Graph Theory

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Module No. # 06
Lecture No. # 36
Probabilistic Method: Markov's Inequality Ramsey Number

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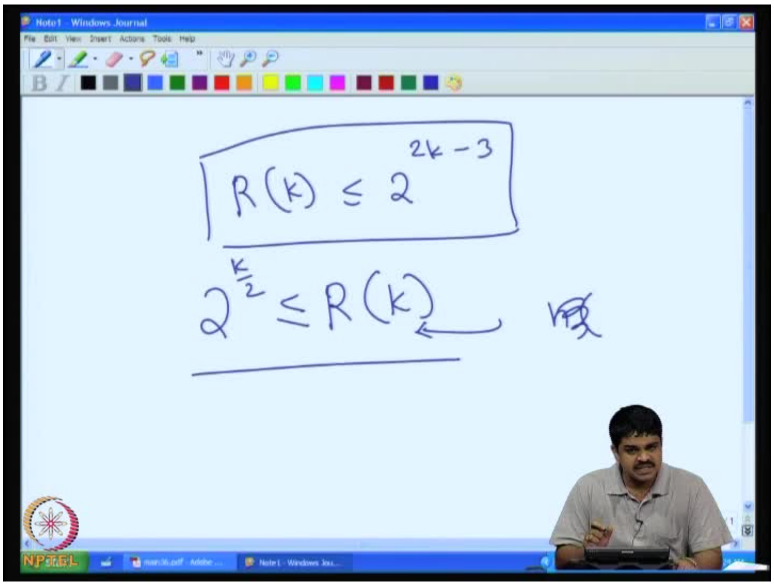

Welcome to thirty sixth lecturer of graph theory. In the last class we considered the question of designing an upper bound for the Ramsey number and getting lower bound for it. In the last class we show that R of k is less than equal to 2 to the power 2 k minus 3 is using a common interval argument. Now, in this class we are trying to get a lower bound of k R of k is greater than or equal to 2 raise to k by 2, this is for intension today.

We call that the Ramsey number of k is the minimum integer n, we can say n of k so R of k such that, if the number of vertices in the graph is greater than or equal to this number then, we are guaranteed that either a complete graph of k vertices or an independent set on k vertices is available, the graph. Now, the question is to decide R of k, finding it exactly it is difficult show. We show that but, this number has to be less than 2 to the power 2 k minus 3 yesterday, by more sophisticates argument we can probably improve a little bit but, we are not interested so in that now. So as of now, we just happy by saying that the reason upper bound the reason indeed the number like that R of k. Then how do will show lower bound for this step?

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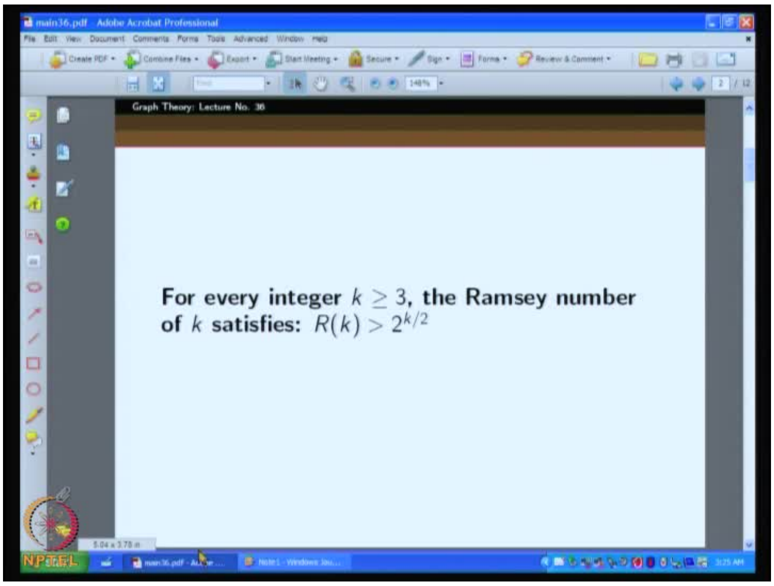

So, when I say 2 raise to k by 2 is lower bound for of k, it means that; I can show a graph on 2 raise to k by 2 vertices. Such that, neither it has a k clique in it nor it has an independent set of k vertices on it, such a graph exist. How do you show this? So, it turns out that the best way to attack this problem is by using this probabilities method. So, this is what we are going to do. So, we will show that for every integer k greater than or equal to 3, the Ramsey number of k satisfies: R of k strictly greater than k by 2. So here strictly great.

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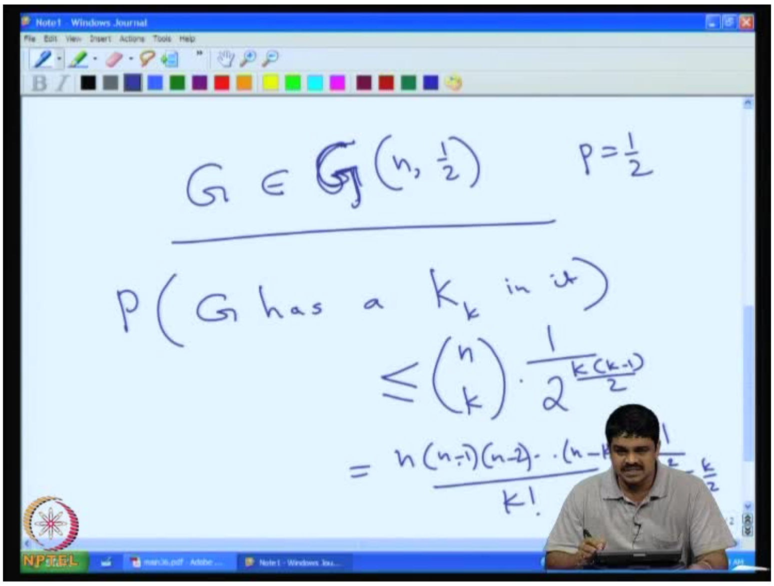

Now we will prove this thing, now to prove this thing we will consider the probabilistic distribution of graph G n half. Here, we are taking P equal to half and then, so this is, we can write it like little call G. So, we are taking graph from this distribution randomly, a random graph we are considering with respect to probably from this probabilities space. And then, let us ask this question, what is the probability of having an independent set of cardinality k in this graph? And then, we ask the question, what is the probability that G has a complete graph one k vertex in it? See, both these probabilities are less than half then, the probability for one of these two evens to happen will be less than half plus half namely 1.

So that means, the probability of the compliment even happen namely the probability that, the graph has no clique of k vertices and no clique of no independent set of k vertices will happen with some positive probability, none zero probability. Which means that, their indeed access one out come, one graph in this probability with respect to this probability space. With this property namely; no clique on k vertices, no independence set on k vertices.

So, have to prove this thing, we will formally consider what is the probability that G has K k in it. What is the probability? How do you, we have already found a way to upper bound to this thing, this is less than equal to n choose k, because there are n vertices and from this n vertices we can select k vertices, n choose k vertices. Now, this probability that, this k vertices indeed form a indeed form a k clique is 1 by 2 raise to k choose to that k into k minus 1 by 2 is indeed.

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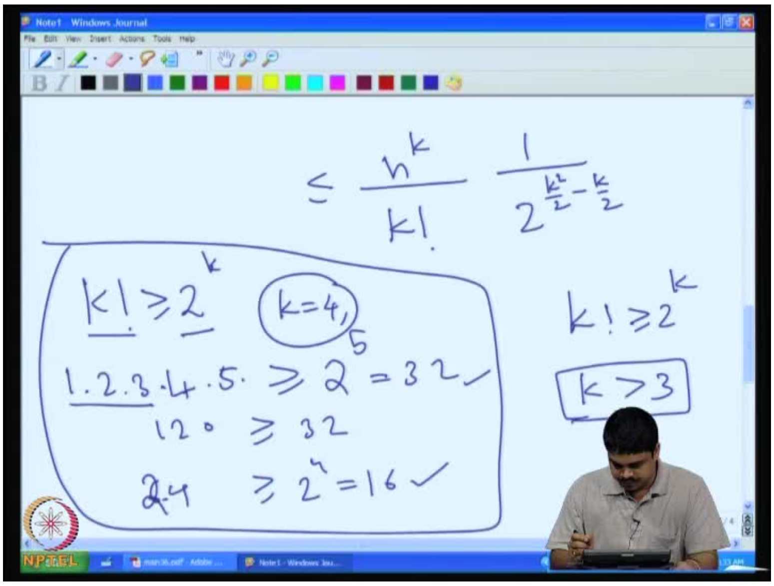

So, this is essentially how much as these. This is n into, n minus 1 into, n minus 2 into, up to n minus k plus 1 divided by k factorial, into 1 by 2 raise to k square by 2 minus k by 2. So, this numerator here n into, n minus 1 into, n minus 2 into, n minus k plus 1 can be substituted with n to the power k, because there are k terms here. And each of the terms n minus 1, n minus 2, n minus k plus 1 all are less than equal to 1, so I can simply write, this is less than equal to n power k by k factorial into 1 by 2 to the power k square by 2 minus k by 2, that is what I can see.

Now, k square is greater than that case, k factorial is greater than equal to 2 raise to k. Why is it true? Because, for instance k equal to 1, this one factorial is need not be need not be greater than equal 2, it is not greater than equal to 2 raise to 1. So but, k equal to 4 k this is 2 factorial that is 2, how can be greater than equal to I am saying so we can.

So, when we put 3 here we get, when we put 3 we get 2 raise to 3 is 8 then, this is 3 factorial is 6 therefore, we can substitute by, so we can say 4 fact (( )) 4 fact. So, 1k factorial, for this is in fact 1 into, 2 into, 3 into, 4 into, 5. So, say for intense 5 then, we can say this is 2 raise to 5 this is 32. How much is this? So, this is 6 into 4 into 5 so we have 120 here, so 20 into 6 is 120 so this is bigger.

So now, with 4 we have 1 2 3, so this is 24 and here it will be 2 raise to 24, 2 raise to 4 is 16 so 24 is greater than equal to 16, this also to with k equal to 4 onwards this is true, k equal to 4 onwards this is true, will k equal to 3 what will happen, this is 6 only, while this is six only while this is 8, so this is not true. So, we have to assume that k for this is to be true, for assuming that k factorial is greater than equal to 2 raise to k we should assume that k is greater than 3. Let us assume that.

So, what will I do for k less than equal to 3, that we can easily verify because R of 3 is equal to 6, R of 2 is equal to 2 and so on. So, when you put 2 raise to k by 2 all this things, it will be immediately clear. For instance put k equal 3 2 raise to 1 point 5 is less than 6, so put k equal to 2, 2 raise to 1 is less than equal to 2, so for k or greater than equal to k greater than equal to 3 we will prove that otherwise, so we have to put R of k greater than equal to 2 now.

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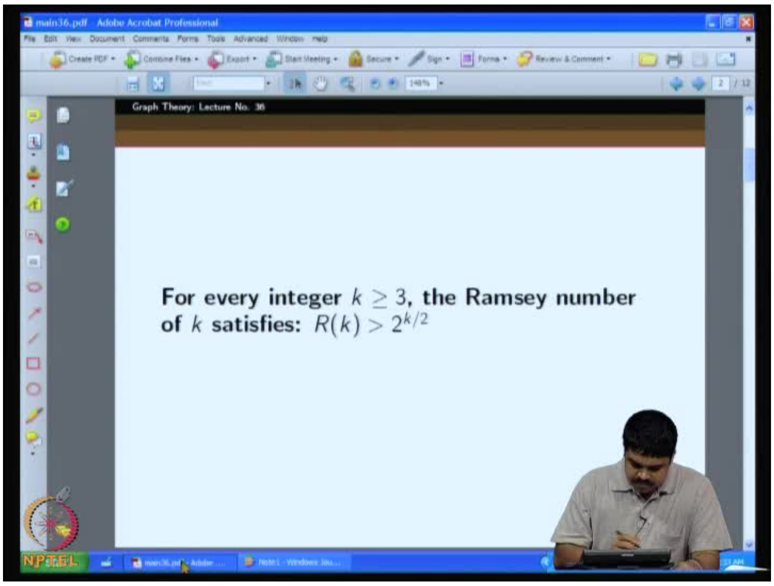

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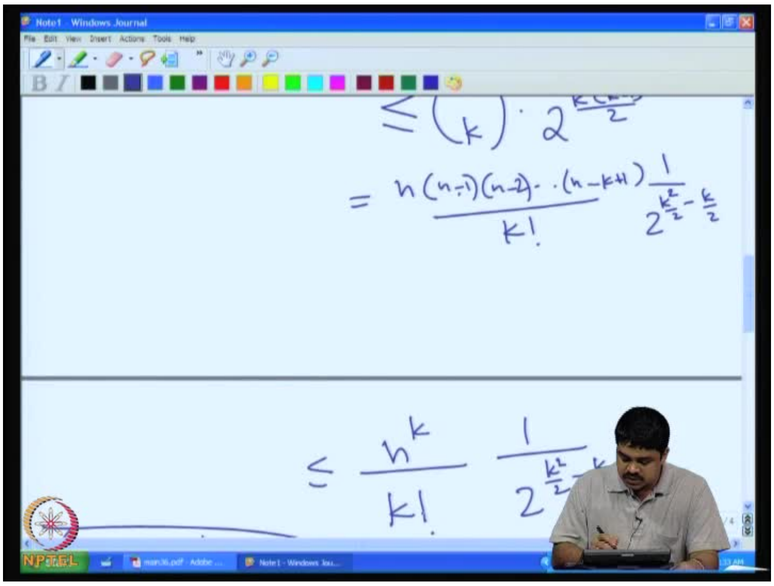

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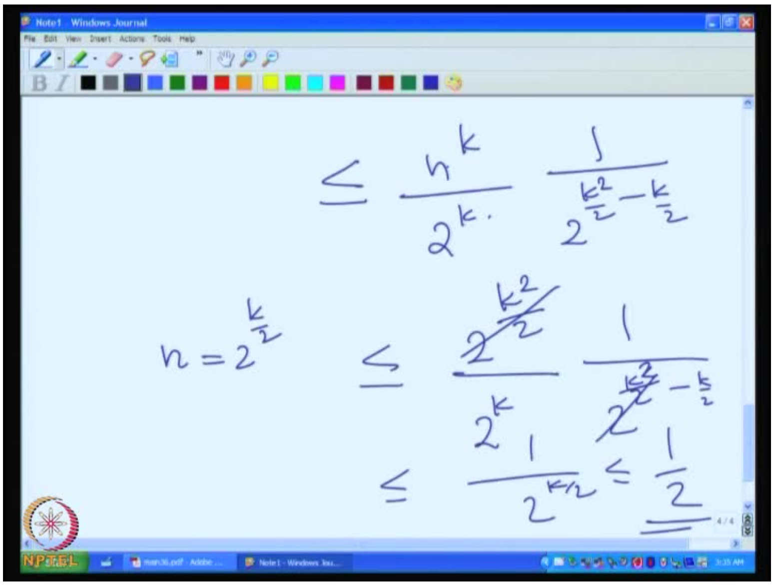

So our statement of the theorem only is being prove k greater than equal to 3, for k equal to 3 we can easily check it may, because R of k equal to 6 for k greater than equal to 4 we will do this thing by observing that, so in this k factorial can be substituted by 2 raise to k, because k factorial is greater than equal to 2 raise to k for k greater than or equal to 4. Now, we will we will get this thing, so like this is less than equal to n raised to k by 2 raise to k into 1 by 2 raise to k square by 2 minus k by 2. When I, now suppose n is an number which is less than equal to, so suppose I taken n equal to 2 raise to k by 2. So, let us take this number now I can substitute this by so this is less than equal to 2 raise to k square by 2, because I am putting n equal to 2 raise to k by 2, so this is by 2 raise to k into 1 by 2 raise to k square by 2 minus k by 2.

So this will be, so here canceling this and this minus so we will get less than equal to 1 by 2 raise to k by 2. Now I have assume that k is at least 4, because k equal to 3 case already considered so this will be less than equal to so if I put the smallest possible value here k equal to 4 is 1 by 2. So I get that, less than equal to 1 by 2.

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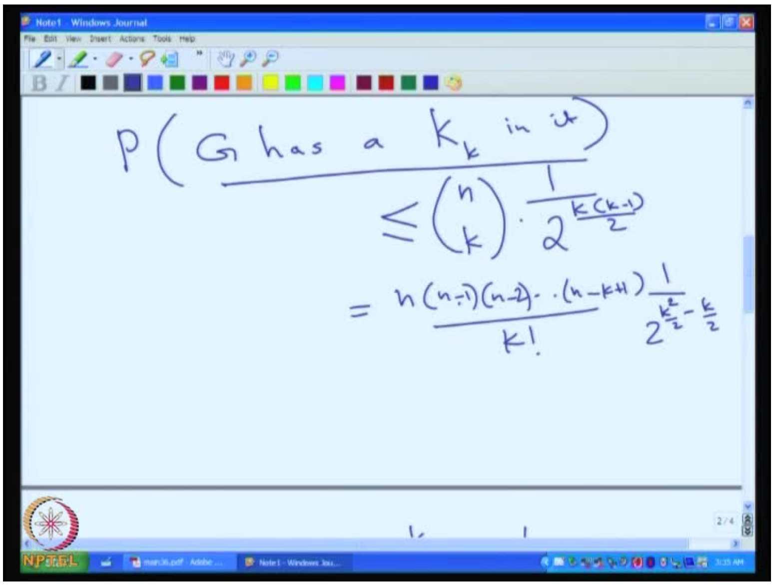

So what we have now done is the calculation of the probability of this event, G has k raise K k in it, induce complete graph on k vertices in it. So, we just did it by considering all possible case subsets and we calculate probability that it should be, suppose given a k subsets what is the probability that there is a clique on it that was these probability, because k into k minus 1 by 2 edges 1 by 2 is the probability that an edge occurs. And then we multiply by n choose k, because we are summing up are all possible case subsets if any 1 of them has clique then, when we have a k clique then, we can upper bounded by so this can be this actual probability can be much smaller than this total sum but, then we are we can upper bound this way and then we substitute this thing. This is replace by n raise to k, this is replace by 2 raise to k for that we calculate so we needed the assumption that k is greater than equal to 4, because k equal to 3 n less is not happening.

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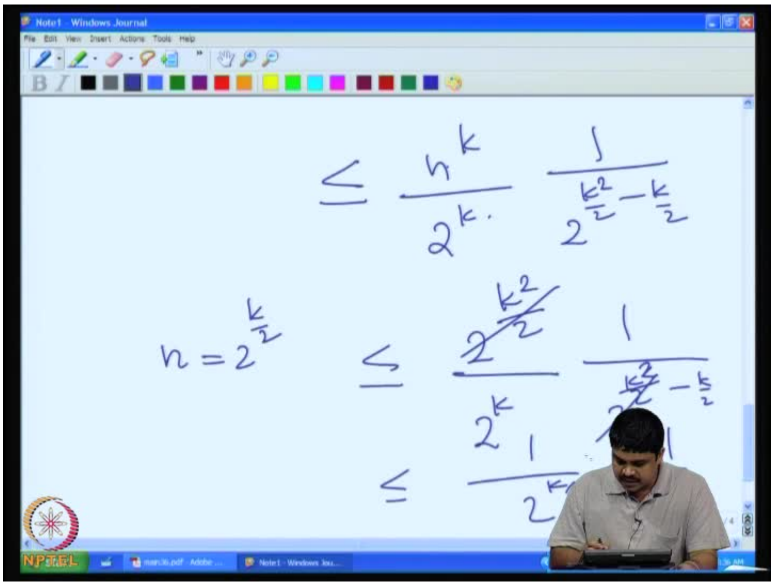

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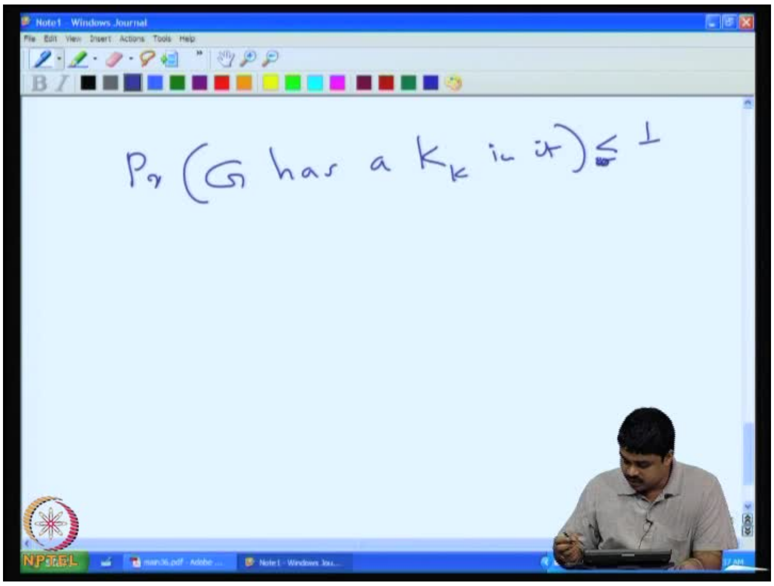

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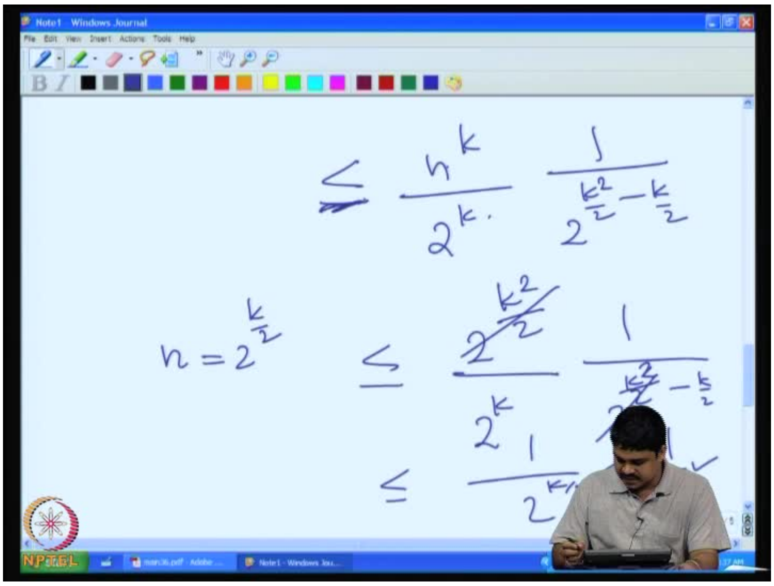

So therefore, when the but, then below it is clear that 2 raise to for R of k we know the value therefore we do not have to bother about it, now substitute it and then we canceled all the things and then finally, so you also have substitute n equal to 2 raise to k by 2 for instances, if take a graph 2 raise to k by 2 vertices then we can substitute n equal to 2 raise to k by 2. So, then we show that, this cancels out and is, because we have assume that k at least 4, we can see this is less than equal to 1 by 2. So, this probability that G has G has a K k in it, is less than equal to half; strictly less than half, so here, so this not less than equal to this is strictly less than half, because we have substituted with bigger values here, less than strictly less than so here somewhere here so for instance when you substituted by an raise k itself we can put strictly less than.

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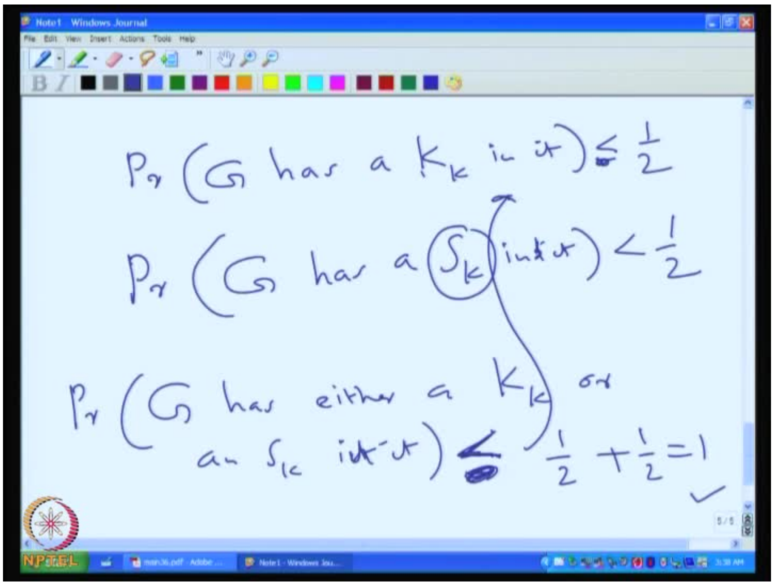

So strictly less than half now similarly, the same the we can calculate the probability the G has a S k in it. What is the probability that S k is independent set of size cardinality k, that will also be strictly less than half. So the probability that G has either a K k or an S k in it, is actually less than equal to sum of the this two probability this and this probability that it has a K k in it and the or the G has a S k in it. So sum of it, this is one so this strictly less than half so, because this was the less than half or this was the less than half so the total will be one. So this is the probability that either K k or S k in it will be strictly less than one. So there is a non zero probability, namely 1 minus this value that G has neither K k or an S k in it. So this is, because we have taken n equal to 2 raise to k by 2.

The upper bound is this much, now we can inform that, the desired event namely there is no K k and no S k in the graph has a probability of probability which is non zero, that means there should be graph with the desired property, otherwise we want get it the non zero probability of event. Recall that the events probability event corresponds say set of outcomes the set of graphs in our case, when I say this probability has a non zero probability or this event has the non zero probability they should be sone graphs belong to that otherwise, we would be able to get along non zero value for that it means there existence a graph with desired probability.

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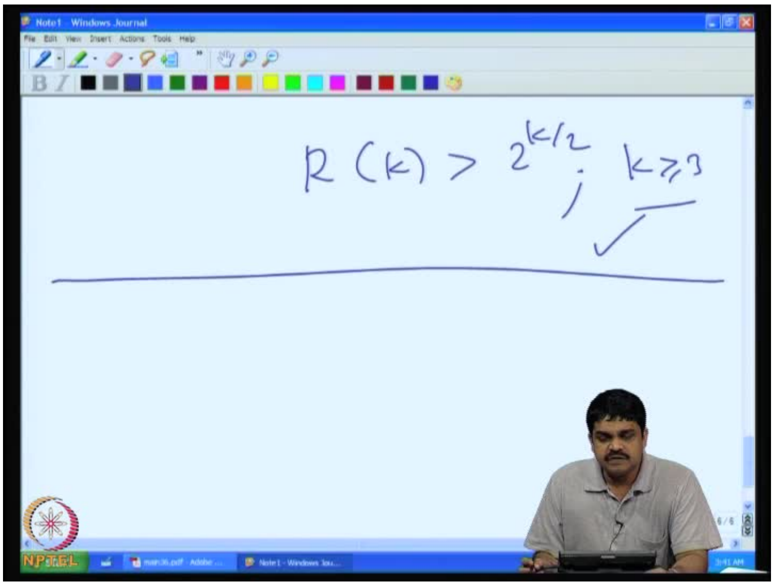

So the inference is finally that, if you fix your n equal to 2 raise to k by 2 or may be less than that, less than equal to 2 raise to k by 2. We can always find a graph with the property that there is no K k in it, no S k in it. So the Ramsey number R of k has to be strictly greater than 2 raise to k by 2 has long as k has greater than or equal to 3, this is what we have to inform. So this is the basic way we apply the probabilistic method, so here we wanted a particular graph with two desire properties that, there is no K k and there is no S k, so if we what we did as we looked at the complement events namely the recycle K k there no S k, so one of them happening, the probability one of probability that one of them at least happening is less than strictly less than one so by we could add up because we could either this, because that so then, because it strictly less than one should be probability for that a desired event.

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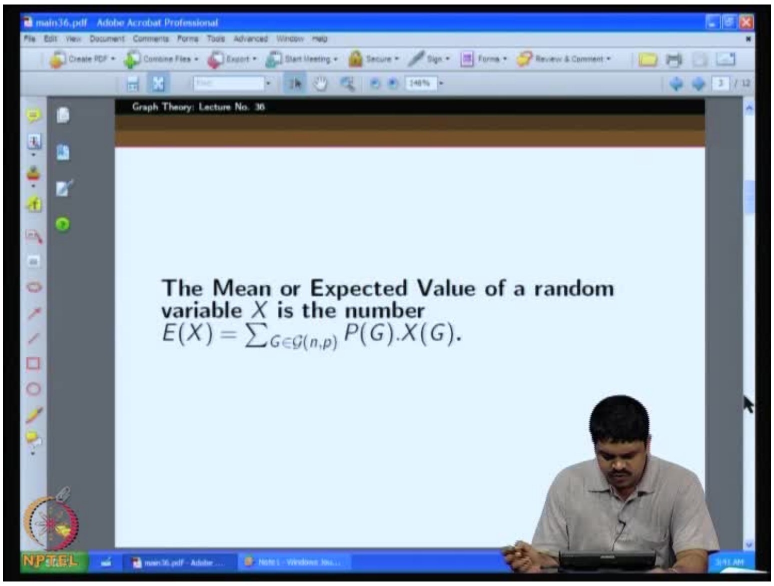

Now our intention is to introduce some other techniques of probabilistic method, so we will start the motion of expectations. So let us say, so what is the mean or expected value of the random valuable x, so let us not get too much the details of probability theory but, let we remained that random variable is real valued function but, other case when we considering the G and P model where especially when we are dealing with graphs usually our random variables will be some of the graph parameters. It can be the chromatic number, it can be independent cardinality of the maximum independent set or any such a usual graph parameter which means that typically they we will be none negative value 0 1 2 3 like that, it can be either cardinality of the maximum clique.

So or even of edges, so usually we will have this X taking values 0 1 2 3 extra. And the expectation of the random variable X is define to be sigma of G element G of P here for our case G and P. P of G into X of g, so for a given graph in G and P let X of G be the value for one outcomes say for one graph X of G be the value of the random variable multiplied with the probability of that particular out particular graph, and then some overall such outcomes possible outcomes in g and p that will be the exceptive of X.

So this is the usual definition of expectation, we just got it in terms of G and P probabilities space that is all. And now, so we will just look at some examples of finding how do we find expectations. So there are, one way of finding expectations is to look at each possible graph and find out the probability of that graph then, find out the value the random variable in it and multiply with the probability with the random variable and sum it over all these things but, many times these can be very tough so usually we go in a different way so to illustrate this point we will take up the following question.

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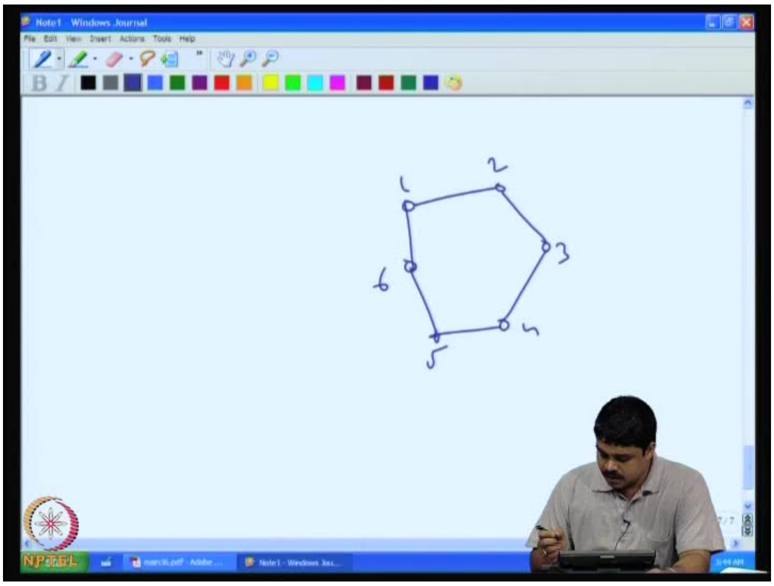

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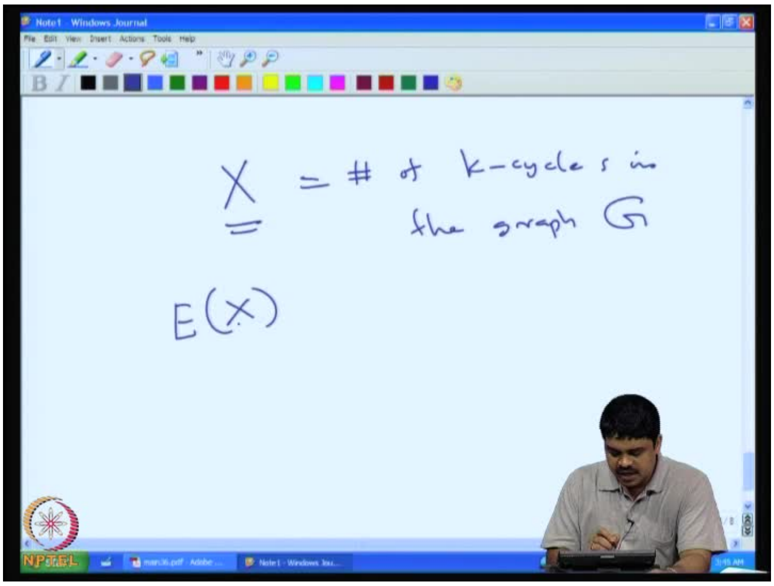

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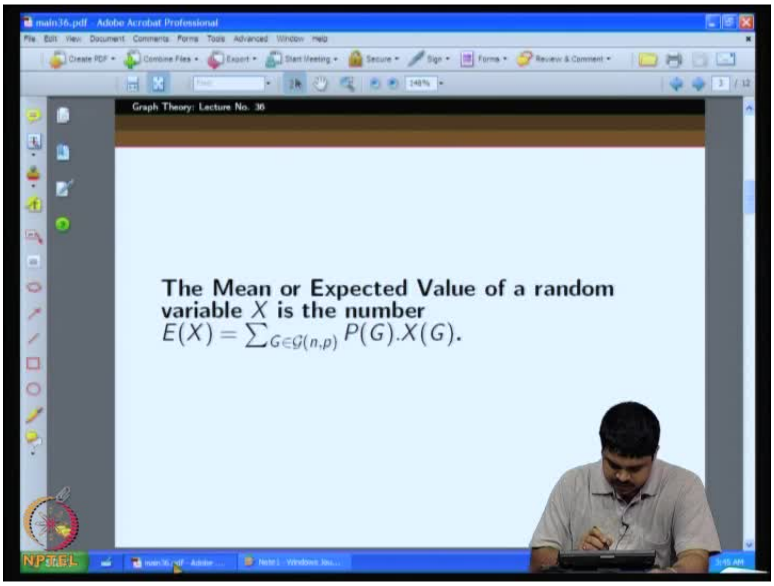

So, let us consider this question of counting the number of k cycle in the graph. What is the k cycle? A k cycle means say cycle with exactly k vertices in it like this, so this is simple k cycle so 1 2 3 4 5 6 this is 6 cycle. In a given graph G how many k cycle are there? Now, I am interested in this parameter x, so that is the number of k cycles in the random graph G. So this is the random variable now, how do you find the expectations E of X? One possible way as it is according to the to our definition of expectations the way we have define the expectations, so we can we have do X we can we can consider this probably for each G and then find then value of the random variable namely how many k cycles there in it and multiply them together and sum over all possible graph but, then this is we tedious it is not possible to do that in many cases.

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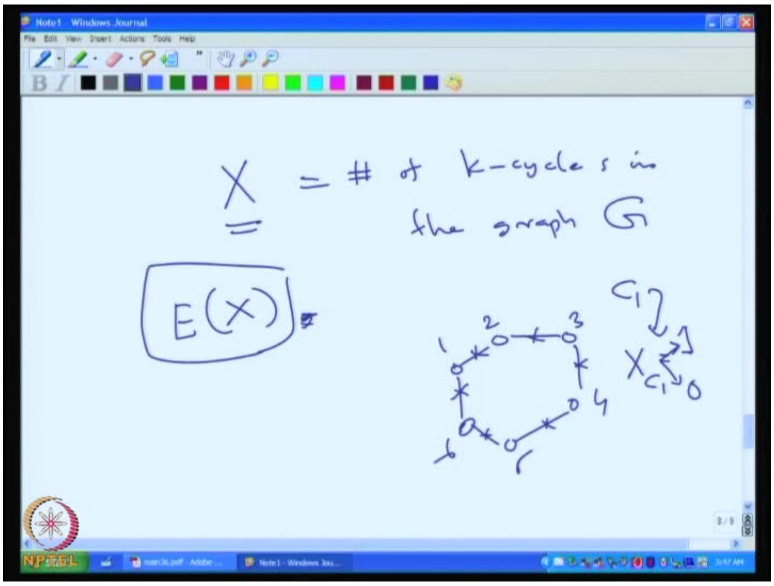

So here, we will find out different way. What we going to do it is to define it is so called indicate random variable namely so given a graph G so we can find out each possible cycle, each possible cycle means see for instance this can be a possible cycle so 1 2 3 4 5 6 this is a possible 6 cycle, if all these cycle edges are present in the random graph; in the graph we have selected then, this k cycle is present in it. So for instance we can call it C 1 we will associate with C 1 then I can define a random variable called X C 1 an then I can find that X C 1 equal to 1 for all the possible edges are appeared here otherwise, I can say that the random value of the random variable is equal to 0, it is 0 or 1 variable. It is an indicative variable which indicates whether this vertices, this collection of vertices in this order for provides say cycle or not, and then it is very clear that if you can sum up the indicated random variables for all the possible such cycles, because this cycle also occurred it will count one otherwise it will count zero. When you sum up we will get exactly the number of cycles that have occurred.

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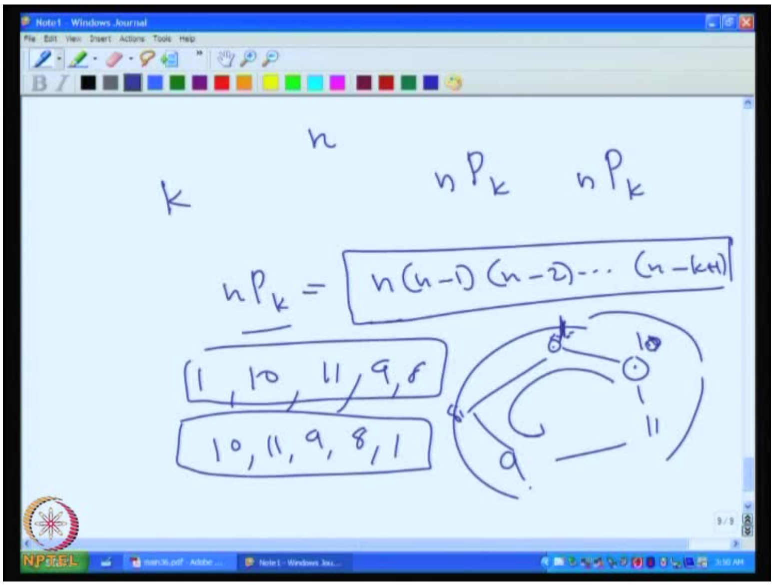

So there are two issues here, how much such possibilities we have to consider. So one possibility, so for instance we consider n vertices then there are k cycles we are interested in. So, from n vertices we can get n P k k (( )) or may be the sequences sequences of length k. So for instance I can take k objects from n things and then I can order then in a certain way n P k ways we can order them, that is n into n minus 1 into n minus 2 into like n minus k plus 1 ways we can order them. If you order them. but then, will it will it always corresponds to all of them corresponds to distinct cycle for instance if I get, this is 1 this is 10 this is 11 say this is 9 and this is 9 and this is 8 so for intense k is equal to 5 so this is cycle but, then you can see that so if I had consider so for sense I can say that 1, 10, 11, 9, 8 can gives rise to a cycle but, then if you had considered 10, 11, 9, 8, 1 that is also gives rise to the same cycle for as the same order but, then we starting from different place that is all.

So we could have inform us for in the case k cycles k, we could starting from any way of the k any of the k number you could have order them, that all of them correspond to same cycle, for instance for instance this sequence and this sequence corresponds to this same cycle this, because the same cycle will come but, just that we started enumerating from here or say first we start numerate 1, 10, 11, 9, 8 the next time the enumerate 10, 11, 9, 8, 1 that is all the another possible way the same cycle is an occur any numerator said 10, 1, 8, 9 like this in the opposite order, the opposite order also or may be this can be could have been enumerated as 1, 8, 9, 11, 10 so for starting with each vertex I could have count I could have listed the members either in the clock wise order or in the anti-clock wise order there are two k possible ways.

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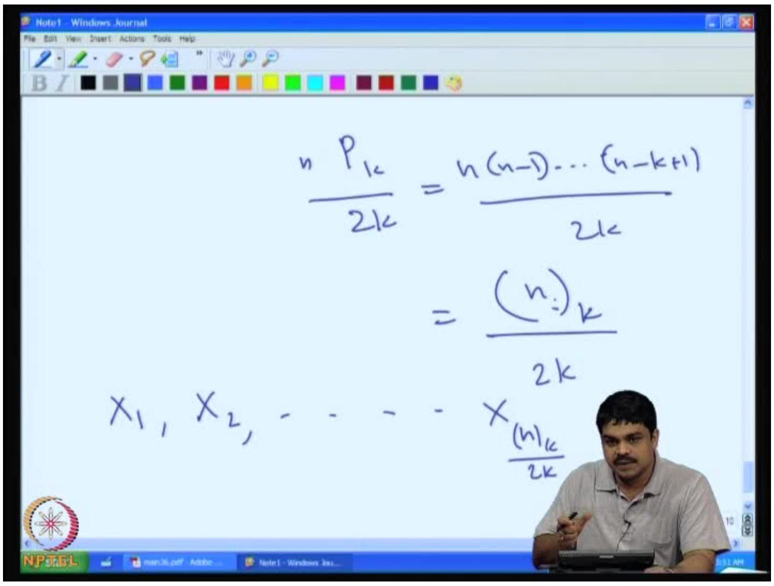

So this is an over counting for the number of possibilities you have to divided by 2 k, because each actual cycle may correspond to 2 k different such a sequence therefore, n P k by 2 k possibilities are there, n into n minus 1 into n minus k plus 1 divided by 2 k which we can even we can also write it as n k this n P k is an another notification n P k divided by 2 k possibilities are there. Now, for each of them I can define a indicator random variable say X for the first among this 2 for second among these, so X so n choose k by 2 k for the last one. And this each of them become zero or one depending on whether that cycle appears or not, whether that, depending whether that cycles appears in your hand random graph or not.

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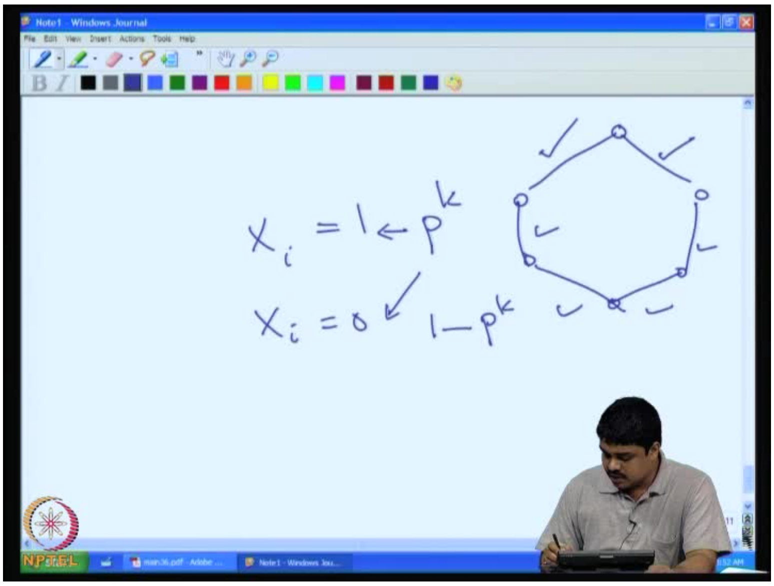

Now the question is what the probability that X I is equal to 1? What is the probability that X I equal to 0? For x I equal to 1 the probability is calculated like this, because for this collect this sequence from vertices we need this edge to be present, this edge to be present, this edge to be present, this edge to be present, this edge, this edge, and this edge to be edge present, all the edges have to be present. That will happen with P to the power k probability and this will definitely happen with 1 minus P to the power k possibility, because this edge occurs the probability P, this edge occurs with probability P, and this also, this also, this also, this also. And there should all occur simultaneously and, because they are all independent event the probability that where all occurs simultaneously is P to the power k.

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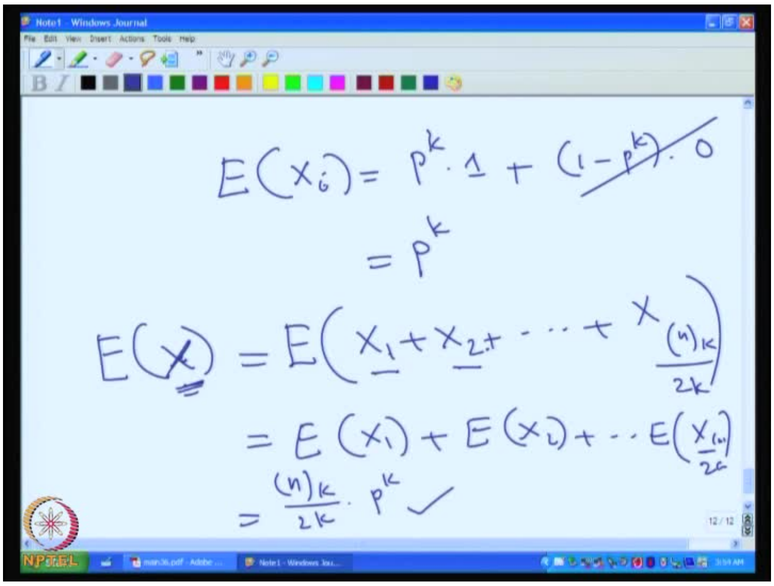

So now what we see is the expectance of X I is equal to P to the power k into 1, because random variable takes two values and 1 minus P to the power k into 0, and this terms does not contribute anything so the expectation is just P to the power k. Now what is the expectation of X? So, expectation X is essentially expectation of X 1 plus X 2 plus and so X n choose k by 2, n P k by 2 k so all the random variables are to be added, as we have observer already this random variable essentially is the random variable which counts the number of cycles and each of these random variable indicates whether a given possible cycle has occurred or not, and this many possible cycles are there. When I sum the up all the this things which have became one will contribute, that means, the actually how many cycles have occurred they will be counted and that is exactly this X therefore, X is equal to X 1 plus X 2 plus this thing is the trick, and then the expectation of X can be counted by finding this thing the one good property about expectation is that it is leaner that means this expectation is essentially the sum of the individual expectation. The expectation of the sum of the random variables will be like we can calculate like this, where we can simply sum the expectation of this things.

So this is expectation of X 2 k, so this is this is the way so if you sum of all of them. So which means that each of them is p raise to k, so I can simply multiply by p to the power k, so n P k by 2 k into the P to the power k will be the expected value. This is an easy way of the calculating the expectation of X, so what even now showed is a method to calculate the expectation of this random variable X, which is which is counting which count the number of cycles in the random graph k cycles in the random graph.

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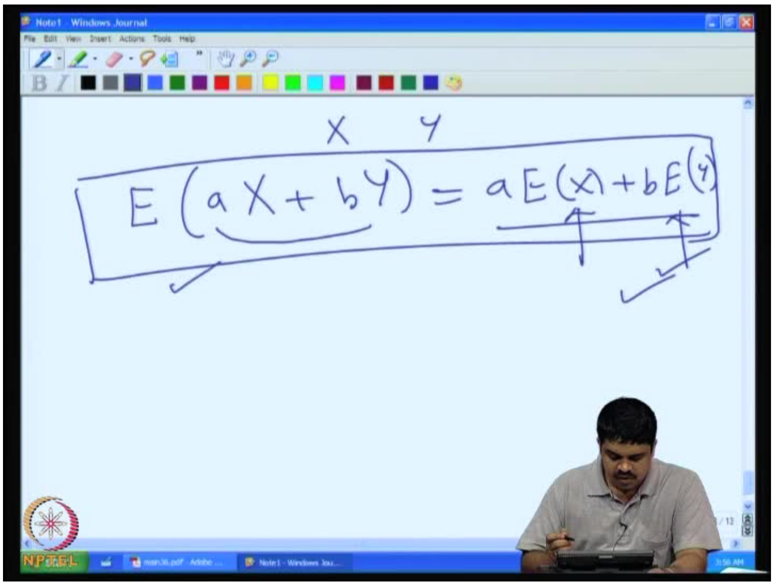

So the again to repeat the trick is rather than using the direct definition for each outcome, find the value of the random variable and multiply the multiply with the probability of the random that particular outcome, and some over outcomes. We rather do a different count kind of counting that means; within each item what are the possible possibilities, what are the possible objects which can be counted towards over a random variable, for each them we will define a indicator random variable then, we will find the expectation of that and then we sum over that. This idea of indictor random variable is a very commonly used technique, the only new loosen we have introduced here is the linearity of the expectation namely when you have 2 random variable say X and y and then we can multiply with a scalar a X and then b into y this an essentially E into expectation of X plus b into expectation of y this proving this is the trivial in fact.

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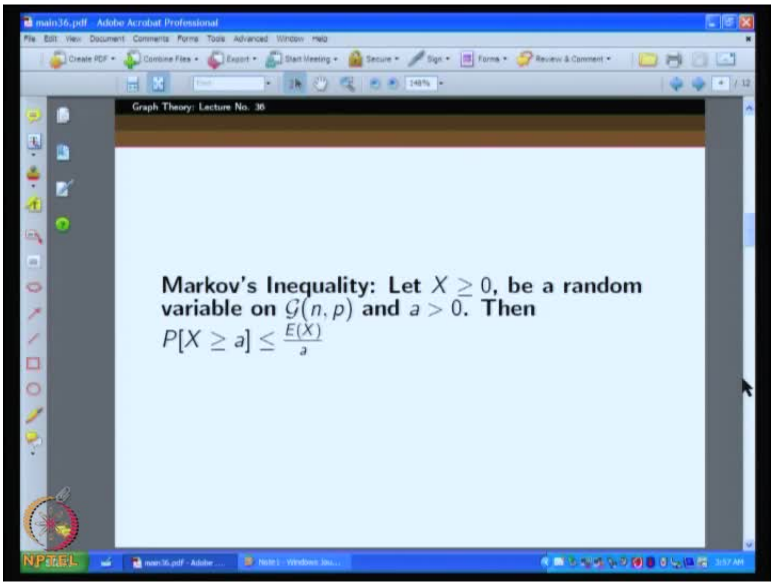

So if you try you can prove it yourself but then, this is very useful property in most of this calculation we will not before incident if you do use this usual definition for this thing may be very difficult but, another hand once you expand like this calculating this and this can be much easier, And then we will be able to calculate this using this, so also this is this property namely the linearity expectations is a very useful property. And now that we have an introduce the expectations we will introduce one inequality related with expectation which becomes very helpful, this is called a the Markov's inequality so the Markov's inequality says; let X be a X greater than equal to 0, be a random variable it need not be in G n P but, let us say it is a positive random variable and then consider a value greater than 0, then the probability that X greater than equal to 0, it is a greater than equal to a, is always less than equal to expectation of X by a, this is the important thing.

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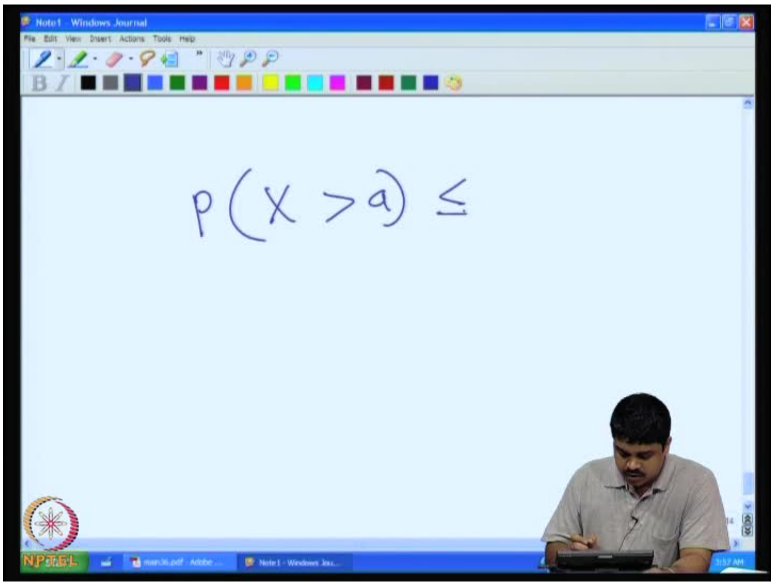

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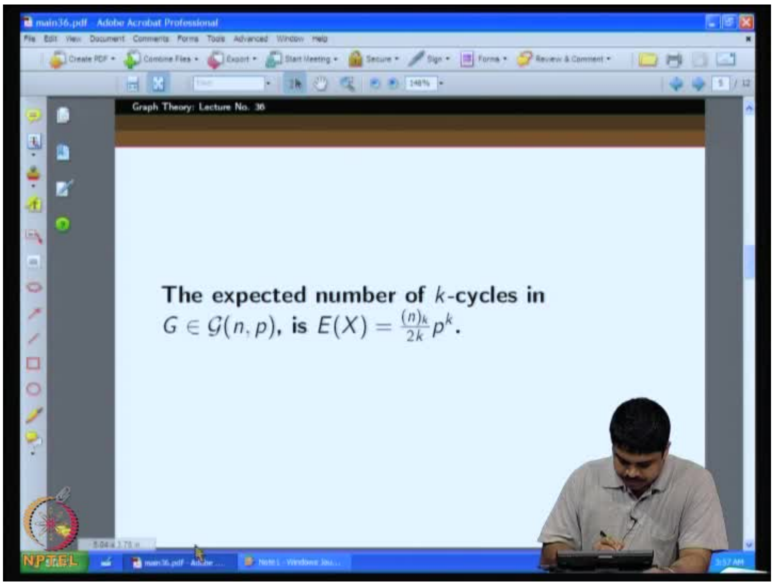

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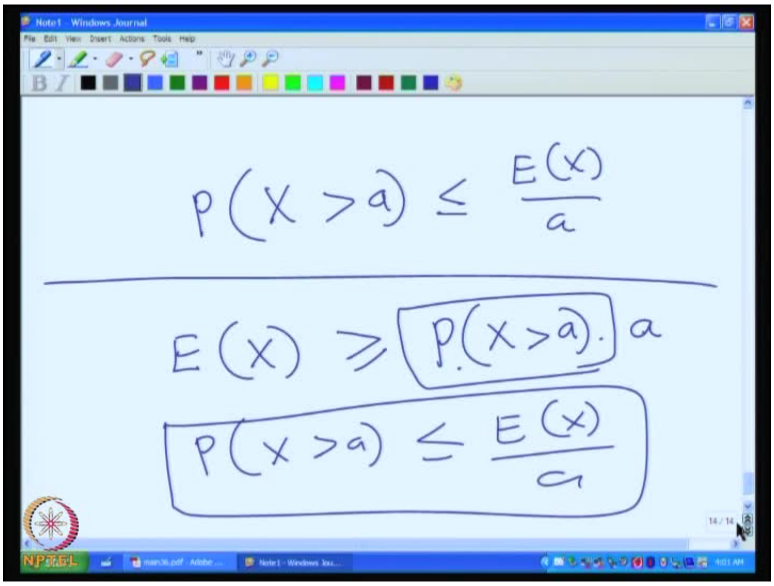

So we are considering random variable it is takes the positive values here, and then we are claiming that the probability, this probability namely probability that the random variable takes value greater than a is less than equal to the expectation of expectation of X by a. expectation of X by a So the proof of this can be easily seen from the definition of the expectation, because this expectation X is essentially you can see that, this expectation is when you sum over all possible outcomes and multiply by the value of the random variable, and then and then and the probability. So we can in fact see this as sigma over the value of the random variable, so the so the probability that, so of case for instance when you when you discard the value with below a, some of them discard it we can easily see that, this is essentially probability of X greater than a, so all the we are consider only the values of X which are greater than or equal to this, and now each of the values the correspond to something greater than a.

So therefore, so this will be greater than or equal to. So what we are going do is the expectation of X, so for each possible value of X we have to multiple with the corresponding probability. What is the probability that X takes this value in to this probability, and then we sum them together? So suppose we drop all the values all the values of this random variable which are less than equal to a, then and consider only those probabilities which are greater than this, so this is the definitely this expectation is definitely greater than equal to probability that X greater than a and into a, because all the vales are at least a here, x is a or more for all of them together the probabilities this much and then I substituting by a here therefore, this will this probability along will be probability of X greater than a, will be less than equal to expectation of X by a. This is what, so this is just from definition of the expatiation and the fact that a is greater than equal to 0, we can get this from there.

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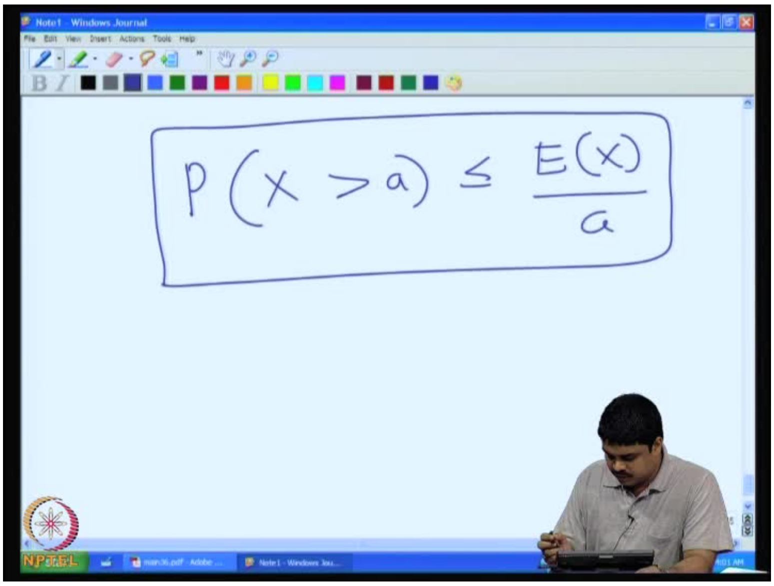

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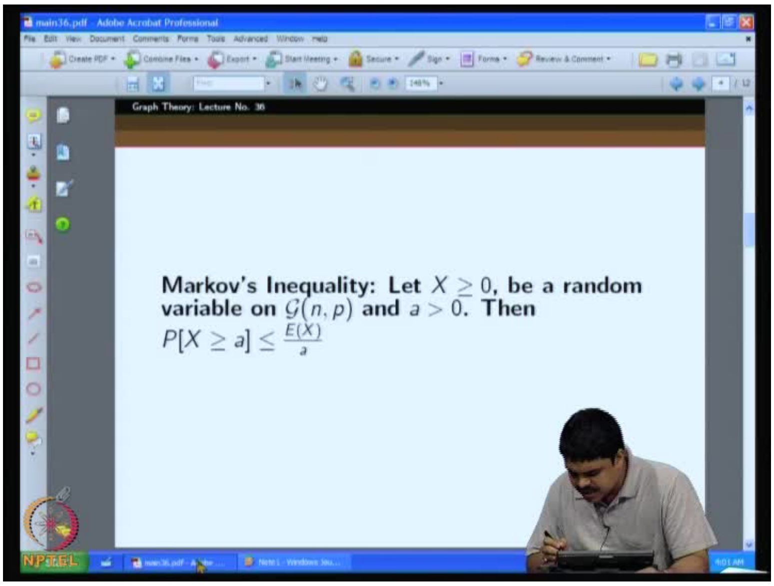

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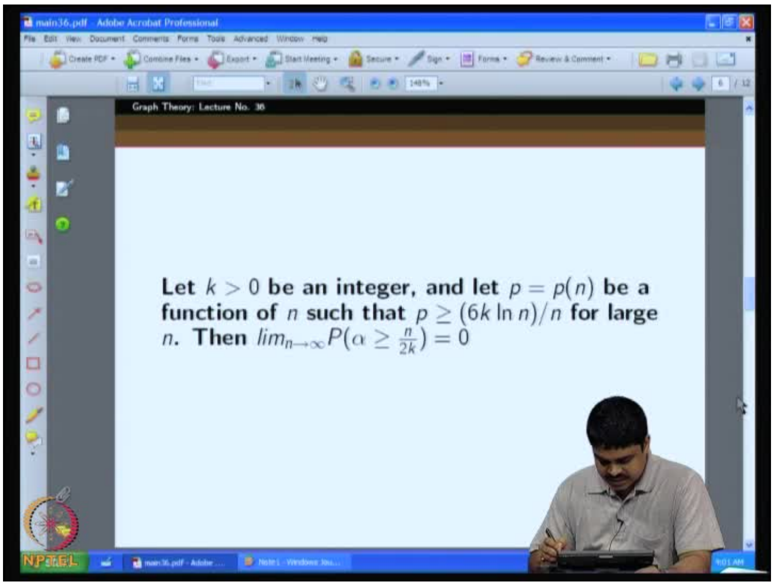

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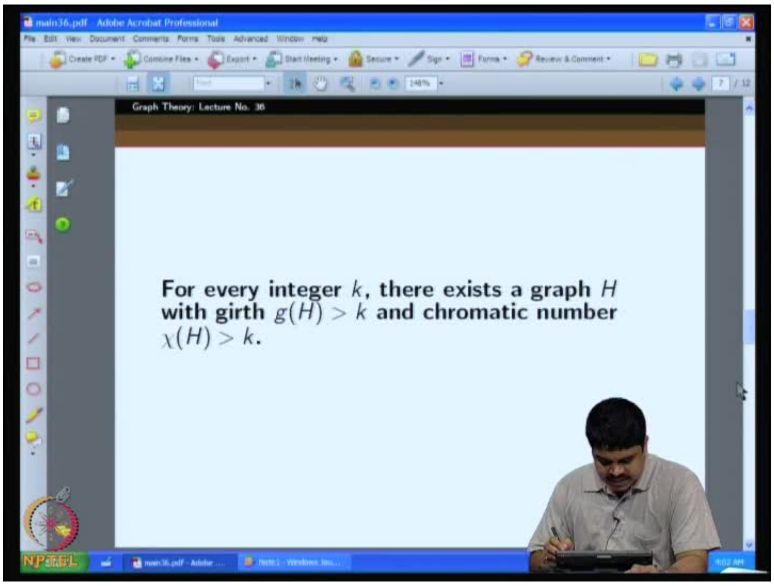

So this Markov's inequality which is a which gives me gives an upper bound for the probability that X greater than equal to a, in terms of expatiation it is quite useful in many situations will be using it. Now the for illustrate all this things, the expectation, the use of expectation, the use of Markov's inequality, what we going do is, we will take one interesting question which was one of the earliest questions in the in this area. Which was proved by (( )). So what is his question? So it consider this question, so do you have consider this question. Suppose we are given an integer k and we are looking for a graph with girth greater than k, girth means what? Grid means the shortest cycle length, the length of the shortest cycle should be greater than k. And also another property is required for the graph namely the chromatic number of the graph is also greater than k, to number. You can always, for instance the question can ask differently can say that want girth greater than k, the chromatic number greater than k but, that is find we can always change by taking the bigger of the two and then ask this question we are the both the both values are same.

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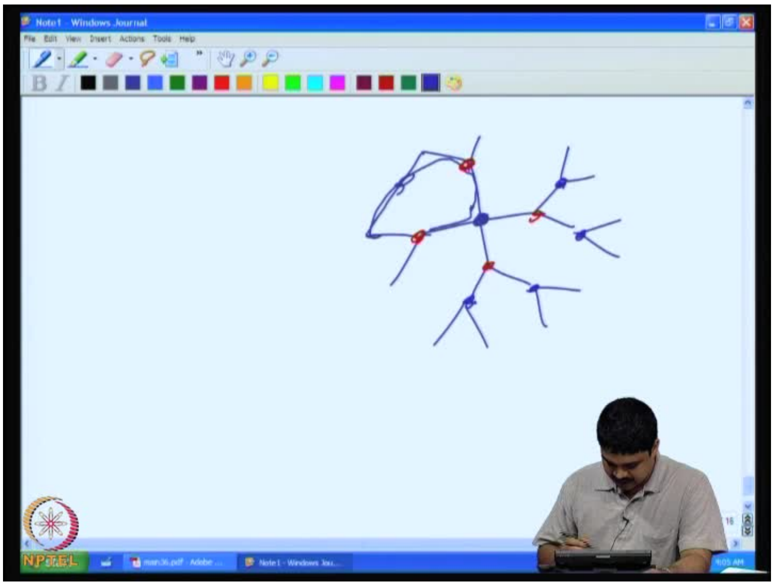

So, I want a graph that the girth is greater than or equal to something in the chromatic number a. Is it possible to find the graph always like that? I am allowing you to make a big, that is not a problem. You can always decide that the number of vertices in the graph is big but, then is it always possible? Why do you think that it may not be possible? Because these are kind of contradictory properties, when I say the girth is greater than k if you look from one vertex, you will see a tree like structure around it, because that is small short cycles so it go some distance and it will look like this, for instance. So if I look from this thing, so you will see a structure like this to some distance until k by 2 they want be any, if you go the if for instance if you see a structure like this that means there is a short cycle here.

So to some distance you would not see any edge blocking the tree. So it means you may look like if you give a color to this thing, you can give a color to this vertices and then you can give the same color here, so it is like a bipartite graph, it is two colorable graph;

locally two colorable graphs. So does not mean the somehow if not two so can I use three color or four color or some constant number of colors, a small number of colors, so where it can be color. Because from looking from one vertex look like there is not much conflict in the vicinity of that vertex, from each vertex it as the case. It gives us the feeling that probably the chromatic number of this thing is high, that is why we ask the question can I keep the chromatic number of the graph also high. I want both this conflict parameter to be high, when the chromatic number is high we may get feeling that the graph is dense and then, because it is dense there may be short cycle add it.

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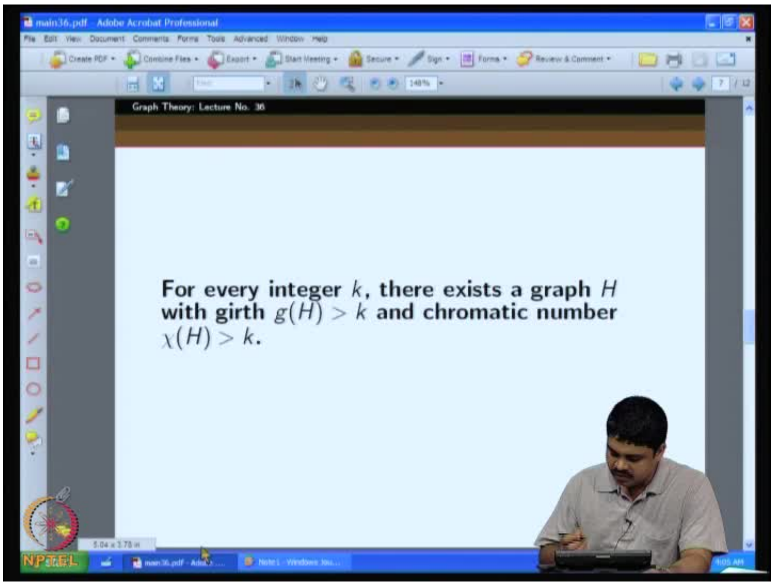

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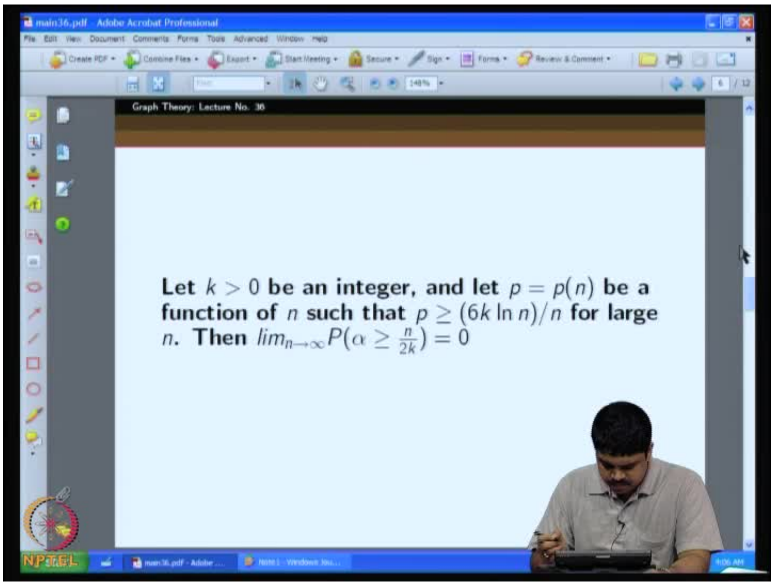

So that is why the other condition is conflicting, that means we want the chromatic number, high and also the girth high there should not be any short cycle in it. So intuitively these are tow conditions which are kind of conflicting with each other. So the this question was not easily answered while it is very easy to or it is not very difficult cook up graph for small values for this parameters but, then when in the arbitrary case when if you if you allow any k, so how do you prove? How do you prove that such graph exists? So (( )) very cleverly use the probabilistic method to prove that such graph exists. Graph exists. This is the next interesting theorem that we want to do. So to prove this thing we be first we will see, how we can use the probabilistic method. See what is the different here from the previous problem? The pervious problem also ask for two properties together, that means we do not want a complete graph on the k vertices in the graph, and also we do not want an independent set on the k vertex in the graph. This we wanted to coexist, both this properties is we want together.

So for that what we did is, we consider the compliment property that is; in a K k in the complete graph complete graph on the vertex, there is a the other event was, there is an independent set of k vertices in the graph. And either this or the other one this evens can happen with probability equal to sum of the individual probabilities at most, then if this some was the less than one then that compliment even may be neither K k is nor S k occurs can happen with certain finite probability. So this so this same technique why does not why cannot we apply in this case? Also here there are also two properties, I want the girth to be high, I want the chromatic number to be high. Let us consider the probability that the girth will be, the probability that the random graph that I have drawn the girth of it will be small, will be (( )) probability for that. And then consider the probability that the independent the chromatic number of it, if at all if I can directly deal with it independent chromatic number chromatic number of this thing also small less than k, and then sum the probability and show that it is less than one.

But it so happens that, if I fix p equal to half this will not be possible but, them why do want to fix P it is equal to half? Try to fix P, the suitable p that is the thing. How can I fix the suitable value of P so that, both this evens will happen together with sum non zero probability? So to do this thing, we to do this thing what we are going to do is to first consider corresponding sum, we have to adjust this probability value P but, it so happens that if p is reduced very much, then the chromatic number willing increase. How do you find the chromatic number is? (( )) so what we, the only handle we have to deal with chromatic number is to show that the biggest independent set in it will be small, because if the biggest independent set and it is small the chromatic number has to be high, because any color glass to be independent set. If the each color glass is small then we need many colors, then we need many colors so we will try to keep the independent set size small but, if the probability P is small for instance if I take P is equal some C by n were C is constant, we can see that there will be a large independent sets but, on the other hand if I keep the probability high, so that I can keep the largest independent set small then, we can see that there is surely we are going to get short cycles.

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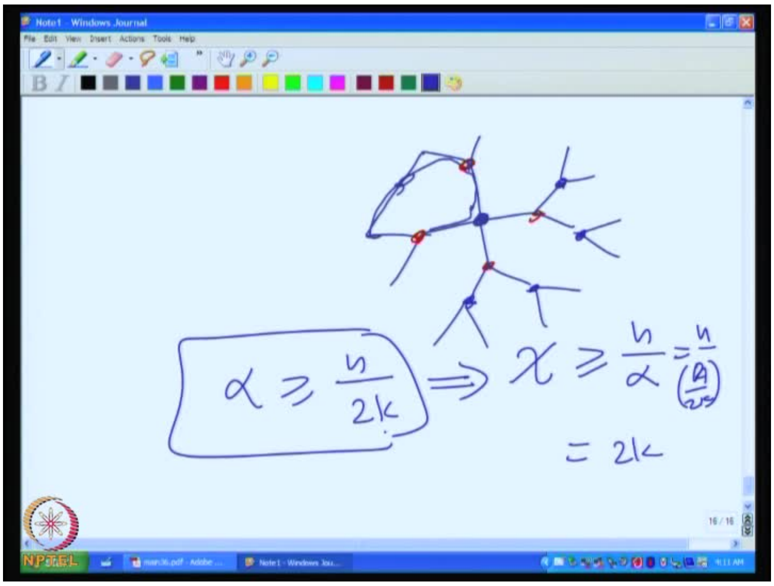

So if we try if you if you work out the calculation of varies piece, we will see that, this such a P will not exit we cannot make it happen together. Both avoiding the cycle, short cycle, and independent sets, so then what will you do? So to get a feel of this thing it so happens that if so when for instance if I want the alpha to be greater than or equal to n by, that the probability that the if I want the probability that biggest independent set size is greater than equal to n by k, should ten to zero suppose if that is what I want, what should be the probability value I should give? So turns out that if I give the probability great P greater than equal to 6 k log n by n, then we can make this happen but, then unfortunately this is already too high for the other event, so why I am trying to make alpha greater than equal to n by 2 k, if anybody is confuse about it this is, because if I proves this, this will immediately imply that chi greater than equal to n by alpha equal to n by n by 2 k n by n by 2 k is equal to 2 k, in fact I do not need 2 k I can by put n by k so even n by 2 k.

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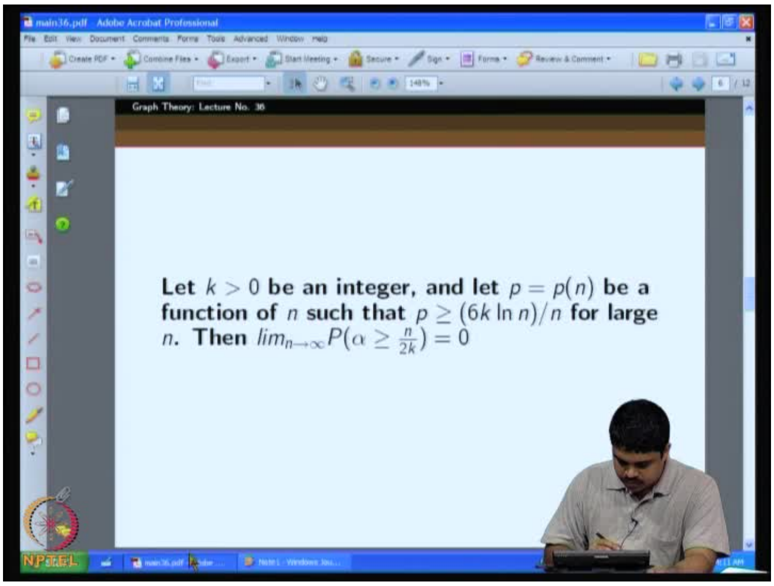

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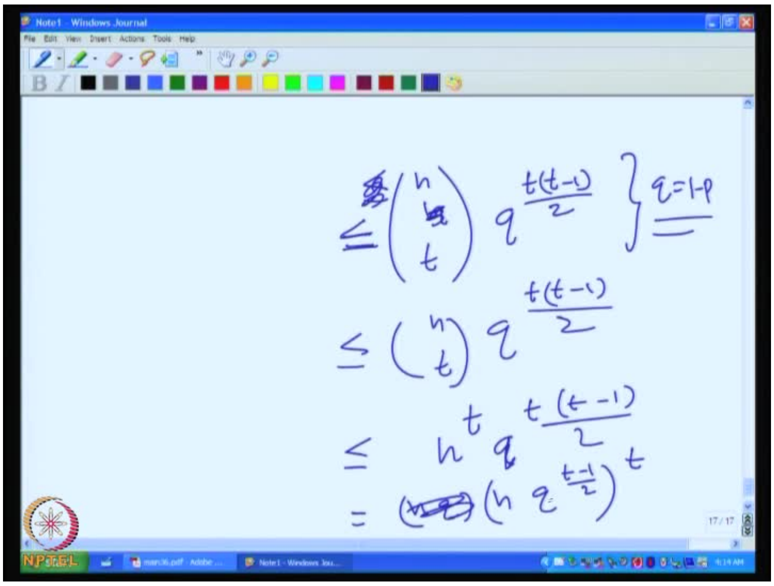

So here, this is what we are going to first prove. But, so to prove this thing, so how do we do that? We have already done certain calculations for that before so the, because the probability that there existent independent set of greater than n by 2 k, that will be let us say greater than equal to some t value, so that will be we know that, this probabilities less than equal to, so we can we can actually calculate it, because so suppose you take the set of size t, lets if I take a set of size t, and what is the probability that there is an independent set so, because we can always say that the all of them so there are t into t minus 1 by 2 edges, so all of them should be missing.

So then the probability q into t minus 1 by t but, then the probability that such an independent set will occur, if at most n choose t times, q to 2 this thing, n choose t times q raise to t into t minus 1 by 2, this is this is the probability. Then we see that, this is the at most n raise to t, so we just discard the n choose t part and this is q into t into t minus 1 by 2. What is q? q being 1 minus, q is equal to 1 minus P here, because P is the probability of an edge to occur, q is equal to 1 minus P is the probability that edge will not occur, this we can write it has n q into n into q raise to 2 minus 1 by 2 whole power t.

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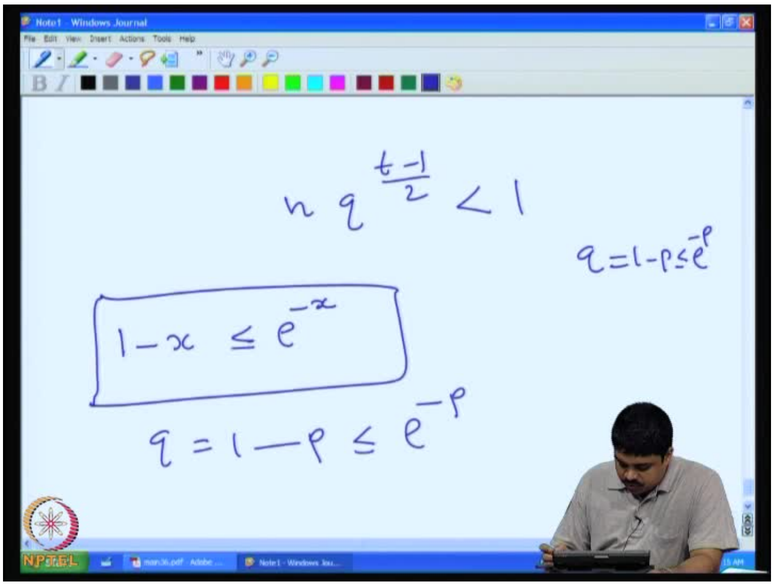

Now we want to know when will this tend to zero, when will this became to zero for large enough (( )) x tends infinite, if I want this probability to tends zero, what should be the value of the q? In fact I am interested in the value p. That is what so It is so happens that if I came take for instance if I want to show that n into q raise to t minus 1 by 2, is less than 1 or I may even show that it is tends to 0, so let us take q is equal to 1 minus P, that is less than equal to e raise to minus p so, because I am using this inequality 1 minus x is less than equal to e raise to minus x, this is the useful inequality which is usually used in the this kind of calculations.

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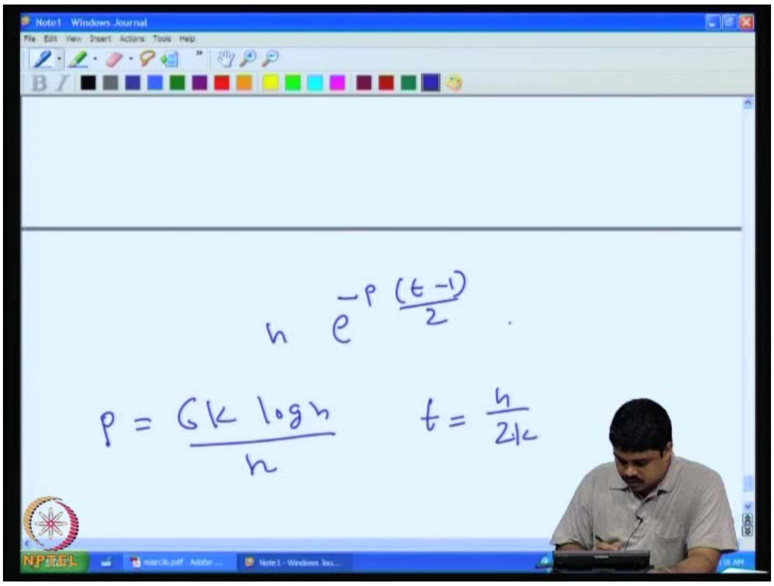

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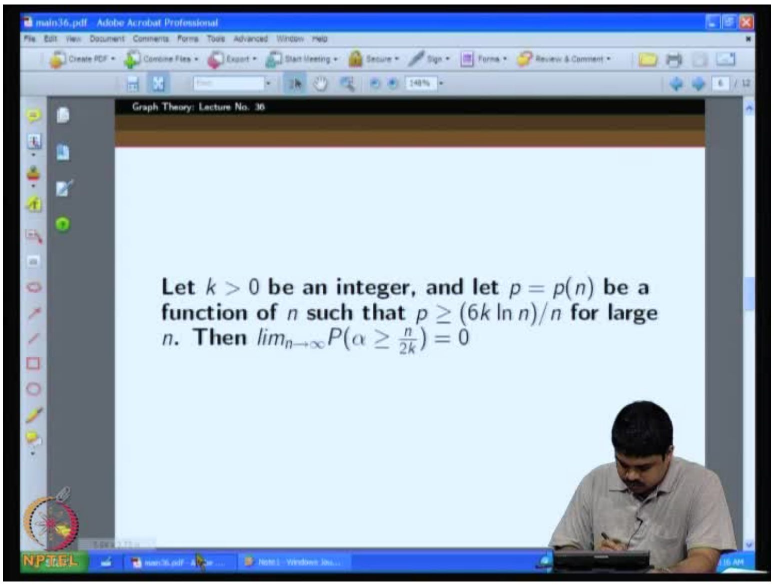

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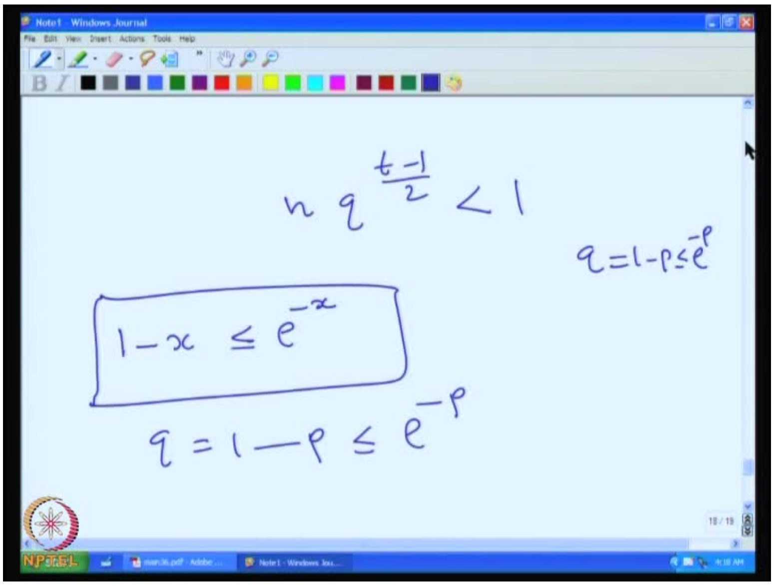

So here q is equal to 1 minus P, that is less than equal to e raise to minus p. So I can substitute here and this thing, so n into e raise to minus P into t minus 1 by 2, t minus 1 by 2 this is what, I want this 2 tends to 0 now, what is the correct value I can put for this so it is so happen that if you take P equal to 6 k log n by n, and t equal to n by 2 k, this will come. We have to take P greater than equal to, any value greater than 6 k log on by n and then so if I want to prove that suppose if I the kind of independent set I was trying to see was seven by 2 k, then we can show that this will tends to this will tends to 0 you have to sub suite here and do the necessary cancelations you can show that, this will be equal to yeah, so we can we can substitute by n into, so if you cancel out we will get n raise to minus 3 by 2 into e raise to P by 2, this will become P, because P suppose one we can put it as root e by root e m and this will definitely it tends to 0 as tends to infinite as tends to finite.

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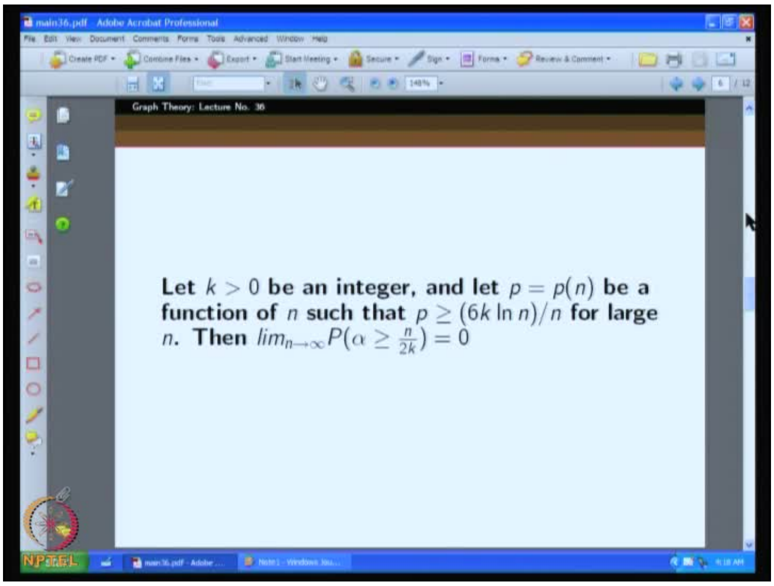

So therefore, so those I have skip from some calculations, so to, because we have we have to finish now so it can it can be easily checked out by the students that when I substitute P equal to this, and t equal to this thing, this will tend to zero. So what we are now told is that if I had selected the value of P to be greater than at 6 k log n, then I am sure with very high probability that we are not going to have an independent set of the cardinality n by 2 k. That means by chromatic number is going to be at least 2 k but, now what we have verify that is that true that the girth also will be high, that means I want be a short cycle but, this will not happen with this value. So we will have to make sure that the girth is high, we have to find a different category that is what will describe in the next class. Since we do not have time in this class we will describe in the next class.

Thank you