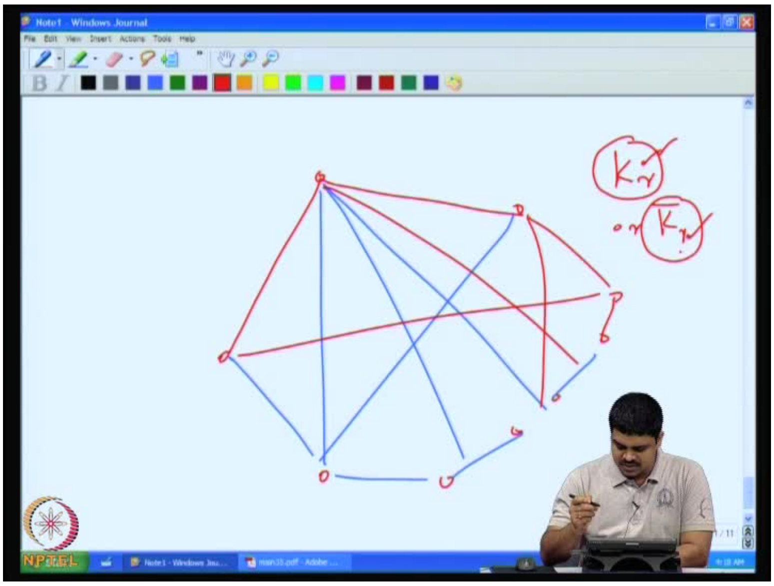

So, this is the idea; now, we can always ask a more general question here; suppose, we can ask this thing, we have a graph, so this can be the graph of several people and persons, so then then on each other we put an edge, if they do not know each other we do not put an edge there or can use a different color to indicate that there is a they do not know each other.

So, there is no edge, it has to possible so like that, so we can think of it in two different ways either it is a graph so there are some edges some edges are not present or we can think of a complete graph colored with two colors - red color and blue color - like that; whenever there is an edge we have the red color, if there is no edge there is a blue color like that; and now, we can ask it is possible that there is always a k r or a k r minus k r bar, that means, a complete graph on r vertices is present or independent set of k r on r vertices is present k r or k r bar.

This is with respect to the graph, when we say that we are just taking a graph on n vertices, we are asking, is it possible that always whichever graph you take the respective of a graph you take can always say that there is a complete graph on r vertices or an independent set on r vertices.

So, if you are thinking here is coloring model, then we are asking, can I always say that there is a red clique on r vertices or a blue clique on r vertices; let us let us go to the graph graph model, suppose we have a graph, we are interested to know whether there is always a complete graph on r vertices or an independent set of r vertices.

So, now to get a complete graph on r vertices, so you will think that there should be lot of edges, if there are lot of edges in the graph then they should be they may be a complete graph on r vertices, but it may not be always the case for instance if the number edges are somewhat less so it may be a possible we do not have any the any graph one any complete graph on r vertices induced of graph; but then will the other exchange come, that means, the complete graph on sorry an independent set on vertices will appear.

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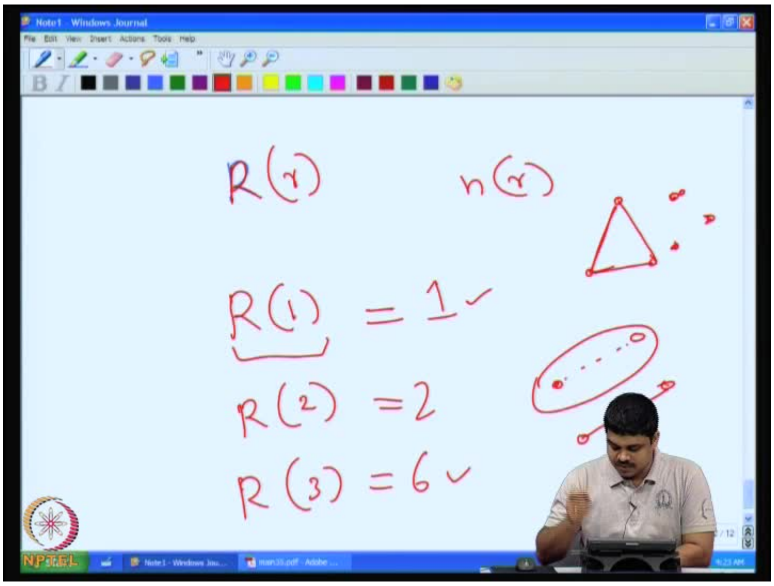

So, very interestingly it so happens that this will as long as the number of vertices is a little large this will always happen; so, we will quickly look at a proof of this thing, why this will happen? So, the here suppose so before that let us let us define a set number called the Ramsey number, because this is an interesting subjecting graph theory called Ramsey theory; so, this is essentially one of the initial question, since in Ramsey theory, since we do not have an much, we would not get into to that topic very seriously, but we will just introduce this basic problem from the Ramsey question namely this is the one.

So, Ramsey theory wants to be show that, if the number of vertices in a graph is reasonably large, that means, if we have a reasonably large number of vertices in the graph, then whichever is the graph you select you will always find a complete graph on r vertices and if you do not find a complete graph on r vertices you will always find an independent set on r either you will a complete graph on r vertices or an independent set of r vertices; and what is a how big when you say that the graph is a reasonable in that how big.

So, it should be so that number have has to depend on r; so, for instance, small number it is very easy, for instance, if you ask of r 1 the smallest number such that beyond that number; when the number of vertices in that graph is more than r of 1, then there exist an either either a complete graph on one vertices on vertices is trivial, because you can take r of 1 as 1 itself in fact in any graph with at least one vertex, you know that there either a complete graph of one vertex and independent set or an independent set of one vertex, because both are same vertices only, but if you look at r of 2, when can I say that, there x is a complete graph from either a complete graph from two vertices or there exist a complete graph from two vertices or there exist a independent set on two vertices, this should because of the two vertices itself either we have an independent set here or if there is an edge between them then already.

So, we can take it as 2; and now, the next question was r of 3, which was the more of 3 serious slightly more non trivial, which we consider when, can I say that there exist a triangle, I mean or an independent set of g size 3, which we had seen if I take 6 vertices are more; you will always able to the find a triangle or an independent set of size 3 6.

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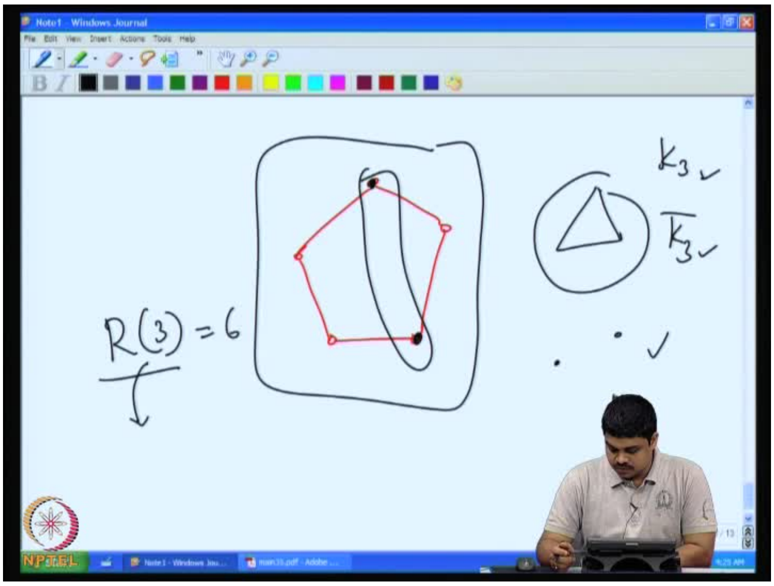

So, that could it have been 5, for instance, if I had taken had graph on 5 vertices, can I say that there is always a complete graph on three vertices or so if you show that the same not the case; we just have to show one example, for instance, we can take this example namely, this is a pentagon, this graph is five vertex graph, here we do not have a triangle, we do not have any structure like this in this thing; now, but we have an independent set of size 3 here in this know if you take this then we can take this and then there is no more independent set.

So, independent sets rises maximum two only here; therefore, in this five vertex graph in either have a k 3, now where have a k 3 bar, so the number we are interested when we say when we say r of 3, we are interested in this smallest number, such that, from that number onwards if the graph has so many vertices, then it should be always the case that either we have a k 3 or a k 3 bar; and any number delay this should be such that, we should be able to find even at least some graph at least one graph with the property that neither a k 3 nor a k 3 bar exist in it.

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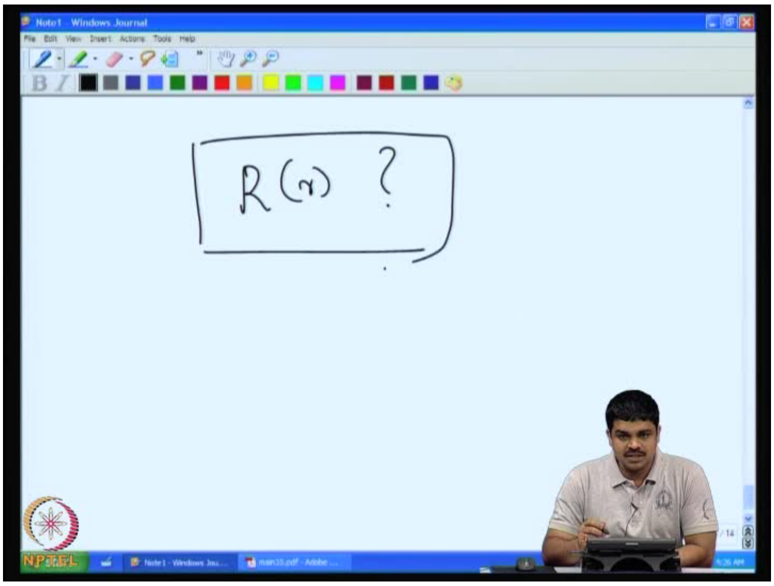

So, here that is why add of 3 is 6, for instance, we have taken and we can find this example neither k 3 or k 3 bar is existing so for any, so that is true; now, the question is, for general case, what is r of r, this is the Ramsey number of r; so, Ramsey number of r means, so what is the smallest n of r, we should say n such that, if the number of vertices in the graph is greater than or equal to this number n, that is r of r, there is its guaranteed where whichever is the graph you take as long as the number of vertices is this much; there is either a complete graph on r vertices or an independent set on r vertices and the and believe that we have for instances when we take a graph r of r minus 1 this guaranteed is not there.

You can always find one graph at least at least one graph, such that, we do not have a case k r, we do not have a k r path, that means, an independent set on r vertices; such that number is the threshold number is called the Ramsey number of r; and now, we are interested in finding, so for instance, is it always true that we have an upper bound for r of r, so why do we think that there is such there exist such an integer for every r; so, it does not look very reasonable, why should it happen? Because, so it is trying to tell us that, when the number of vertices is large, we cannot avoid one of the one of this structures either a complete graph or an independent set on this given number of vertices, that is what as long as the number of vertices, it is not totally reasonable to think like that, but why should it happen?

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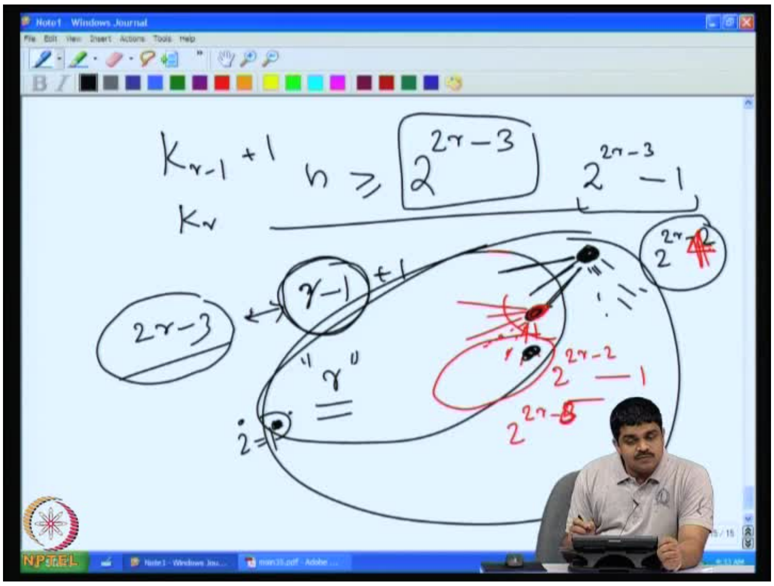

So, we are giving a quick proof that, when the number of vertices reasonably large number, it should happen for instance, if n is greater than equal to 2 to the power 2 r minus 3, we will show that this property will automatically come, why do you think like that.

So, for instance, we can take the vertex, we can consider a vertex, so there are n; so, first from the given graph g pick up two resister 2 r minus 3 vertices, so we can we can discard this; this is the set of two raise to 2 r minus 3 vertices; now, this be a special vertex; now, what we will do is, we will the remaining two raise to 2 r minus 3 minus 1 vertices, we will group into two categories - the ones which are adjacent to it, the ones which are non-adjacent to it; is clear that one of the groups should be two seal of this, that means, 2 raise to 2 r minus 2 of it; you know both of them cannot be 2 or minus 2 minus 1, because then if you sum up you will only get two raise to 2 r minus 3 minus 2, which is not true, which is one of the groups should be at least 2 raise to the power minus 3; so, let us say without loss of generality, this is the group.

And now, from the bigger group, we can pick up another vertex, so maybe we can pick up this vertex, and then here again with in with in this group, we can categories then into two like, the ones which are adjacent to it and the once which are non-adjacent to it.

So, now, let us say, so there are remaining vertices essentially 2 raise to 2 r minus 2 minus 1; and in one of the categories, we should have at least 2 raise to 2 r minus 3 vertices 2 r sorry 2 r sorry 2 r here, this is four and this is 5 2 r minus 5 vertices sorry minus 4 this 2 r minus 5 vertices.

So, we are halving it; so, then here let us without less of generality the known adjacent set of vertices are bigger this time; and then we can we can go into that group, you can pick up a new vertex and then again we can do this procedure till we end up with just 2 raise to 0 is equal to one vertices; in how many steps in 2 or minus 3 steps we will come to just one vertex remaining; but this 2 r minus 3 vertices towards that the pick like this, this they depending on how to relate to their previous vertex, we can put it into two group sorry relate to there the vertices, which are below this; for instance, they are we are selected the non-adjacent side, all the adjacent side we can put them in category say it category or black category like that two category here.

So, since we have 2 r minus 3 such vertices, since we have 2 r minus 3 such vertices, which is clear that 2 r minus 1 of them should be, r minus 1 of them should be of the same category when do I say it same category r 1; for instance, r minus 1 of them belong to the first category namely, when we will look from that vertex and categorized based on it is the remaining vertices, based on its neighbors and non-neighbors, the neighbors were more then this r minus 1 vertices are such that, they are adjacent to all the vertices bellow it which is selected later.

So, if all this r minus 1 vertices are adjacent to each other, that means, they form a k r minus 1; and namely the last vertex the 2 raise to 0 equal to 1, the last vertex will also can be can also be joint to that, so we get a k r that will.

On the other hand, if it is so happens that, these r minus vertices belongs to other type, other type means, if each time they were selected so happen that from the remaining vertices when we categorizes based on its neighbors and non-neighbors, the non-neighbors were higher; and then we see that each of these vertices have non edges to each other, because one was selected we picked up the non-neighbors of it and from that only the next vertex was picked; and then from that only the from the non-neighbors of it only, it was picked, so we get a collection of non-adjacent r minus 1 vertices; and this last one can be added to this to form a r independent set of cardinality independent set of cardinality r.

So, it looks like, so we can see that if the number of vertices in the graph is at least 2 raise to 2 r minus 3, we can always find a, we can always find a either a complete graph of on r vertices or an independent set on r vertices; it is it is always possible to find a complete graph on r vertices or an independent set on r vertices.

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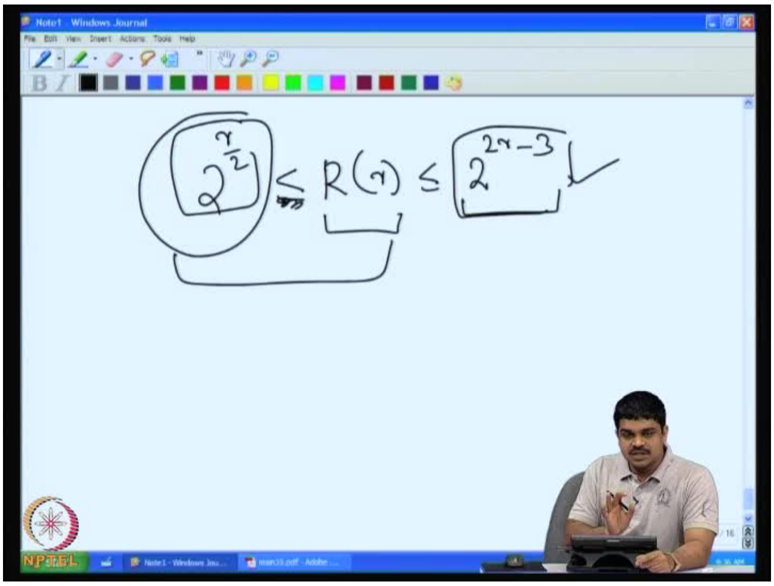

So, therefore, we conclude that, r of r the Ramsey number has to be less than equal to 2 raise to 2 r minus 3; why is it less than equal to 2 raise to 2 r minus 3 it, that is because it can be the number can be smaller, because it is possible that that number of vertices such that the minimum number of vertices, such that, that is a guarantee that either a complete graph on r vertices or an independent set of r vertices occurs is even below this thing - below this 2 raise to 2 r minus 3; but that what we can prove now is 2 raise to 2 r minus 3, we know that as long as 2 r minus 3 number of vertices are there, this is true, but bellow that we are not sure as of now, but so with more effort we should we will be able to prove little better.

Now, that is not our intension, because we just quickly completed describing this proof to convince the reader that, the reason upper bound for these things; that means, it is not that we are talking a question which is just not possible; that means, we were asking of a number n such that, if the graph has more than n vertices, then always there is a complete graph or an independent set of r vertices, but it indeed access that; for instance, if the graph is the reasonable graph is the reasonably large is always happen, but in this when we are talking about g n p model, the random graphs, so our intension more is to illustrate the proof technique of probabilistic method and this is the problem which is very suitable for that.

And what we are going to do is to show that, this r of r is greater than equal to 2 raise to r by 2 also, this is an upper bound and this is greater than this; so, this actual value r of r has to be somewhere in between 2 raise to r by 2; and so, strictly greater than we can say 2 raise to r by 2 and 2 raise 2 r minus 3.

How do we show this thing? This problem seems to be more difficult, so this has to be constructive one which was more intuitive, in fact, if you if you had noticed the technique that we had used in the in the in solving that passel more or less the same technique was extended here also, but this question, how will I prove this thing.

So, if I want to show that r of r is greater than 2 raise to r by 2, that means, if so if I take a graph on 2 raise to r by 2 vertices, then this guarantee is not there; that means, either a k r or a k r bar is present is not there, that means, we have to somehow show one graph on 2 raise to r by 2; even one graph is enough, just an example, graph on so many vertices such that so many vertices such that, there is no k r in it no k r bar in it neither a complete graph on r vertices nor an independent set of r vertices is present in that.

So, here for this proving this thing we will use probabilistic method in the next class.

So, thank you.