Vehicle Dynamics
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Lecture - 13
Ply Steer and Conicity (Part 1)

So in the last class, we were looking at the Lateral Force Generation.

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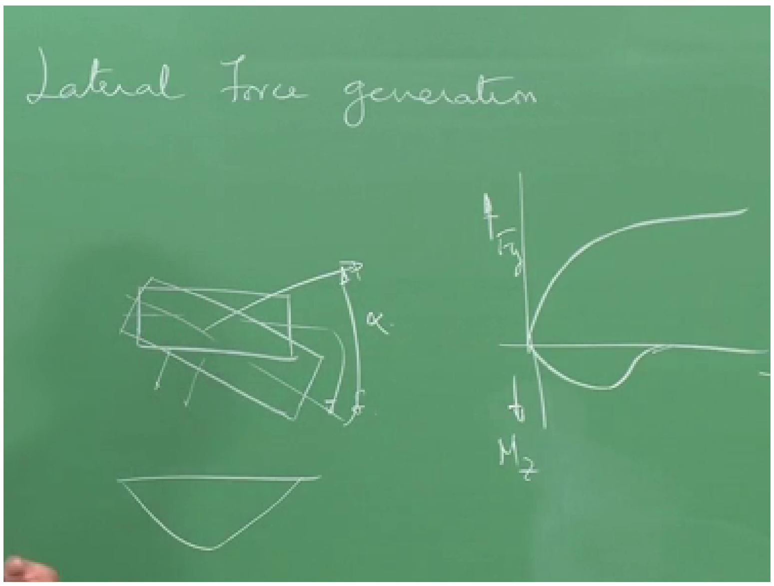

We said that the key factor in the lateral force generation was the distortion which of course to the contact patch, which actually makes the tire assume a direction okay it is at an angle to the direction in which the tire is supposed to be moving, so if this is the straight running tire and if you now let us say that is the central part and then if you now give a steering input okay, which we said as delta and then the actually there is a slip angle produced and so on.

And we saw that the distortion causes that centripetal force that required in order to sustain this centripetal acceleration, in other words cancels out the D'Alembert's force called the centrifugal force. The end of the class we said that we get Fy and then we have what is called as restoring moment, restoring moment is due to the centripetal force not actually acting at the center and that it acts at a distance which we called as pneumatic trail.

And then we had the curve which we plotted in the negative direction because okay I think we plotted it together let me do the same thing here, so that is what we get and that is alpha remember that this is Fy and remember that that is Mz which is the self-aligning torque right, this is what we did. The question towards the end of the class where we brought this to notices that why is this aligning torque? You know why is this taking this shape? Okay.

In other words, why is the aligning torque becoming 0 when the force reaches saturation point, remember that the shear forces okay the shear force which is responsible for that Fy okay is something like this right, so and there is a moment also sorry there is a normal pressure which is we will remove this you know what is alpha now.

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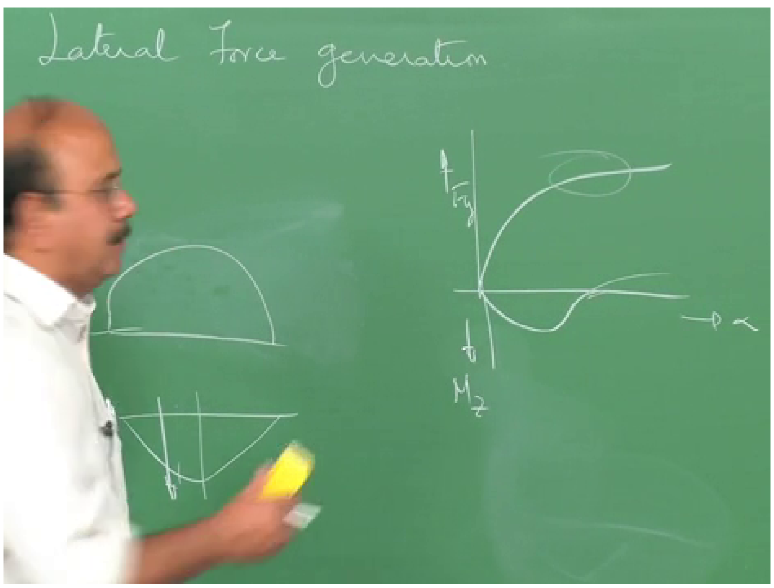

So which is like that now because of the fact that assuming that there is no slip because the fact that it is now shifted to the left in other words there is unsymmetric distribution obviously the resultant force will not be symmetric, but is the resultant force because of all those shears or all those pulls of those bristles which were there they happen to be not symmetric but it is unsymmetric right.

Now imagine that slowly guys are going to slip when it slips what happens it reaches a maximum okay value which is mu*qz right, so it sorts of starts now pushing because all of them are going to assume the value of mu qz and this is being almost symmetric okay this one yes it is not exactly symmetric but it is assuming that it is almost symmetric this guy is now going to move towards the center.

So in other words at this situation where all of them have reached that mu qz okay which is the maximum force that can be delivered by this bristles the resultant force position now shift towards the center and t goes to 0, so if you assume for example a good parabolic distribution okay the mu qz also will be parabolic and hence it will be exactly symmetric and the force sitting at the center okay, so for a brush model with a parabolic pressure distribution the aligning torque goes to 0 right.

Actually, it does not go exactly to 0, because the fact that it is not very symmetric and hence it may actually be something like this and so on okay. Now the question we asked in the last class was that is the lateral force development only due to cornering is there any other means of developing a lateral force that brings us to a very important interesting topic, which the tire manufacturer's call as ply steer and conicity.

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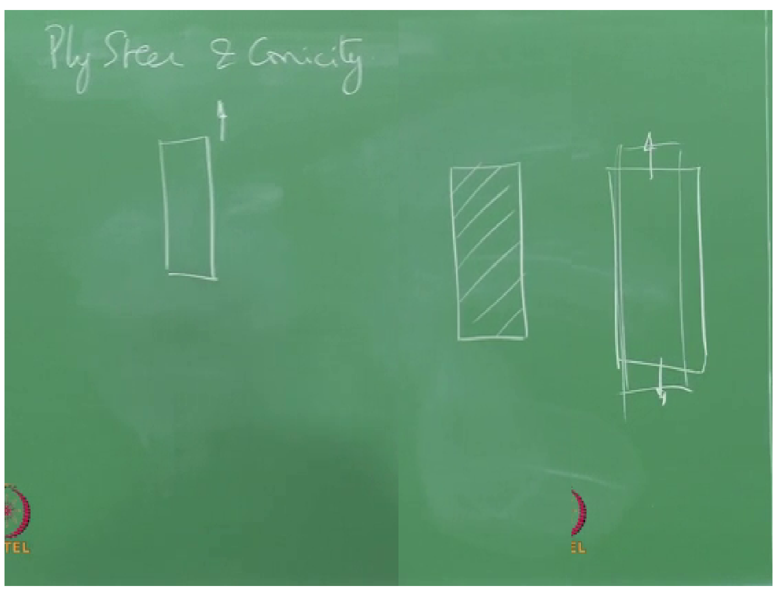

Ply steer, as people call it is something like a steer like behaviour and conicity is something like a camber like behaviour, now let us look at what is ply steer? Yes, any questions? okay. Now let us look at what is ply steer let us say that is the tire which is moving this direction rolling in this direction right, what is a tire?

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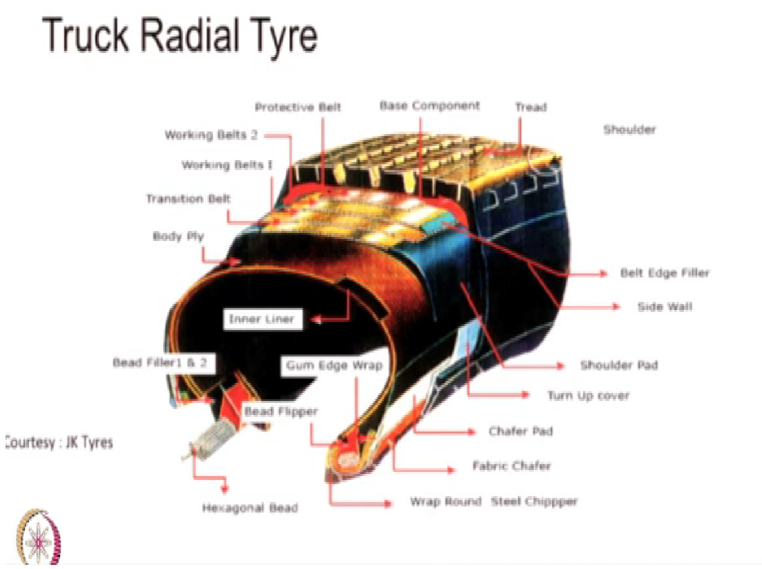

Let us look at this say for example a truck tire similar is what do you have for passenger car tires and you have the tires here okay, now it has what are called as belts okay, in this case it is steel belts okay and there are a number of belts it can be 3 belts, 4 belts and so on right. If you look at these belts closely I will just zoom that here and draw it here if this is one belt I already told you this that there are steel cords which run like that okay.

Which is run like that and that there will be another belt okay where the steel cords run in different directions and so on, there is an angle to this let us not worry about that what is exactly the angle and so on right. In other words, this whole of this belt bundle which is just above what we call as the body ply okay behaves as a composite laminate, and what is the importance of composite laminate?

It has a very interested property maybe you have studied this in mechanics of materials okay, let us do that here, so if I have now a sheet say for example made of steel and then pull it we told this already in last class that it just gets pulled and that there is a Poisson's effect in the other direction okay.

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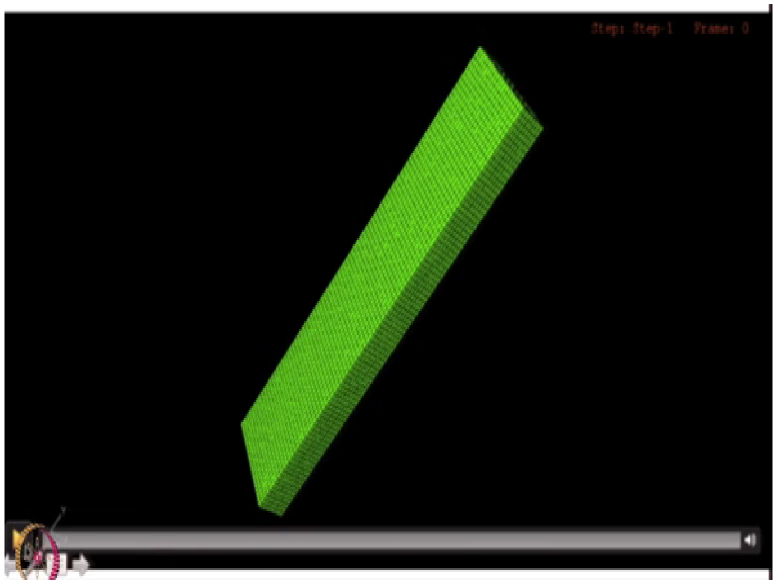

Now let us see what happens when I pull this here in this case, now that is a sheet okay that consists of number of all these what we called as reinforcements this belts let us say that they are all together and now I am going to pull this sheet okay, if it were steel I know what you would expect it would just go like that and this will be in one plane there will not be out of this plane there will not be any deformations right.

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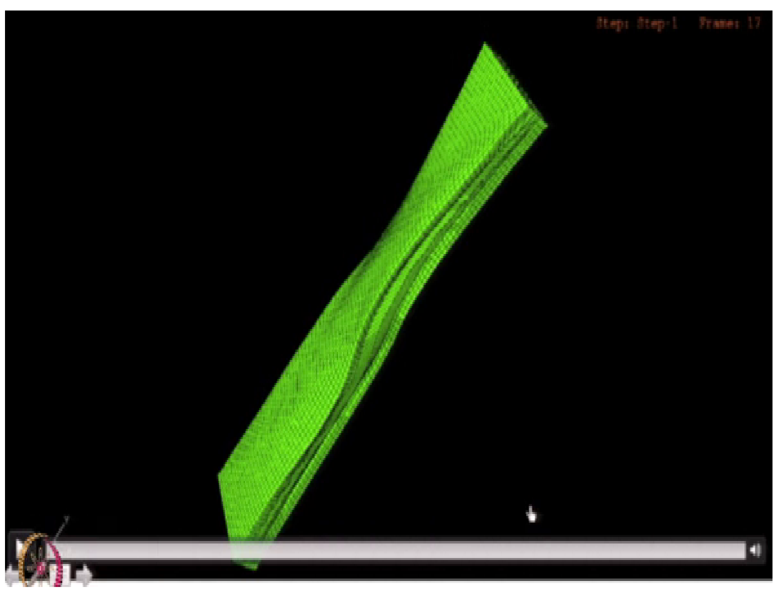

Now let us see what happens here, you pull it I did not apply any force in the other direction, I just pulled it and what happened whole thing twisted okay. So in other words there is a coupling between this longitudinal force or one force to the other moment and so on right clear, so this is one running in one direction if it were to run in another direction then it would go the other way okay, so these are called as coupling stiffnesses okay,

There is in other words a coupling happens between one plane what happens in one plane to another plane, in order to explain this, let us put down let us go into some details on the laminate theory, we will not go too much details into laminate theory but we have to understand this, so we will go into some details of the laminate theory.

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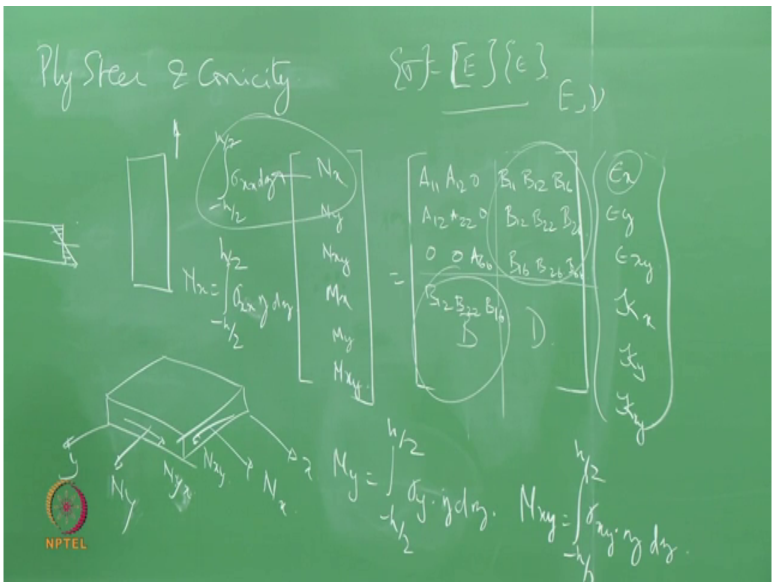

And what does this laminate theory say? Let us look at what we call as the constitutive relation for a laminate plate or what is called as laminate theory right, so what are the forces and moments that, what is the constitutive equation I have in this case forces moment and have some terms here constitutive terms, and this is what I would call as strains and so on right. So let us say that I have Nx force, Ny, Nxy Mx, My, Mxy right okay.

What is this if I have a laminate like this just give a small thickness to it, so that the thickness is say 2h let us say that there is the y direction and that is this x direction, and the force that is happening here is this Nx force here in this plane is sorry Ny force that is happening here or force that is pulling here is Nx, and Nxy is this and Nxy Nyx and so on. Or in other words if you want to write that as Ny okay x is the plane y is the direction.

So if you want to write it like that this becomes Nyx plane direction and Nxy right okay, we know that Nxy should be equal and so on, now what is the right hand side, the right hand side of the strains epsilon x, epsilon y, epsilon xy and then it is a laminated plate laminate theory, so you will have what we called as curvatures, so you have curvature x, curvature y, xy right, now these 2 are related by this constitutive equations.

Let us say that we will divided by 3x3, 3x3, 3x3, 3x3, so we will call that as A, we will call that as B, call that as B and that we will call that as D okay, now let us see what is so there are terms here let us not worry about how the terms are calculated any composite book will give you the terms how it is calculated, so we will call that as A11, A12, 0, B11, B12, B16 okay, so these are terms which depends upon the new values E values and so on.

So let us not worry what it is that is not our intention, so you can go on like that B12, B22 and A, so this is a way of I mean the numbers are important so will write this as B26 okay and this should be A we will say let us be you know correct to a book A12, A22, 0, 0, 0, A66, B16, B26, B66 and that B gets repeated there again okay, same way you have D11, D12, and **"Professor-student conversation starts"** (()) (14:37) yeah they are values okay which are coefficients.

What are they? Very similar to what do you know for example in plane stress and plane strain okay say for example you have sigma=E epsilon right. **"Professor-student conversation ends"**. So you have a constitutive equation so which relates stress and strain okay, so this what do they depend for example in isotropic material what enters into this E and mu, so you require 2 quantities for isotropic material E and mu okay.

So here this is not this is a laminate so this is not an isotropic material okay and hence you have okay a matrix which is similar to this, that is why I said let us not worry let us not going to the details it is not to explain laminate theory but to explain why it happens and how that causes what is called as the ply steer. **"Professor-student conversation starts"** Sir, what is Mxy, good so Mx My and Mz are the results of they are the moments okay sigma x multiplied by z okay dz -h/2 to +h/2.

Similarly, My –h/2 to +h/2 sigma y*x*dz right and what is Mxy? Sigma xy multiplied by z so Mxy is okay **"Professor-student conversation ends."** So the moments are the results of stresses that are acting in this plane, remember we had a simple beam remember that we had a beam bending that is the beam say for example in your earlier classes on mechanics of materials so when the beam bends you would have okay the how do you have this?

So you have for example bends like this okay a moment that is created and that would be stresses that are acting okay sigma xx is neutral axis and that these stresses cause equilibrium at the moment. So that is what you get right okay, it is exactly like that it is a plate what is a plate? You can assume that a beam is extruded in the other direction, so you have a plate and the plate is extruded in the third direction it becomes a beam extrude in the third direction becomes a plate. Okay right.

"Professor-student conversation starts." Any questions. Notation based on sigma to Nx? Yes right this is the way you express I did not say it is the same I said it is similar okay, so this is the way you express for a laminated plate okay so express it in terms of force express it in terms of strains as well as curvature right okay, and those forces are of course related to the stress through those kind of relationships.

This Nx is same as sigma x? Exactly not the same that is exactly what I am trying to say that there that is a force that is acting Nx is the force acting Ny. Sir Mx is that sigma x or, when we define Mx I define sigma x or sigma xx if you want. That is not a sigma? No this is sigma xx okay. Sir sigma xx*what is that? z right, that is what creates the moment okay you have Nx, Ny and Nz right that is the force that is acting in the x, y and z direction okay. Any questions? Right.

So the left hand side represents the stresses of the moments obtain, the right hand side the extreme matrix represents the deformation? Yeah please note Nx is due to sigma x which means that what is Nx? How do you define Nx? Just integrate -h/2 to +h/2 from the same thing if this Mx is like this Nx is that is it clear okay, so note the difference the way a plate is written and regular constitutive equation.

Call this as constitutive equation yes you can keep debating that because constitutive equation is stress and strain for a material now you are talking about plate of course I agree right, any questions. **"Professor-student conversation ends."** So this is the equation which you would look at, now what do you mean by okay having understood that what do we mean by coupling? When I apply epsilon x or epsilon xx right.

When I apply epsilon xx I would expect a force in Nxx or Nx that is all I would expect okay. Now there are in the same fashion epsilon y and so on okay, but these coupling terms here give a different picture, so for example here I have B12, B22, B16 and so on right, what is the picture it gives? What is that you get from this? Okay I give you a minute to understand that, so what is it that it gives? Elongation also gives moment with force correct that is it.

So in other words there is a coupling between the 2 in the sense that elongation also gives a moment okay or shear gives a moment okay Mx. **"Professor-student conversation starts."** It is a vice versa? Yeah of course because that is how it is populated right that is how it is populated is it not right **"Professor-student conversation ends."**

So in other words if I change the curvature term say for example Kx term if I can change then it is not that if I change this term it is not that I get only a moment for changing the curvature, you would imagine that I have a plate like this you know I change the curvature or the curvature is changed only by giving a moment, this is what usually you think but it not only does that happen. In other words not only is the moment of course moment has an effect.

But you would see that there are terms which would also give rise to your force clear okay. So in other words that is why what happened here is that when I stretched twisted and so on okay, how do I apply this? So what is that we are talking about why are we talking about this changing curvature gives rise to a force fine. So why so what happens so why is that I am worried about it, suppose that is the belt okay forget about directions forget let us understand the physics.

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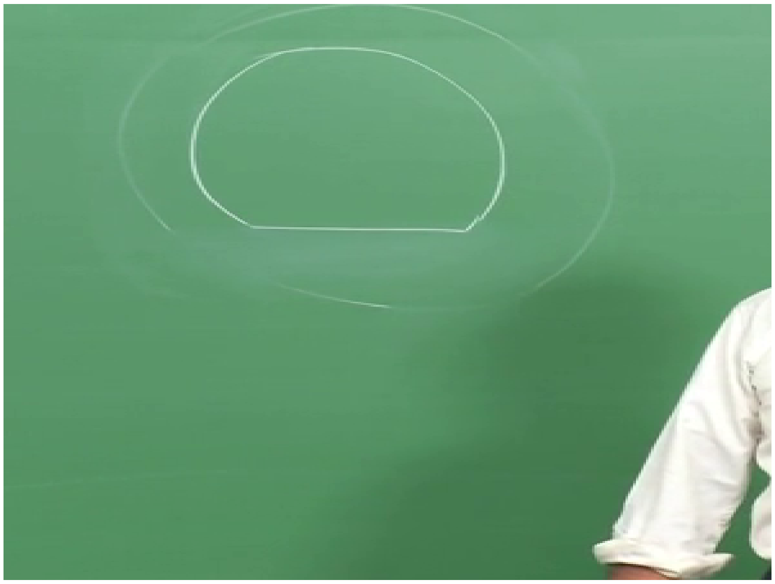

Suppose, this is the belt okay of course you know that it is a circumferentially going I am looking at a cross section okay that is the tire and that is the belt, so when this tire hits the ground or when it is sitting in the ground in effect what I am doing I am changing the curvature okay, I am making this guy flat okay. In other words, I am sort of giving a moment to make it flat when I give a moment to make it flat okay what am I attacking? I am attacking this I am attacking that.

But when I attached that or in other words when I changed kappa x if I call that as kappa x the curvature changed, then there is one time which is sitting here which I will multiplied by kappa x in order to give Nxy right, I change this I give a Nxy term it is a force term, change this here is one guy who is going to give me a force Ny term and so on right. So when I change kappa x it is not just moment, so you would think that fine.

I have a curvature like that I am making it flat how do I make it flat? Maybe give a moment like that and make it flat right, but that is not just what happens and you have forces. So in other words I am producing a force here right and I am producing a force in this direction. **"Professor-student conversation starts."** Sir why some terms of the A matrix gets 0? Yeah that is why I said that let us not why is that 0 okay. There is no coupling between those elements, of course why?

Because there is no coupling time between the shear and the longitudinal force okay, Poisson's effect is only x to y direction, so gamma xy is g* sorry tau xy=g*gamma xy okay, go back to your earlier class on mechanics of materials right okay. **"Professor-student conversation ends."** So that is in other words obviously there is a shear term does not have any effect on the force term okay, vice versa right.

In other words, there is no coupling what it simply means is that I will I would eliminate all these guys, these 2 guys will be done with 0 no coupling and I will have only these 2 terms these terms are your familiar terms for a minute for forget about all this, so you would see that just force and well known okay A11*epsilon x, A12*epsilon y, 0*epsilon xy or gamma xy or whoever we call it okay right that is all.

So the shear force is equal to or the shear stress this result here is nothing but this multiplied by this shear strain right okay. **"Professor-student conversation starts."** That is that clarify your doubt that there is a shear term that is involved that has nothing to do with normal force terms okay. **"Professor-student conversation ends."** Let us come back a lot more to look at, so kappa x I am going to change now because I am going to make it flat when I make it flat obviously I am introducing forces okay.

I am introducing forces because of this coupling term, and those forces introduces the lateral force okay the lateral force and a moment. So simple fact that as the tire rolls in the ground and a part of the tire becomes flat results in a lateral force called ply steer okay.

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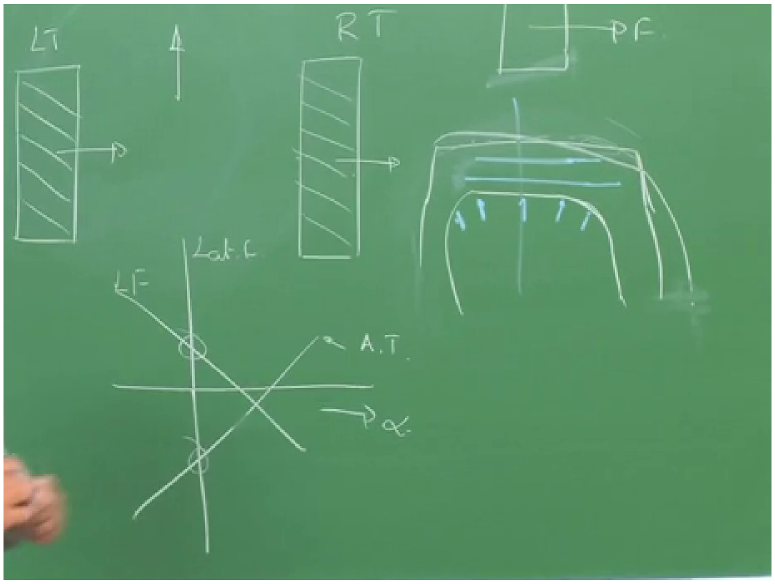

So if I have these are the 2 tires which are running and it is right, so this produces as a part of its belt flat and hence there will be a force that will be acting. **"Professor-student conversation starts."** Sir why cannot we cancel it? Yes, very good, so the question here is that does it get cancelled it is a very important question can it get cancelled see let us say that I have this belt structure okay now I have this belt structure.

In other words, what happens if I rotate the tire okay if I rotate the tire and keep it okay you have to be very careful in this, there is a lot of confusion for people in ply steer on that, suppose I have this tire okay how do I cancel it if I have this force act in the opposite direction can I ever make it happened like that happened that way, can you do that you rotate it you whatever you want to do you do you rotate it okay you rotated the structure will exactly be the same.

You rotate it like that it will be like this put a point here okay that point will go here this point will come here so it is not you cannot cancel that okay, **"Professor-student conversation ends."** But if you rotate in the opposite direction the force will be in the other direction okay, you can change that in other words by the structure of the ply in the 2 direction, so there is always a confusion that ply steer changes direction whether you rotate it in clockwise or counterclockwise true.

But you do not rotate the tire okay you do not run the vehicle in the counterclockwise you run it only in one direction, so you call this as the right tire for example and the left tire okay, and now superimposed on the say for example I plot alpha versus both the aligning this torque which we called as the aligning torque that happens it is due to the Nxy and lateral force Nx term okay, so that happens to be like this.

And it so happens that because of the directions that you will get if this is the lateral force not left force lateral force if you want to call it like that lateral force okay that is the aligning torque, and these are in other words when they are 0 not when alpha=0, in other words when alpha=0 okay what is alpha? Slip angle we said slip angle alpha is required in order to produce a lateral force so when it is 0, you would see that there is a lateral force and a corresponding moment clear.

Hold this for a minute we are going to come back to this graph could explain it again is this clear, so when you when it is free rolling okay on the ground nothing to do with the ground you will get a lateral force okay. Now there is also what is called as conicity, what is conicity? Name indicates that conicity, as you had seen that there are what are called as belt we saw that just now okay.

So assume for a minute that the belts are not aligned exactly at the center, in other words suppose I say that I cut this tire okay let us forget this part of the tire okay, we will concentrate on only this part of the tire top of the tire right that is where the belts are, the belts are supposed to be nicely placed that is the center line, the belts are nicely placed like this okay they are nicely placed with symmetric about the center right.

I inflated there is a stiffness that is given I inflate the tire this is a stiffness that is given to the tire because of these belts okay, so it assumes a uniform say radius or in other words it or the deformations on either side is the same right, let us say that that is the deformation due to inflation okay of the belt inflation with the presence of symmetric belts. Let us do a thought experiment let us say that the belts are not symmetric about the center they are not symmetric about the center they are not correctly placed.

So I will have say let us say that there is a belt what we called as belt offset, so this guy is like this okay left and the right placement of the belt are not symmetric. So what happens what is that you conclude looking at this left hand side and the right hand side so radius in the left when I inflated will be higher because this is going to be stiffer, so this these guys sitting here at least these material elements are not going to move to the same extent as to the left.

And hence okay my tire is going to become something like that, in other words my tire is going to look like a truncated cone right. So now you have a truncated cone let us say I mean just exaggerated that and roll a cone what happens? So there will be a force generated. **"Professor-student conversation starts."** Sir if the bottom surface is flattering? Yes, bottom surface will be flat but still I am going to okay like it will be like this there will be a conical shape.

In other words, the pressure distributions on either side is not going to be the same, so with the result there is cone as if there is a cone sitting and then rolling. Sir any time the wheel and tire endras with the road the surface is only flattering? Correct agree with you surface is flat but the pressure distributions are not going to be the same okay that is exactly what happens in the cone, the cone is also when it is placed okay it assumes the surface and one side becomes flat it is not that it is like this it becomes flat.

So when you rotate it why is that it is not going straight it goes like this right exactly the same thing right okay. **"Professor-student conversation ends."** So the cone you are placing it on the ground which means that please note that one side of the cone is sitting like this, so the cone will be sitting like this okay fine. So this is what is called as the conicity effect that is giving rise to a force right. Now this is one thing which is not necessarily is one side or the other side

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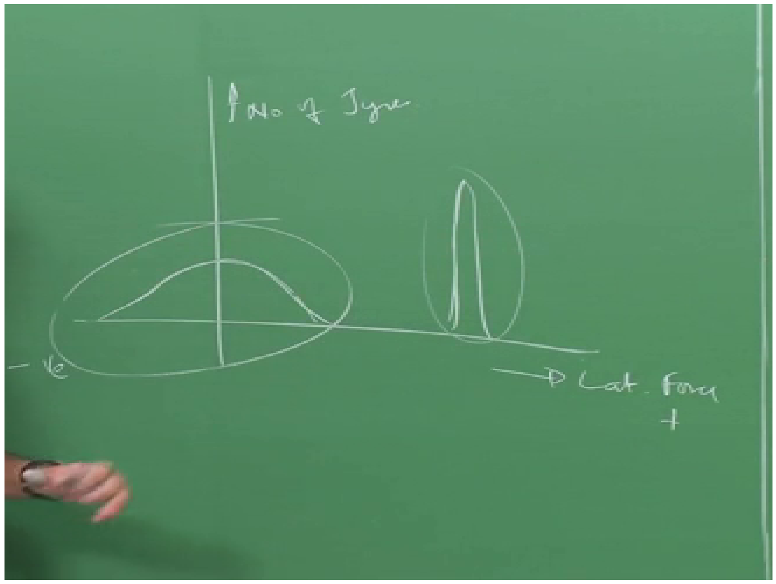

Because it depends upon which side it is shifted, so if you now take a number of tires you go to a one of the manufacturing plants tire manufacturing plants, you take hundreds of tires, now and then okay say that I want I measure hundreds of tires I measure the lateral force, let us say that it is the lateral force let us says that is positive and that is negative and let us say that this is the number of tires and I take 1000 tires.

What would happen is that you will get suppose I get a peak like this a very pointed peak okay, and say I get a peak I get something like this okay right, I have tires which are so if I get a peak like this suppose I get a peak like this in the tires, if I get a peak like that in that tires which I have tested which this means that this peak is what? All the tires whatever I have tested okay this falls let us say that it falls within this gap, all the tires which falls within this gap small gap almost the same.

Then that lateral force is ply steer because ply steer is due to the design okay, here if it were if it so happens that the tire the force distribution is something like that, then there is that is not ply steer basically because it is distributed on either side and so the belts are offset this side or offset the other side right okay. We will come to the effect this diagram in a minute let us forget let us go and look at are there any other things that cause a peculiar behaviour of the tire right.

So in other words straight running tire is it going to be a nice guy who is going to run straight well he is going to do other magic other things. **“Professor-student conversation starts.”** Yes. Sir conicity is manufacturing defect or it is something? Yes, it is most of the most instances it is this belt offset that is going to cause conicity okay it is a defect. **“Professor-student conversation ends.”**

Now that is not the end of the story, there are other things that are happening which are peculiar and which is not covered by all your courses in mechanics of materials. Now what are the other things that are happening okay, let us look at that now.

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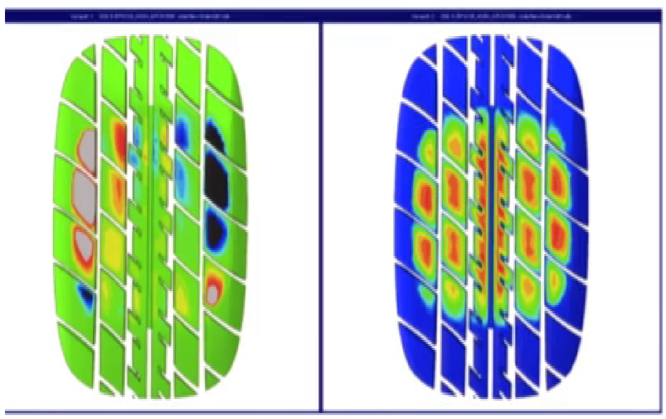

Now let us look at the contact pressure distribution okay and the lateral forces that take place or that present when I roll the tire okay, now you see 2 things there so that is the lateral force distribution right, there is a lateral force distribution that is happening. What is the importance of this lateral force distribution? Okay, now why is this important? And what is that we are going to look at?

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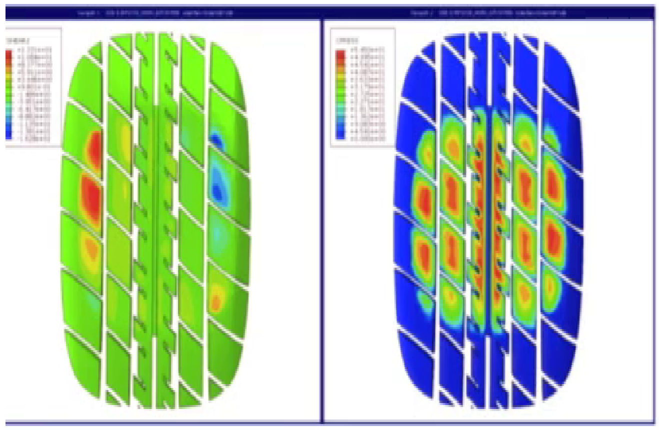

Actually that is the previous picture is just to zoom in and show you the values are the inner ones okay.

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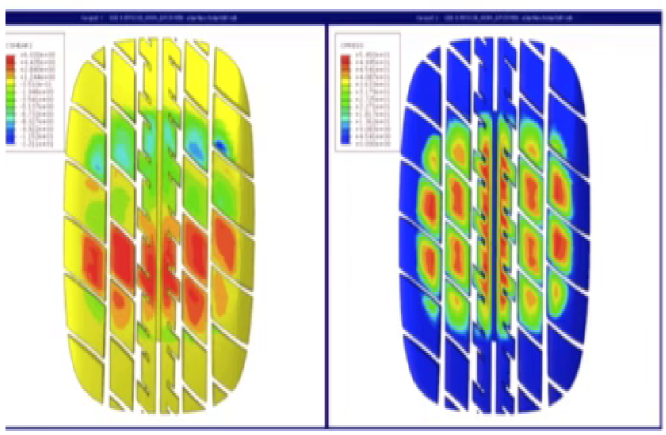

Now that is in the longitudinal direction.

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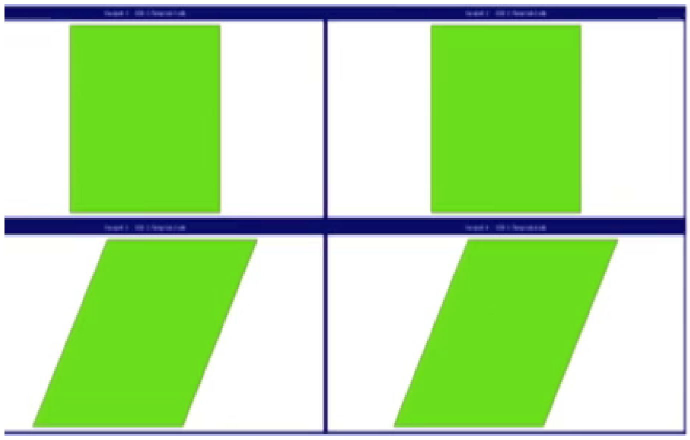

In order to understand that picture we will come back again to the lateral force and longitudinal force. Now let us assume that the tread is a block okay, now if you look at these blocks they are not straight you know squares like this is a plan view.

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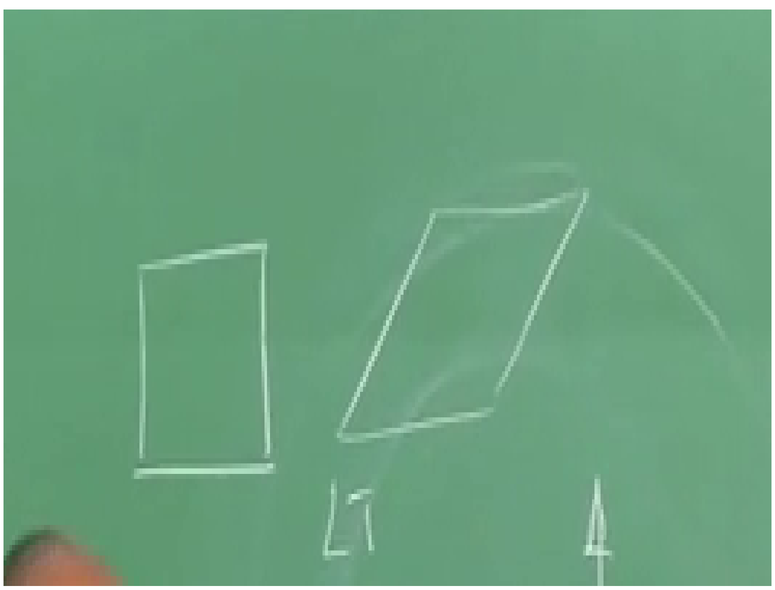

And so they are not nice squares like this okay they are so rhombus type of figures okay, all those blocks are rhombus types of figures right, and we will explain this in a minute we will come back to that. So in other words they are not what is there at the top okay they are more a rhombus type of figures right. Now what happens when I apply see let us say that let us now model this blocks, this is a plan view okay.

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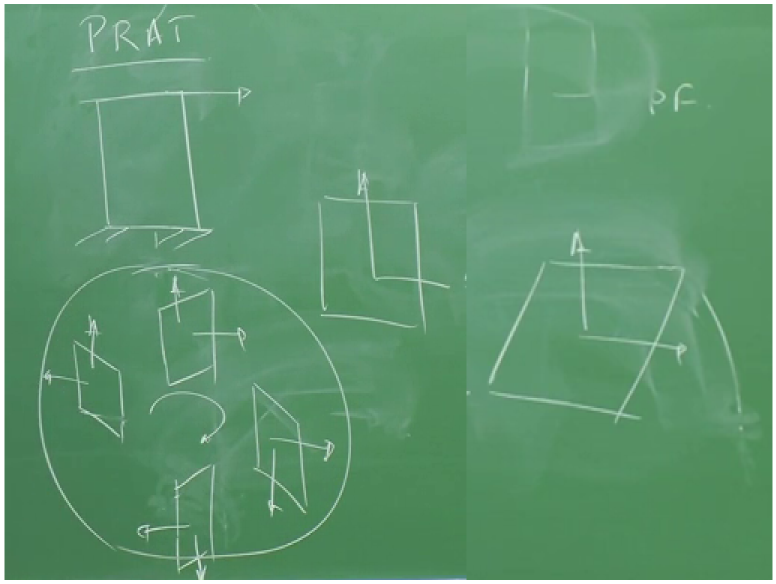

So now let me this model this block okay and apply a force say a square block apply a force in this direction, I am applying a force in this direction okay for both these things right, now let us see what happens okay.

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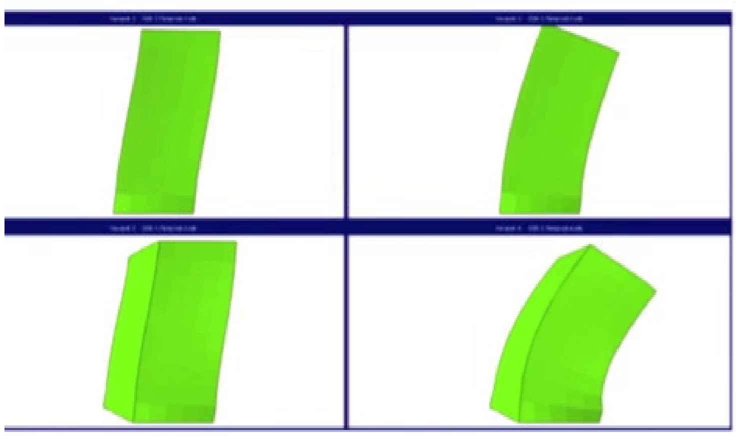

So I can do 2 things, one is that both of the bottom and the top are fixed not to move in the z direction okay both of them are a guy stuck between 2 plates this is what happens to a block when it is in the middle of the contact patch, at the edges of the contact patch one guy is free okay, so I can free him I can free him okay we will apply this load, let us say that I have a square cross section I applied this load okay what do you expect?

You expect it to move the direction bend in the direction in which I am applying the force F for a square block, I change this block to rhombus you would see that there is a difference okay.

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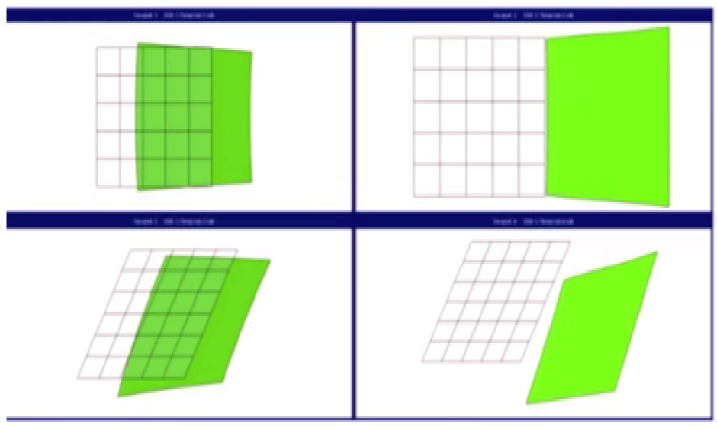

Now that is the top view see what has happened let us concentrate on the left side, so left side clearly shows that this is a finite element picture is just an isotropic rubber block, so it shows that when I apply a force F the displacement Fx let me call that as Fx, the displacement is only in the x direction for that rhombus okay which is very similar to that there in the tread block, if I apply a force Fx.

There is also a displacement look at that there is a displacement in the other direction y you can see that at the bottom picture civil engineers long ago recognizes this and call this as unsymmetric bending, in fact when you take this next course on automobile structures we are going to talk a lot about unsymmetric bending okay, they realize that there is a principle axis directions okay along which there is no coupling.

And again there is a coupling factor that is involved, because I am not applying the force along the principle axis direction which happens to be the symmetric axes okay that you see an example in the first picture. So if I now have a block whose plan is like that and if I have symmetric axes okay, it is symmetric bendings so simple cantilever beams bends, but of course it is not a cantilever beam as you know or you have studied because it is a short fat fellow and hence shear deformation become very important.

There are other things that are happening and a simple beam theory which you learnt a cantilever beam theory cannot be applied okay that is one thing. Now if this plan view happens to be not so symmetric say now it is something like this, and I am applying a load here in that direction that is not a symmetric axis and hence the coupling happens in the other direction okay, and hence there is a push okay.

Now that in other words that results in if I stop it that results in a force, if stop this that results in a force okay. So if now go back to my first picture will explain that in the next class again, now I have shears that are happening okay, let us say that I have this kind of rhombus blocks sitting like this okay, and there are going to be even if when its rolling remember that there are going to be lateral shear forces that are going to happen okay.

That would be the lateral shear forces that would happen or that would act on this treads and that lateral shear forces results in forces that are acting in that direction okay, and hence would result in a torque, which is called as the PRAT Ply Steer Residual Aligning Torque I had to be careful in this but anyway that shape is you know there will be this is a very simple method simple thing, but there will be a because of this shear forces that are acting okay.

As it goes straight the shear forces that are acting okay, remember that the way the contact pressure shear distribution was there, so because of this shear forces that are acting, there are forces because of this coupling there are forces perpendicular to the shear forces okay at the ends there are shear forces like that, and this is the shoulder region where there are shear forces which are happening like this, and at the end of the contact patch you have shear forces that are happening like that.

And because of which you have coupled forces which come along with this shear forces and the coupled forces which are acting perpendicular to the shear forces, so if there is a shear forces like this this is the that is the contact patch say that is the end of the contact patch, and so the contact patch you have okay shear happening like this, here it is happening like this and these 2 they are happening like that, and so there is always a perpendicular force that is happening okay that is the result of this coupling.

So when you have perpendicular forces that are present and that results in torque okay towards the center of the tire which is this is the center of the tire okay, in other words you see that that is the okay, so that is the center of the tire and so there is a moment exactly, so this is another reason why there is an aligning torque that is happening when you are going straight okay. We will come back to this and we will continue with this and the other derivations in the next class.