Vehicle Dynamics
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Lecture – 11
Tire Brush Model

In the last class, we were looking at how the tires behave during longitudinal run. And of course, we started lateral aspect but we will come to lateral in a minute or may be after sometime. We will look at the longitudinal force development through a very simple and very nice model called the brush model.

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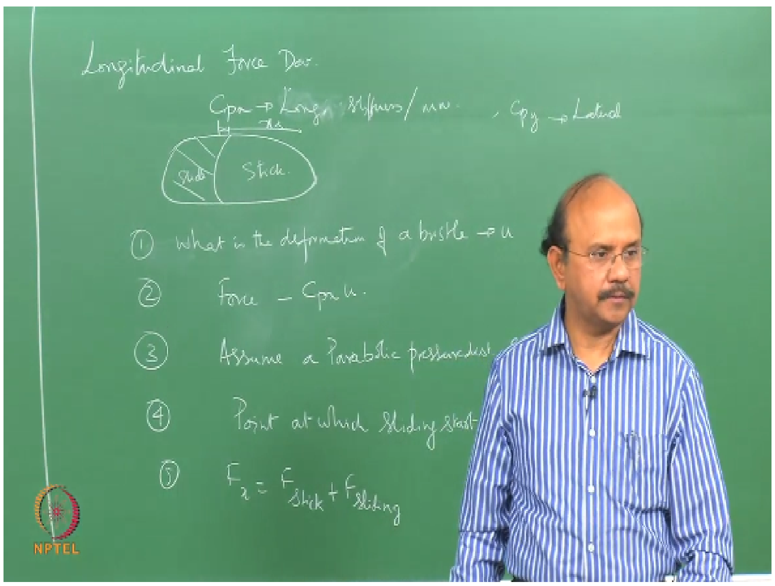

The brush model is very nicely explained by Prof. Pacejka in his classic text, Tire and vehicle dynamics which is gone I think, it is a third edition. It is a very nice book, mathematically very rigorous. It would be very nice if you can have a look at many of the mathematical models that are developed in this book on tire mechanics. Though we will not be able to go into those models in this course and we will be talking about very simple models.

Because we have to move into vehicle dynamics and discuss lot more things on vehicle dynamics and discuss lot more things on vehicle dynamics. Tire mechanics being extremely important, I am going slow and we are looking at how actually the forces are developed and so on. There was a question just before the class whether we will have some mathematical model in this course?

Yes, we are going to do a simple mathematical model which is going to be called or which is called as the brush model. But they are very complex models that are available for the force development in tires, of course I will give you a very broad picture in the next class about what all tire models are available for modeling longitudinal as well the lateral forces. But this model brings out the physics of entire behaviour in the longitudinal direction.

One of the things that we show in the last class is that there is a pressure, I am just summarizing what we did. There is a contact pressure under this tire and there is a longitudinal force. In another words, contact pressure can be converted in force and so on but the total f to z. So, there is a contact pressure and there is a longitudinal force due to which there is a shear. Now, we also show the difference between breaking interaction.

So, we said that there is a region where the tire is going to stick to the ground and there is a region after which when the shear stresses exceed the frictional mu*fz. The normal force multiplied by mu or pressure multiplied by mu then we said that there would be a slip. In another words, if this were to be the contact patch then there is a region where tire is going to stick to the ground.

This is the region where it is going to stick to the ground and this is the region where it is going to slide. And I said slide remember that these guys are going to just come in a bit, till again equilibrium as reached and so on. So, in another words a maximum force that could be withstood is obviously in a simple model of Coulomb's model mu*fz. What is our goal? Our goal is to find out how Fx has developed mathematically express it and what is it related to?

In passing, we define what is called as slip and we will see how this slip actually enters into the mathematically model and what is the relationship between fx, the force developed versus the slip. This is the whole idea. So, how do we do this? Simple, by means of what is called as a brush model. These are the bristles which are sticking outside for example from the carcass of the tier.

And they have a stiffness lets us say lateral stiffness, say lateral stiffness per unit length which we would call as Cpx which is the lateral stiffness per m number unit length. So, what we are going to do? This derivation is that I am going to find out what is the deformation. So, the first step is what is the deformation of this of bristle or (()) (06:11) of bristle which I would call as u. So, once I know what is the deformation the next thing I am going to do is to find out or multiply this to find out the force which is nothing but the Cpx*u.

The third thing I am going to do is to assume a pressure distribution because one is left hand side and another is right hand side of the equation. So, I will assume pressure distribution which in this case we are assuming to be parabolic. Assume a parabolic pressure distribution and call it as qz. The next step, I am going to write down the equations but I want to give a broader picture. The next step is obviously, what is the next step?

So, once I do it I have the left hand side of the equation, what is this force? The longitudinal force and this is the right hand side of the equation I can find out mu*qz. I can find out the left hand side and the right hand side and then what do I do? I find out point at which the slip starts or sliding starts carefully use the word slip because we are going to give a very specific meaning for slip. So, in another words let us call that xs.

"Professor - student conversation starts" Yes, this is a Cpx, the longitudinal stiffness. Good question. Please understand that this bristles have 2 stiffnesses. You know, what is stiffness? Stiffness is here in this direction and stiffness can also be in the direction which is perpendicular to the plain of the board. So, which you would call as the lateral stiffness so which is Cpy. So, Cpy is the lateral stiffness. Sorry, longitudinal stiffness.

So, Cpy is the lateral stiffness I did not noticed that sorry. Though usually it is assumed that Cpx = Cpy it is not so there is a 50% difference between the 2 and so on. Good. I am sorry. I hope it is okay. **"Professor - student conversation ends"**. So, the point at which sliding starts then as a last step what I am going to do is to find out the total force that is in x direction which is the sum of the forces that is generated in the sticking region + the sliding region. Sticking region + F. So, F sticking + F sliding, all right. So, that is the total force.

This is how our derivation is going to progress. The derivation is going to be exactly the same when we look at the lateral forces. We will come to that the same similar model is going to be used in lateral forces. But I am not going to derive this in detail. People who are interested in this derivation which is going to be exactly the same can refer to Pacejka. Actually, he has derived the lateral force and said that the similar (()) (11:00) I am going to do that other way about so that you will have both the derivation with you.

I am going to derive the longitudinal force and then say the lateral force is going to be very similar. Of course, we will discuss the physics of that a bit later. Just before we proceed it a word of you know a comment that though we are discussing longitudinal and lateral separately there are conditions under which you will have both lateral as well as longitudinal forces develop. So, in another word when you are cornering and breaking for example then there is a combined lateral and longitudinal deformation.

And hence there is a force which is both fx and fy. We will mention this as we go along and again we will not be able to derive that completely but at least we will just see this like this and see what are the final equations? So, with this background let us go ahead. So, it is always better that before we derive, better that we write down what are the steps we are going to do? And maths just follows Physics.

Now, I am going to make some assumptions because as I have told you this is a simple model. So, there are a number of assumptions which we are going to make to keep the maths within our bounds. This is a figure from Pacejka we redrawn it in order to understand how the deformations are going to take place. Let us assume that there is a point S.

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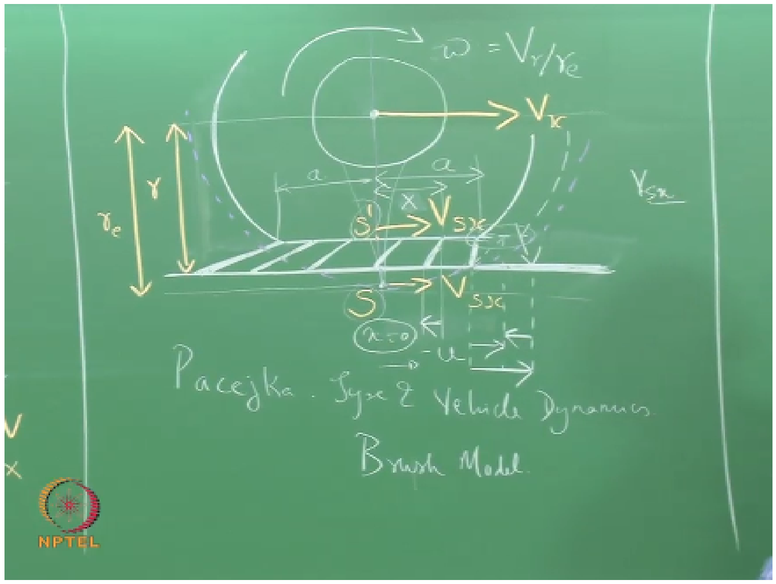

This is a point which lies on the rolling radius r which is such that it rolls on an imaginary circle without any slip on an imaginary ground like that. So, in free rolling, the velocity of S in one direction this direction which is omega*r e = the Vx so it does not slip, point number one. On another hand, when there is breaking or when there is acceleration the velocity of S is different from Vx we show that.

And hence there will be a movement of Vs or there will be a velocity of Vs which we would call as Vsx. We will see what this definition is in a minute Vsx depending upon breaking or accelerating it would be one direction or the other. We will use the same thing like the V which is in this direction for the tire can be assumed to be V in the opposite direction for the road and that is rotating at a stationary position.

Let us assume that there lies a point there are slight assumptions here because you may be confused as to about the r. One of the assumptions we are going to make is that these radiuses are so large when compared to the lengths of these bristles and so on that we are going to write down this in terms of radiuses which we have defined. In another words, we are going to make some assumptions on the radiuses to make again things simple.

Let us say that S prime it is a physical point. It is a point in the tire which moves with the same velocity as that of S. We assume that this bristles that are sticking out though we have discrete bristles actually you can assume that they are continuous but then continuous bristles do not give that physic and hence you can do that and later we will integrate it we will make it continuous. So, these bristles have a base.

2 things one which is sticking to the, you can call as the tail which is sticking to the carcass. On the other which is sticking to the ground. So, there is a difference between the 2 what is sticking to the ground is going to move with the velocity of the ground. And what is sticking to the bases is going to move with the velocity of the tire the tangential velocity of the tire since these 2 are going to be different in breaking and that is what we are going to see now.

There is going to be a deformation of this bristles that is what we said first that we are going to look at the deformation of the bristles. So, the first thing first so let us look at what is Vsx.

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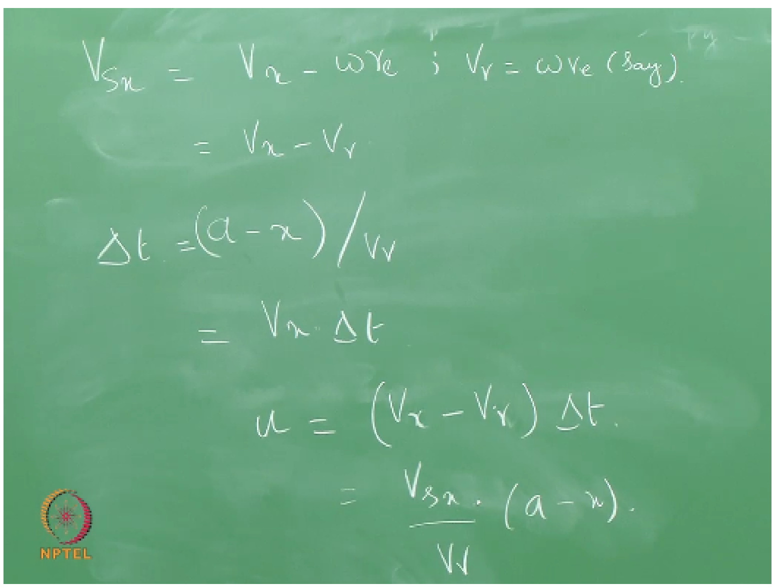

So, Vsx is as I told you is the difference between the velocity that is we will write that as Vx, velocity of the ground - omega*re and that is where s is sitting which we can call as r so let us say Vr equal to omega re say so that Vsx is Vx-Vr. **"Professor - student conversation starts”** Any questions? What is the difference between slip stage? Slip is a very technical quantity which we defined in the last class. **"Professor - student conversation ends”**

We are going to define it again. English word slip means it is slipping. In tire mechanics slip has a very specific definition. So, though you can say that slip and slide are synonymous. So, you can use the word interchangeably. In tire mechanics, slip is a definition. I am going to define that again in today's class. Sliding is the word which we use where when the force exceeds the mu*f it slides, it moves, all right.

So, do not get confused between the 2 I know this is a common confusion. So, I am going to define slip now just wait for a minute. Now, the first thing is that I want to know how this tail has moved? What is that it moved? So, in order to do that let me introduce a coordinate system with this as the bases sorry this as zero x=0, positive in this direction. I know this is a very busy picture we will understand this picture slowly.

So, x=0 and this is the positive x direction and that this contact look at that this contact batch +a and –a so that is again a on either side of zero this is +a and –a. Now, the first thing is that since these points here are moving with S prime which is moving with the velocity of Vr the motion of the base point let us to a point which is x from this point which is x from here. In another words that the base point travels a distance of a-x at a time which is =delta t.

The base point is traveling with the velocity Vr and hence the base point moves with the velocity sorry at the time a-x*Vr. At this time, this is the base point moves this is say for example this has moved from here to here. At the same time, the tip of that bristle or the head of that bristle that has moved a distance of at the same time the distance moved by the head = Vx*delta t, all right. So, at the same time the distance moved is Vx*delta t.

So, you can say that the distance moved from here to here suppose I say that here to here this distance or let me put it like this. Okay, this distance is V, this is a-x and this distance = Vx*delta t. So, because simple there is not even a figure is required that the base is moving at the velocity Vr and hence that is the time that is required for it to move and the head is moving with the velocity Vx and so the distance moved = Vx*delta t. All right any questions?

Simple. So, what is the deformation? The deformation of this bristles at one hand it has moved this on the other hand that is the distance so the deformation is nothing but the difference between the 2 so that you can write down the deformation u to be Vr-Vx* delta t. **"Professor - student conversation starts"** The point is that at which velocity is it moving? It is the same as S, S prime velocity same and Vsx. So, the question may be is it sitting at Re?

That is why I said initially that we are going to make some assumptions. We are going to say that r is so large that there is small difference okay does not matter and that is how we are defining it. **"Professor - student conversation ends"**. Substituting the expressions so I can write that to be Vsx, what is Vsx? Is nothing but Vr-Vx or Vx-Vr because it is going to be you know just a difference so let me define that as Vx-Vr or sorry this can be Vx-Vr.

So that is actually the deformation that is = Vsx*delta t and delta t is a-x/Vr (()) (23:34) a-x. So, we are considering breaking you have to careful correctly we will write. So, Vsx*Vr a-x. Since, the deformation is in my coordinate system is positive along this direction deformation is actually in the negative direction so I have to write the deformation to be -. I am going to define 2 slip quantities.

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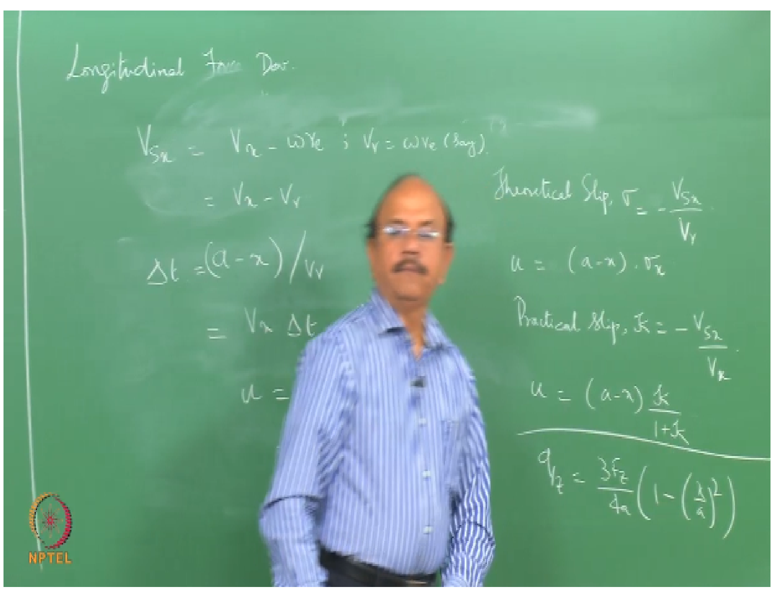

One of the slip quantities called theoretical slip sigma term given by Pacejka that it is a theoretical slip is defined as this difference Vsx/Vr. So, that u can be written as –a-x* sigma x.

We also define a quantity called practical slip kappa which is used by the industry, usually people in the tire industry talk about kappa which is what we would call as the practical slip is given by –Vsx/Vx.

This we will also put - we use the same quantity that are +, -es there are so many definitions. I stick to that definition. So, that you U can now we written as sorry I will remove that I will put a - there. Can be written as in terms of kappa as well you can find out the relationship between sigma and kappa from this and can be written as a-x kappa*1+kappa. So, the first step is over so we have found out the deformation.

I hope my signs are correct because one more thing that I am deriving, breaking if there is any sign difference. Hopefully it is correct and I will correct it later. My next step is of course finding out the force we will come to that in a minute. My next step is to assume here a distribution for the normal forces. I said that we will assume a parabolic distribution so that qz very common distribution that is assumed all through = –it should be in terms of total fz and in terms of a.

So, we will write that as 3 Fz/4a*1 - what should be the quantity there. It should be you can take a guess. x/a whole square so that when x=a or –a the term in the bracket goes to 0 and so q is close to 0. So, it is maximum when x=0. So, let me now get to qx so these 2 are fine.

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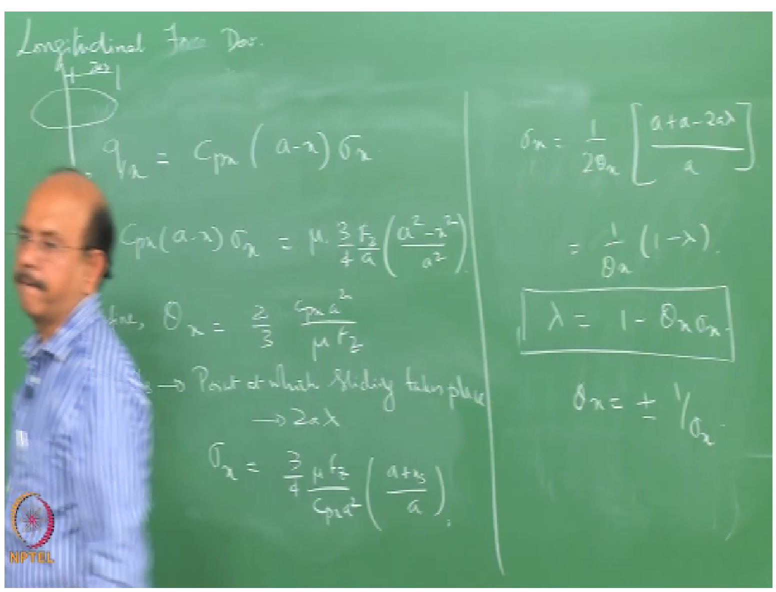

Qx is quite simple so what is the way we write it. So qx is what? I know the deformation multiplied with stiffness Cpx*a-x * sigma x or kappa/1+kappa. So, the point which I was trying to say is that this is what we define a slip that is why you have to be careful and use the word slip in tire mechanics. So, the next step for us is to find out when does the slip happen? Is it slip? When does the sliding happen?

"Professor - student conversation starts" What do you think? Yes. So, mu n. Now, q is the time now. qx for unit length so qz so it should be equal to qx when it is less mu qz there would not be any slip, sliding and exceeds that will be there. So, I will write that condition = mu*qz. Let us define a quantity called theta just to make things easier for us to write let me define a quantity called theta, theta x because you define similar one for, why are you writing?

This one? That is the assumption I make as to how contact pressure is distributed. You know the contact pressure is like this. Why? Basically because when I integrate this expression I should get Faz integrated from –a+a. I have to get like this. This expression, clear? **"Professor - student conversation ends"**. So, that is the expression. Let us define the theta x to be 2/3 Cpx a square mu*Fz. Let me simplify this expression now. Can you do that?

Write a-a square – ax square* a+x*a-x substitute for theta and get me a value of relationship between theta and sigma x. So, the first thing, let me - there is nothing, great, let us say that x is the point at which sliding takes place. Let me also call this as 2a lambda following Pacejka. It is a fraction of 2a, 2a is the total length of the contact patch, 2a lambda is what we show as xs.

Now, let us take this expression and write down this expression substitute for theta and write down what is xs. So, you just do that and it is very simple 3/4 mu fz divided by Cpx a square*a+x axis/a. What I did was just this is a+x and da-x. My goal is to find out when total slip takes place. When is the total slip take place? Lambda*1 that makes my job easy. Now substitute for x axis to be 2a lambda and that is the point at which this lambda.

This is the contact patch that is what we called as 2a delta so that lambda =1 = 2a. But in terms of our x not this carefully there is a difference. Because we are looking at x from that point from the center, this is at x=0. So, in another words this point is x=+a and that point is x= -a. So, let us substitute that in this expression x axis is a-2a lambda that is what is x in another words that is x.

That is a - 2a lambda and hence I would write sigma x to be 1/2 theta x*a+a-2a lambda divided by a which I simplify. I am not going to do every simplification you can do that is why I am writing it and hence lambda the point at which it starts is –theta x sigma x. So, lambda =—I said that lambda is =1 where the complete, the sliding takes place to a this is completely the sliding.

So, you can write that so theta x to be + or – depending upon. So, let us now find out the force Fx. As I told you before the force Fx is the force developed in the sticking region + the force developed in the sliding region.

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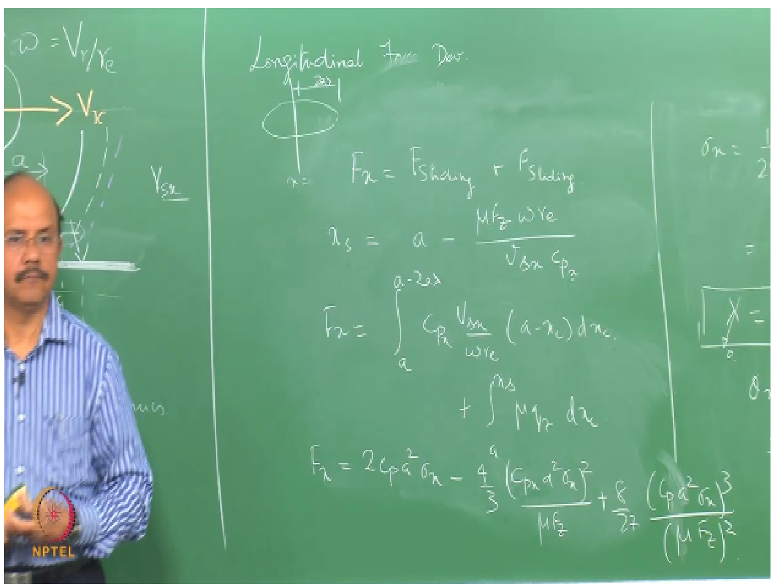

So, I can write down xs from my previous expressions whatever I have done to be very simply you can see that fz omega re divided by v sx*Cpx. Very simple whatever I have done just substitute that. So, fx can be written as a to a-2a lambda sorry lambda = not 1, 0. Okay, the lambda should be equal to 0 for this that is why you get this. Yes, we awake. Lambda = 0 is where complete sliding takes place, not lambda = 1.

So, this is why you get theta x sigma x =1, from here that is the point from this side, sliding this moves like that. What is that I did? Simple, Cpx sigma a–xc dxc + this one and x axis I have got from this expression that is all. All right, integrate this and I am giving you a final expression. I am not going to do the integration it is not a very difficult integration. So, you would notice that this is the expression all right.

You would notice that I can draw Fx now in terms of sigma. Now, If I now plot Fx versus sigma my graph would look something like this.

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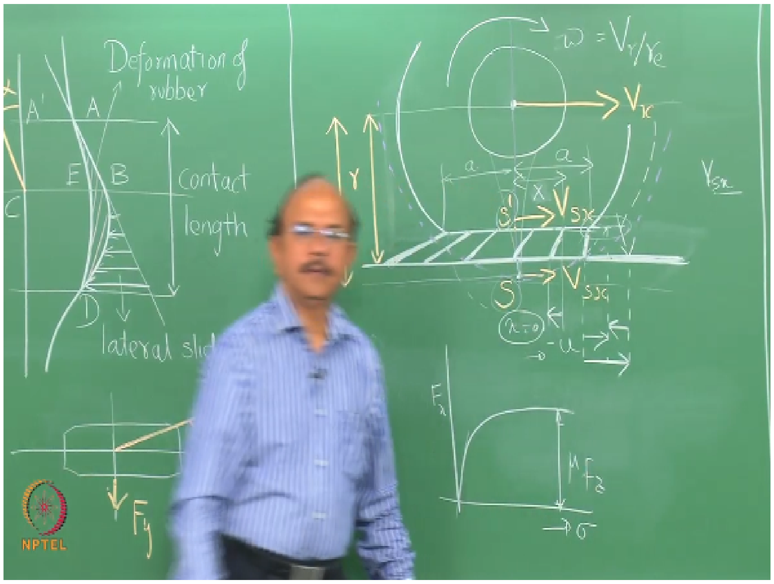

This is the sigma versus Fx. For very small slip sigma you can assume it to be = Kappa, first term gets into picture. One of the quantities of interest to us we will see that later is always the slope here which can be approximated by if you look at kappa it is approximated to be 2Cp*a square. And what is this quantity? That is the maximum force and the maximum force would always you need not even look at any of these things maximum force would always be equal to mu*Fz.

So, this is would be mu*Fz. So, essentially this is the brush model where we find out the deformations. We found out where the sliding takes place, the place where the sliding happens and we just found out the force due to just deformation and due to sliding and then we wrote down a complete final expression. Look at this. This is slightly different from what is given in Pacejka as an expression but I think this is correct have a look at it.

Now, practically this may not be a straight line and that is because we have assumed mu to be a constant that is going to be heating, and there is going to be dynamic friction and so on and usually the mu here as a constant may not be valid and you would have a drop here. We would talk about that again in the next class but before that let us now look at the lateral deformations. Any question here?

When we equated (()) (44:18) The slip condition at which the slip happens that is what we said, the condition under which the slip happens is = –this is the condition where both of them are happening = the qx per unit length, force developed by unit length = mu*qez. This multiplied by dx that qz is what I have written down there that is per unit length. So, my*qz =qx develop when both of them are the same then would be slip.

So, this may happen say for example at this point. If it is completely sticking this is how it would happen. It would look like when for example here is where that axis where sliding is going to happen then this guy would actually start slipping, sliding rather. He is going to slide. So, this is how it is going to slide. Sir, actually qz, (()) (45:38) qz is the normal. See what I am using is nothing but mu*f or normal force very well know = that is all I am using.

The only thing I am using it per unit length. This is actually an inequality. There is not equality. This is an inequality. So, whenever F is greater it would not slide that is all whenever it is equal it will slide. Sir, we used the same derivation for both sticking and sliding rate. No, sticking is a portion which is before when it reaches what we said is when it reaches mu multiplied by this quantity qz when it is = qx it slides that is the point by which we found out lambda or sx.

Clear, so when they are equal so in another words if this is going to be the contact patch, sorry about my diagram, then we said that this is where it is sticking and this is what we defined as 2a lambda or this is what we defined as xs which is = a-2a lambda.

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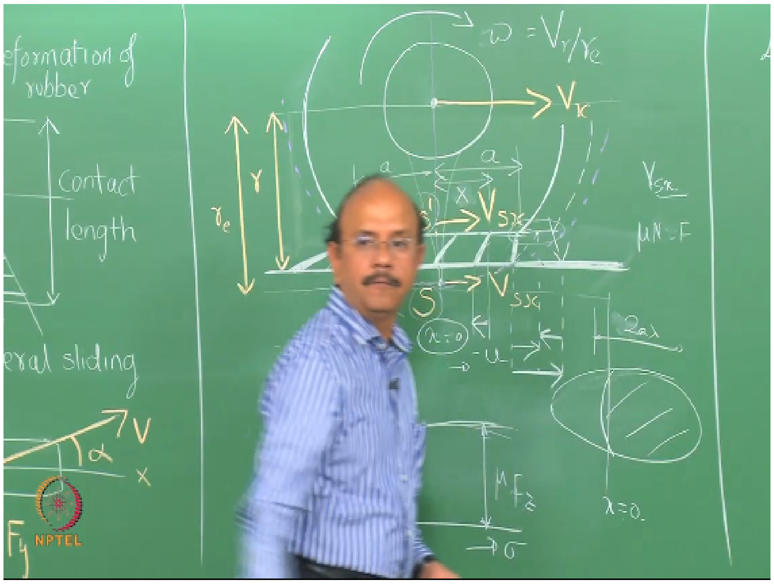

Sorry, I should not say sx, 2a lambda and this is the x =0 and then this distance is what is called as the sx. So, we equated it to find out that point at which sliding takes place. So, at this point we assumed that the force is given by mu qz per unit length dx*force per an infinite length dx then we integrated it throughout. Actually, it is x*integral xs-a which is, all right. Strictly speaking I should write it like this. Xs to -a. Any questions? Clear. Okay.

Now, let us look at quickly the lateral force development.

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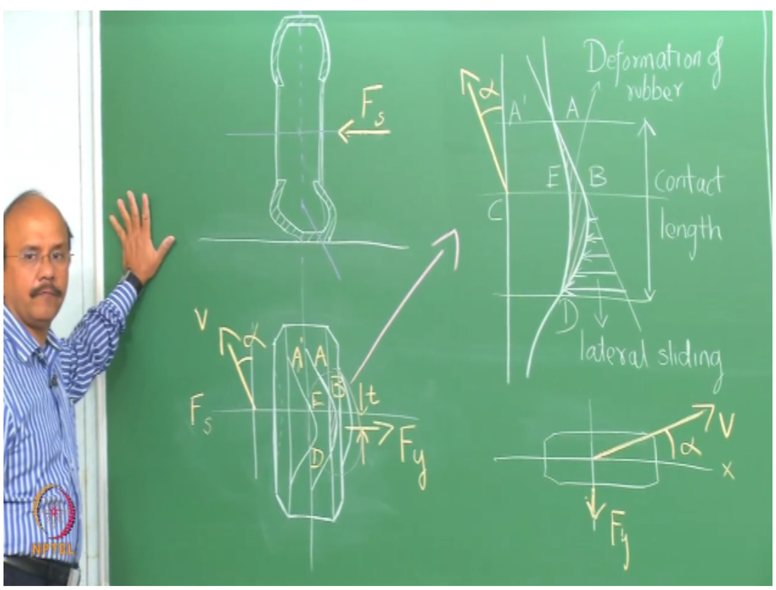

I think we have to get back to this figure again in the next class. Let us do that in the next class. Apoorva has drawn it so beautifully, I feel bad that I have to rub this and we will do that in the next class.