Vehicle Dynamics
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Lecture - 12
Lateral Force Generation

Okay, let us start with the Lateral Dynamics. I thank, Apoorva who was done all these you know drawings on the board, so she is my TA and thanks a lot for taking a time off to draw this, she has done it very beautiful. So we will look at how lateral forces are developed now, remember what we did? We are going to extend that and we are not going to derive again lateral force separately.

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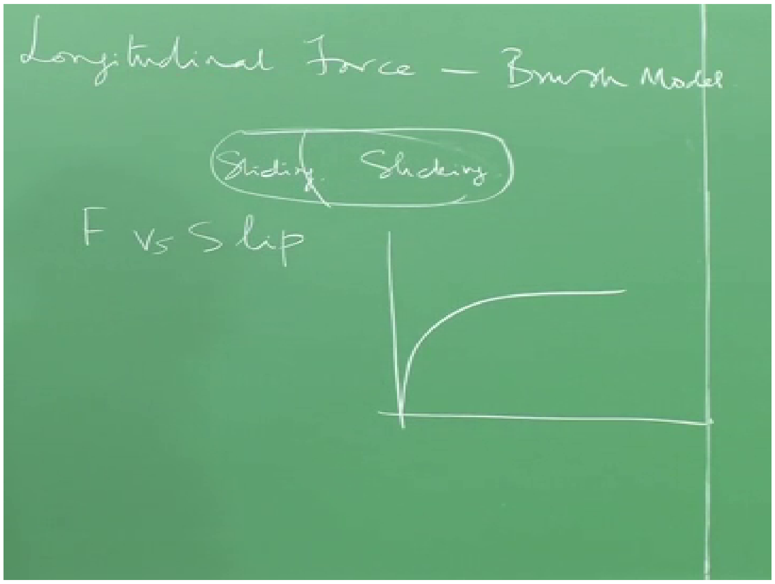

We looked at in the last class just to summarize what we did, we looked at longitudinal force development and we did that using what is called as the Brush model. We recognized that the contact patch is divided into 2 areas and we said that if this happens to be the contact patch, then you have what is called as the sticking region and the sliding region, we developed the quantity called slip it came naturally in our definitions okay.

And ultimately we saw that the force the Fx the longitudinal force is a function of slip and it happens to be something like this okay, now we have to come back to this portion we will do that a bit later okay, we got it straight mu*qz, the question is whether this is going to be a straight line or not, but I do not want to waste time she has done drawn it again let us go back to the or go to the lateral force development okay.

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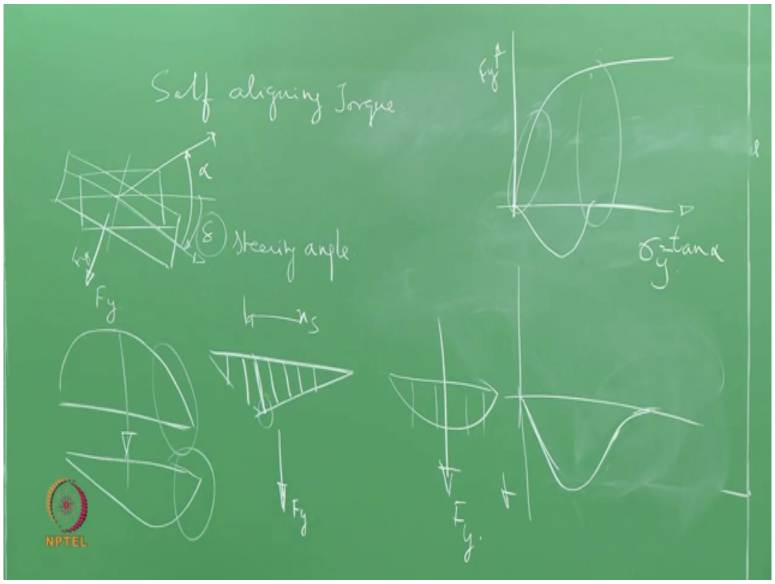

Now what do we mean by lateral force development, slip angle and restoring torque, pneumatic trail these are the terms that we are going to look at in this lecture okay. Now let us say that this is the picture of a single wheel, let us say that the vehicle takes a turn okay, so you give a steering angle when I say steering angle its referred to the wheel let us say that delta is the steering angle of course it is not the angle with which you steer the vehicle that there would be a ratio between the 2 so I call that as the steering angle right.

Now so we are going to view it from this point of view okay, this how I have steered it okay so that is what we are going to view.

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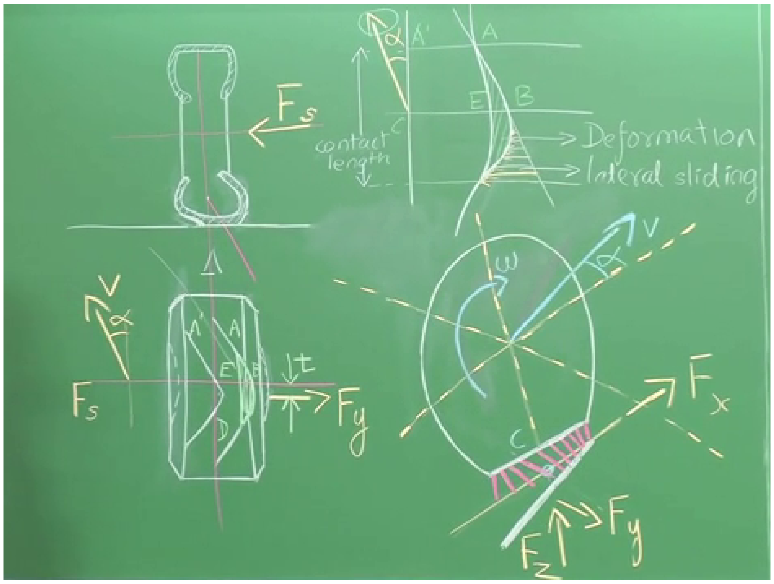

So in other words this one what this is the plan view or you would view it from the bottom of the tire and that is this direction okay, we are going to see what this direction is or in other words this direction is okay, we are going to give a name to that a slip angle, so we will see what it is but before that get the global picture that when we talk about slip angle it is the angle from the steered angle.

So what we are going to do is to sit here after you are given that steering input and then observing what is happening at that point of time clear have that coordinates clear right, so what is that required when you take to cornering we have already seen this I need a centripetal force okay in order to accommodate the centrifugal or centripetal acceleration whose D'Alembert equivalent is centrifugal force and the centrifugal acceleration okay.

So I need one Fy clear, now let us see how Fy is developed a very quiet complex much more complex than what happens in the longitudinal case okay. Now imagine that this whole tire is rotating right rotating, now what is that I need? I need a force here that is the force I need and that is the force which is pushing it out which is the what we call within quotes D'Alembert force or centrifugal force right.

So in other words this becomes the sticking region okay, the vehicle the tire sticks to the ground like before and that is the force which is now going to push the wheel out, this is the force which is going to push wheel out right, so that is the D'Alembert equivalent force right, so both these forces now have to compensate. So how is this force developed similar to what we did in the longitudinal case.

Now I am going to have these kind of sticking elements okay and as this wheel is pushed out in the y direction, these sticking elements are going to be stretched they are going to be stretched, and this stretch producers a tension which is going to pull the wheel inside. So in other words this stretching in the y direction of this tread elements what we called as tread elements of the bristles or the brush model that gives a push in the y direction and the result is that I get the centripetal force.

But then things are not just that all of them are going out and things are not as simple as just getting up pull, so as you roll now look at this as you roll because this plane is now getting pushed out, so as you roll you hit the ground note this carefully as you roll you hit the ground at this place okay at one place, now that place is ahead of this sticking place, so this is now rotating the corresponding point is here right.

Because of the fact that this sticking makes it not possible for that to roll in one plane okay, the contact patch gets a very skewed appearance, so what happens is that this point comes in so this point later gets to this point that point later gets to that point and so on, because it has to stick and when it comes out it has to go back to the plane to which this wheel has been pushed right. So it is not one straight line so it goes like that and comes out like this clear.

So that is the shape of the contact patch when cornering takes place that is the first thing I want you to understand. **"Professor-student conversation starts"**. Any questions. Sir, can we say it is the trace of the base of the tread? Yes, exactly, so it is a tread so what is this? This is nothing but if you sit in a tread okay with respect to the ground okay first you will move you will come down like that then you will hit the ground at that point right.

So then the next guy is going to because there is going to be a pull you know you are going to be here the guy who is in front of you will go and hit that point and so on, the same fashion as the contact patch moved above you in that case here also the contact patch is going to move above you, but the contact patch is not a straight patch but it is inclined because I have to scattered to the needs of 2 planes.

One plane sticking to the ground or one part sticking to the ground, the other is that the wheel is going out so it comes like this and goes like that clear okay that is how I can do it fine. Any questions. Sir, bottom part cover we did not disconnect sir actually when we should turns (()) (09:28) we hit the ground so when next guy comes and hit it pulls like this direction. Yes, so what happens here is look at this diagram.

Initially, when it was rolling straight or if it were to roll straight this whole plane would have been like that right okay, so now because of this force the plane gets like this right clear okay, so that is what is manifested as this twist in more simple terms with this twist clear, when you get back you have to get back to that plane, you have to when you are rolling you are in one plane you are sticking you are in another plane okay, so you have to come from here to here okay.

And then get back and go to this plane that is what is seen here clear, so that is why this skewed appearance **"Professor-student conversation ends"**. What is the result? The result is that the tire just does not roll like that the velocity there are 2 velocities now because there is a lateral velocity and there is a longitudinal velocity with the resultant velocity is not like this but is like this, which is equal to an angle which would call as alpha.

So the wheel now moves actually in the resultant velocity direction which is alpha to the plane this plane which I would allowed the tire to move, so I am not going to move and I do not want to move because if it moves like that all my pulling does not exist, so I am not going to get the lateral force. **"Professor-student conversation starts"**. Sir what is t there? What is t? We will come to that in a minute right. **"Professor-student conversation ends"**.

Now go to the next step this creates a obviously a shear force which has to be compensated by mu*n, or in other words this guy who is sticking he is pulling these are the bristles which are pulling okay if that friction the same old guy which we saw in the longitudinal direction if that friction is overcome what will happen? So it will slide in right same concept nothing new right. Now let us see what happens let us just put that what happens during the sliding? You get the normal view okay.

Let us say that okay that is those are the deformations okay, you know now by now you know that there is a race mu*n the normal force as well as the frictional force right we know that right. So let us say that at a particular x which we would call as a sliding this is overcome, so there is a normal force you know that and there is a tangential force, obviously, as I move look at that as I move towards the rear of the contact patch the pull is going to increase, the shear force is going to increase so I am going to go okay.

Then later I have to go back to that original plane, so my pull is going to decrease in there so I have something like that right okay. Now let us say that up to that point the tangential force is able to hold, so it will be sticking after that point it is going to now slide at that point there will be sliding right okay. So in other words after x S let us say that x S then it is going to slide, in other words the development of Fy is not symmetry, how do I get Fy? Integrate this right.

And in other words it is the sum total of this pulling and obviously this pulling is not symmetric about the center, the pulling is less in the front and the pulling is more as I come down right, so this is Fy if this where to be the contact patch, if this is the center of the contact patch okay, where does the Fy act? It will not obviously can it act here will it act here unless it is symmetric it is going to act at that point right, it is going to be shifted right.

So where will it be shifted to the rear and that is what we have here, that because of this unsymmetric distribution I have an Fy which is not symmetric which is at the center, in other words in this diagram I cannot draw it exactly the at center, if this where to be the center so that Fy will be displaced clear, so that is what happens here, and that distance what is the effect of that distance exactly so it would create a moment, what does that moment do?

That moment rotates the tire and aligns it back to the straight running position okay, hence, this is called as the self-aligning torque or moment, so we call that as self-aligning torque or aligning moment whatever you want to call. **“Professor-student conversation starts”**. Sir, after that straight line instead we have parabola there will be any contact with the ground? No, it is see please note that it is not that it is not in contact with the ground.

Because its contact patch is this, it is in contact even if it slips or slides rather that is the correct word as I said even if it is slide it is not that it is away it slides and still it is in contact with the ground. It has some pulling force. Yes of course okay that is the maximum force which is mu*qz correct, so it is though the figure looks like that it is a 3 dimensional figure and it is not that it has lost contact right, this whole thing is the ground clear okay. **“Professor-student conversation ends”**.

Now please note that it is both these guys are you know reducing towards the end and this race between 2 reducing quantities, so it start slipping, as it starts slipping at one point of time okay, it my so happened that the slipping may engulf and that would be the force distribution of all this bristles, and so Fy will be uniform when sliding completely sliding takes place okay of every bristle that is there, then every bristle will have that q*mu*qz and then it will be completely symmetric right okay.

So now if I plot this graph that will be my Fy and what do I plot it against? What is the x is? It should be slip angle which I would call as sigma y, I am not going to derive this, this is exactly the same the deformations are exactly the same derive that, so I just replace it I am going to derive a combined slip in which case you will understand what is going to happen okay, so that is what happens.

Now, if I plot now, My so it is a usual practice to plot as -My for the simple reason that if this is positive this is positive okay, x, y, z should be inside the ground. **“Professor-student conversation starts.”** Yes. Sir (()) (20:19) 2 types of bristle some bristles are sticking and then they will moving what is the basical difference between sticking bristle and sliding bristle, because even the sticking bristle is moving right?

No is sticking that is exactly what we said it is only deforming, it is not moving this is not motion this is only sticking here and it is not moving okay, it is just the head is here this is exactly how we developed in the last class, the longitudinal force okay, so this is the guy who is sitting in the car caused moving this guy is here is sticking okay, so that is what the difference between that is what caused the force clear okay, go back and look at last class we did exactly that and that is what it is. **"Professor-student conversation ends."**

So we will come back here and usually it is the minus sign that becomes I meant that is what is used because z is in this direction but actually the M is in that direction so this being positive that being negative so usually it is plotted as like this, so it should be something like this so where should it come to 0, and I think it is better that we plot here where should it come to 0 full sliding, so when you reach this point that should be 0 right.

So that is the region where sticking, few sliding and complete sliding okay fine. We will now look at a small simple mathematical model to understand the derivation for Fy, Mz as well as Fx. In other words, let us look at a situation where we are going to have both longitudinal force as well as the lateral force, in other words you are braking while cornering a common thing that would happen braking and cornering.

"Professor-student conversation starts." Yes. When the tire starts sliding we lose the control over the vehicle? No, we are not right now when I say sliding a part of it is sliding, during (()) (23:16) okay yes that is in other works in this region itself there is some parts which are sliding okay here is where it is full sliding okay, now what is loss of control? That is the question okay. **"Professor-student conversation ends."**

We will get back to this question what do you mean by loss of control? In other words, what happens in the in what we are going to call as the unstable region okay. In order to explain that we have to start with the Fx, I will just make a passing remark now, but we will come back to this later in the course maybe I mean after couple of days okay we will come back to this. And let us just go back to Fx I thought I will do that later.

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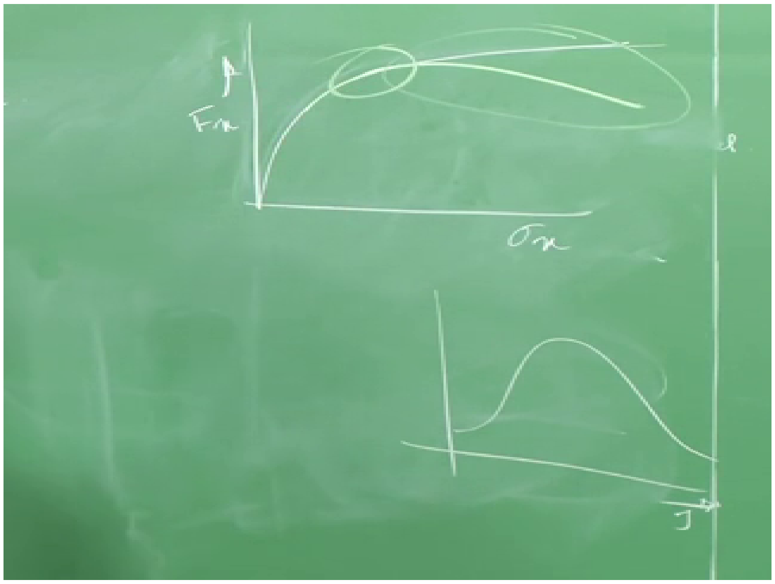

Because I do not want you to leave with this doubt, so let us do that now so if I now plot actually sigma Fx which we had done okay using that brush model remember that we got a graph nice straight line like that, in practice in actuality in reality this is not straight line like that and it is not a flat line rather maybe a straight line but it is not a flat line, because of the temperatures because of the fact that mu is not a constant as it slides.

When it slides the temperature are very high flash temperatures let us called as the temperatures can reach hundreds of degrees, so the reason paper say that it can reach up to 200, 250, 300 degrees okay. So when there is the temperature increase this mu is no more a constant, we just said that mu*Fz is this okay that is no more a constant because temperatures play a major role right.

Remember our first graph with viscoelastic material behaviour we had 3 regions okay that this region with high temperatures we plotted that temperatures at higher temperatures where the viscoelastic properties drops or hysteresis drops and so the friction coefficient which is a function of these hysteresis would again be affected when we are in that region and that is what happens when I move towards the region that is what happens in here after I reach the peak.

With the result that my graph is not going to be a nice graph like that but would go like this and start coming down, so here what happens the wheel has not come to a halt but what happens what do you mean by when it starts coming down. So in other words this is an unstable region the larger you brake you know this force is going to come down, and that region where increase in sigma results in loss in Fy okay it is the unstable region and warrants the application of abs okay Anti-Lock Braking System right.

So if I have to brake I would like to brake here that region rather than going to this region, so the best way is to get back to this curve okay so that my braking force will increase that is the braking force Fx okay rather than stay in a region where the braking forces going down fast clear okay that is what happens in a FOS well okay. Some physics before we take this take onto this derivation.

As I told you before that there is going to be a race between these 2 quantities, now we are going to remove this, so this is the slipping this is where you know there is a sliding region that is the sliding and these are the deformations and that is the sliding region, so very nicely brought out in this picture thanks to more, friction of Pneumatic tires we have a very good picture of what really happens okay.

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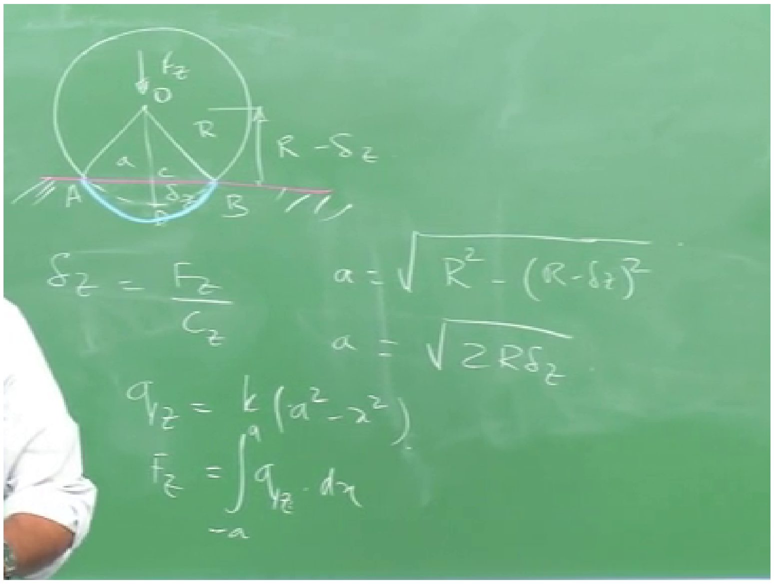

Now let us start with combined lateral or combined braking and cornering let me call that as combined braking and cornering okay, before we proceed to this derivation there are some questions on how we can find out that a remember that we had a, what is that a? 2a was the contact patch, the question on how do I get A? How do I get qz? This was some other questions at the end of the class very simple.

Let us say that, that is the tire and let us say that, that is the ground in other words that is the tire which is being deformed. So in other words if that is the center of the tire that is the R I think I used R right I hope I used that R, let me call that as A, B okay call that C and call this point as D, so let me call that DC as the deformation in the z direction or perpendicular okay, so that is the ground how do I calculate that?

A very simple expression delta z= the force Fz or Fz that is acting divided by the stiffness okay let us say that stiffness in the z direction is Cz right okay, then what is a? so what is this now R-delta z, so a= root of R square -R-delta z square okay, which you can expand it leave out delta z square term and you can see a=root of 2R delta z, this is a very I would say an approximation okay nice approximation and that is what you can use if you want to use a.

And the other thing is how do we get qz? Right that was the another question what is this qz? okay we got a very nice expression for this how do we get this very simple again we assume let us say that, that is a pressure distribution parabolic pressure distribution so write down an expression for the pressure distribution K* write sum K that is I want to find out how do I find that out qz*dx integrate this from -a to +a, substitute that here do the integration and you will get the expression 3/4 Fz 3 Fz/4a*1-x/a whole square okay this is what we got.

"Professor-student conversation starts." Sir here you assumed as symmetric parabola. I fully understand I agree with you that when rolling tire you just said that you this is not okay very symmetric distribution as we make an assumption here in other words rolling distance is not taking into account in this okay. **"Professor-student conversation ends."** Precisely for this reason we have moved slightly away from all these brush models.

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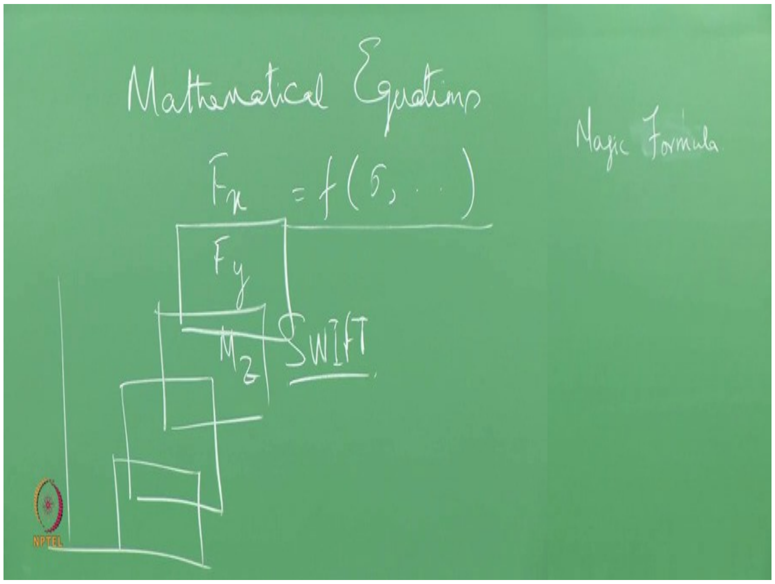

So I will just since you asked this question we have to talk about models I will come back to this combined braking and cornering in a minute, but we have to talk about what are called as tire models. These brush models I mean they have been extended they have been beaten to death with some very smart analysis but still it is a long way to go okay to get the correct what happened exactly in the tire.

It is not very easy because of the complexities that are involved, you are going to see more and more complex I am going to spend some time on tires because this is one of the most important topics in Vehicle Dynamics and it is slightly a difficult topic to understand this is the reason why we are going to spend some more time the rest of them before we go to lateral dynamics the rest of them will not as difficult as this topic.

Because here we have to get the physics and get a hang of this mathematics, so this is the reason why we are spending some more time okay. Tire models are extremely popular I know some of you would have used packages like Carsen or maybe even Adams that is which has a number of tire models, tire model is nothing but an equation a mathematical equation I will devote a class or 2 for different tire models that are available okay they are mathematical equations.

So model means of mathematical equation which gives as Fx, Fy, Mz which we just now saw okay as a function of say sigma and other quantities okay which we would see, there will be some stiffness and other things CP and so on right, so it is just an equation which gives Fy. These equations can be derived by various fashions can be experimentally determined as well, what we saw in brush model like very nicely pointed out that what happened to all those things that you said you know suddenly you say it is a parabolic distribution okay.

So the brush model is a simplified model but it brings out all the physics you know if you want to understand it, it is a very good model because you now know friction coefficient how it affects, you know there is sliding, there is sticking and all that. So it brings out all the physics but it is a very fundamental model okay, it agrees to a certain extent does not agree there are been a lot of things that is happened okay so that is the fundamental model.

So if I said this is the model that is the brush model but people are not happy with that model, so they go to what is called as an empirical or semi-empirical models, in other words they express this as an equation, we will see that typically what is called as magic formula models we will see that in the next class, we will see that by first put forward by Pacejka it is sometimes called Pacejka tire model.

And magic formula model is the next level where you write down an equation and simply do a nice curve fitting of experiments that are done, so it is a semi empirical model and in other words you do an experiment I have a curve and do a curve fitting and then express this as an equation for Fx, Fy and Mz and so on. So happens that it is called magic formula because this is one equation where the structure of the equation is same whether it is Fx or Fy or Fz.

So that is the next level of formulas right, but then these models have again some limitations, so as that tire speed increases okay and you want to look at ride okay the roads which are not very nice and smooth and you have all the undulations on the road okay and so on, then I have to move to a different level of models. I would like to now look at the dynamic behaviour of the belt I want to know actually how the belt vibrates.

That would give rise to noise that would give rise to vibrations and so on. So I would go to next level of models which also would take into account road, road profile the envelopment characteristics of the tire and so on maybe up to a certain frequency, so you have in that category so this is the second category and that is the third category even more sophisticated, as we go up our models have becoming more and more sophisticated again we are capturing this with an equation.

So at the next level you have what is called as short wavelength intermediate frequency trial models or swift models a lot of work done by Pacejka at Delft University you know we are having we have a number of models like this, of course we will talk about other models bird's-eye view of other models, there are other models as well okay. Now lastly we have models which are based on say finite elements okay which brings out all the physics okay carefully done brings out all the physics that exist in tan mechanics.

So the finite element of tires is actually I would say is a very sophisticated finite element analysis it has all the shall we say difficulties or challenges, because the material that we are going to use is geometrically non-linear, materially non-linear in sense that they are hyperelastic, viscoelastic at least if you want to look at hyperelasticity it is a hyperelastic material, and you have what is called as geometric non-linearity because this strains are extremely high okay.

You have contact you have reinforcements, we saw that there are belts and radial plies, so the model becomes very complex but because of the fact that even contact is taken care of well and even the friction coefficient need not be a constant it will change it with respect to pressure as well as the sliding velocities and so on okay, the model becomes as sophisticated as you would like it to be of course you have got to do lot of more work for it.

You can predict temperatures and if you want do look at mu as a function of temperatures that is also possible okay, so these are the models that are available in this sophistication okay. There are other interesting things okay let me before we get in 2 more comments before we get into this before we started this and I would like to start it in a new class because this derivation is going to be involved in a lot more things I have to say maybe we will start in a new class or whatever I want to say to the end of this class and the next class I want to say it now right okay.

Now this is fine you know I have an alpha I have that alpha and that alpha gives me deformation and so there is an Fy that is generated right, the question you may ask is I am sorry at the end of the class you are going to ask, I am taking a turn okay this you say when I take a turn say a constant radius okay I am driving a constant steering patterns it is called I take a constant radius okay around this pad I agree with you whatever is happening.

On the other hand I am entering a curve or maneuvering you know so what happens in other words Fy does not exist you are talking about after the development of Fy, so there is a region where from 0 to Fy I require Fy but Fy is not immediately realized I turn it, then there is a time at which because after all this is also a viscoelastic material, so I have the time lag before this Fy is developed okay.

In other words, there is a transient region before which I get into this steady state Fy okay, so that transient region has its own mechanics where okay you develop theories on the force development overtime and it is usually said that it takes about half a revolution or more slightly more in order that the force is developed and it is characterized by what is called as the relaxation length okay right.

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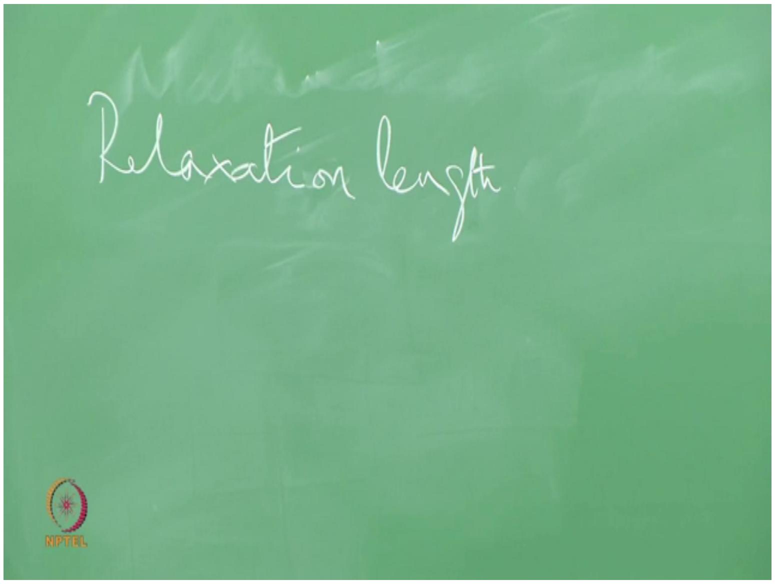

Relaxation length

If time permits, we will talk more about relaxation length.

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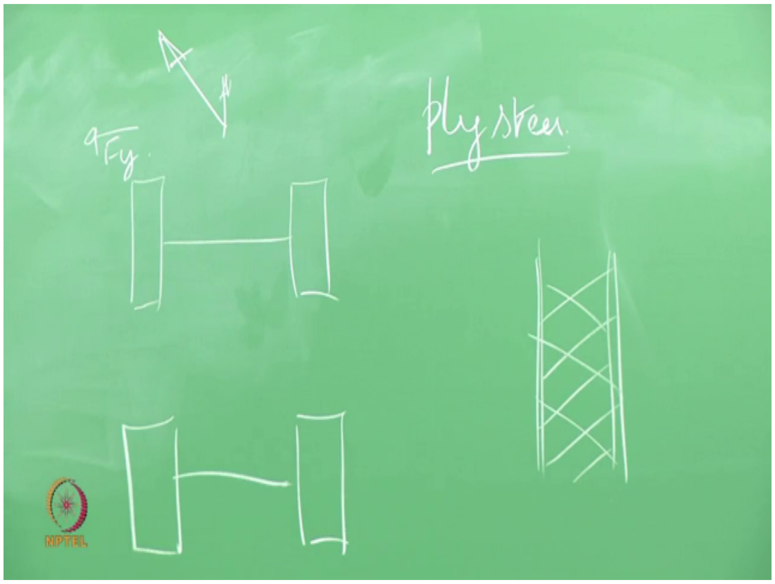

The other very interesting phenomenon is the development of this Fy, now Fy development as we had seen is when the car takes a turn okay, strictly speaking then if I have a car, let us say that is my car I do not want to take much time and drawing it, so it is going straight okay let us say I have a beautiful road to go okay and I leave the steering would the car straight or the car go like that, forget about even an actual experiment.

I do not want you to do this experiment okay do not do this experiment with your cars to this experiment in one of the numerical code say for example Adams you would see that the car does not go straight, in other words even at when there is no camber okay and no steering given that would be a lateral force that is generated okay, this is what is called as ply steer we will be talk about 2 things ply steer and conicity.

But we will first talk about ply steer, this is a very involved concept so let us talk about first ply steer okay. So in other words even in straight run here we have just delinked it, I have an Fy because I am taking a turn I have an Fx because I am braking okay, and both are as I said are modeled using or written as a mathematical equation using tire models fine, but is that all what happens is it that can I get an Fy by just going straight?

You will and that is exactly what is called as ply steer well I get I know you are all surprised that always tire mechanics has lot of surprises that is why this whole topic is extremely interesting, this is Engineering by itself okay anything in engineering you say this is a small roles here in tires, why does this happen? That is because of an effect on composite laminates deformation which is very different from that of the deformation of Steel.

For example, if these belts what we are talking about you know this belts these guys who these are the belts we saw that the guys who run around okay, I said that if I look at it from the top it look like this, I said there are steel cords that are running like this the more than one belt and then there is another belt where replace it on top and there will be steel cords running like that and so on okay, so in other words that makes tire composite.

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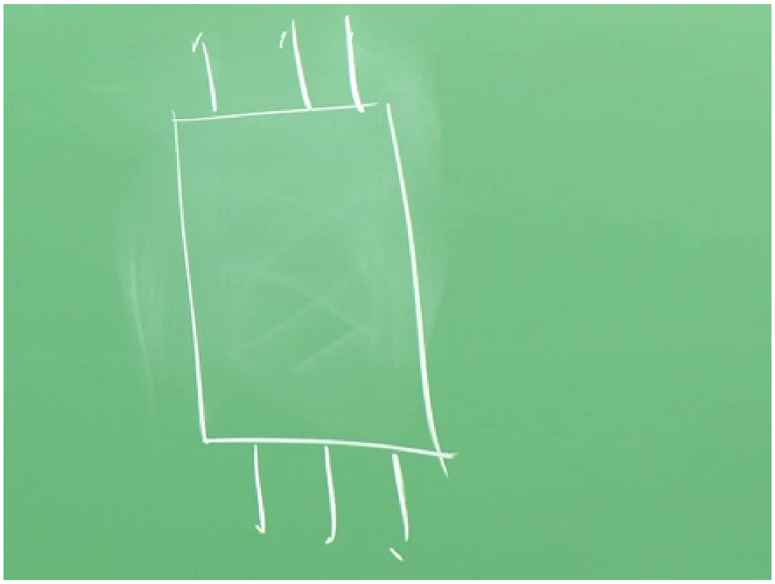

Now let us forget for a moment that it is a composite material, and let us say that is made up of complete steel okay, now this is rubber embedded sorry steel embedded in rubber, let us say that it is steel just steel sheet, obviously when you pull this just pull it what happens there is no twist okay just pull there is thinning which is called the Poisson effect straight forward. What if this material is a composite laminate?

Then the behaviour is not as simple as this, we will see that in the next class what is the behaviour and because of which what happens to the force development and why the car is not going to go straight and so on okay. We will finish that and then we will come back to the combined braking and cornering division okay, we will meet in the next class.