Vehicle Dynamics
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Lecture - 15
Tire Models - Magic Formula

So in this class, we are going to talk about what are called tire models very short introduction to tire models. We go over and look at what happens in the combined cornering and slip a simple model and we will wind up may be in the next class about tire models which are used in this combined slip. Now first of all what are tire models, this is a very important word today in vehicle dynamics.

Because all people who use or interested in vehicle dynamics, they use one software package or the other, be it, Adams or (()) (01:02) or whatever it is. So they are interested in the way the tire behaves, in other words that becomes an input into the software. So they have to give to the software how, for example slip versus longitudinal force varies, how alpha to lateral force varies and alpha and moment varies and so on, these graphs are to be input.

As we can give it as the lookup table which some of the software accept it as lookup tables are through a formula which is called as a model. So a formula which links this slip.

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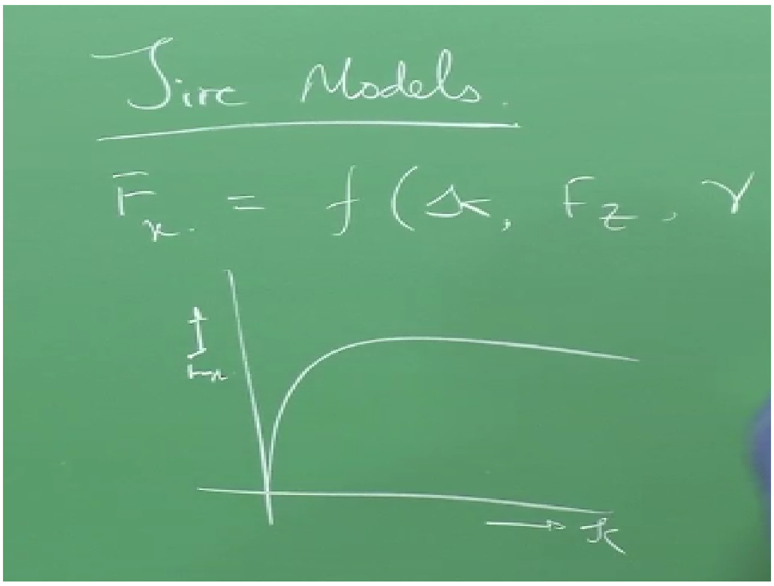

For example, if I write F, it can be Fx, Fy, Fz whatever it is as a function of, let us say that its longitudinal force Fx as a function of kappa or sigma or slip, then it can be a normal force Fz which is acting, so that camber angle if it has an effect for example in Fy it has an effect, so in Fy it has camber angle will have an effect and so on. So in the other words, this tire model is nothing but a mathematical equation.

There are a number of ways in which these mathematical equations are arrived. The easiest of them is to do a curve fitting, which we would call as empirical equations. For example, I can determine these curves using experiments for example it can be kappa versus Fx and then fit an equation to this curve. The foremost tire model used extensively by people is what is called as the magic formula tire model.

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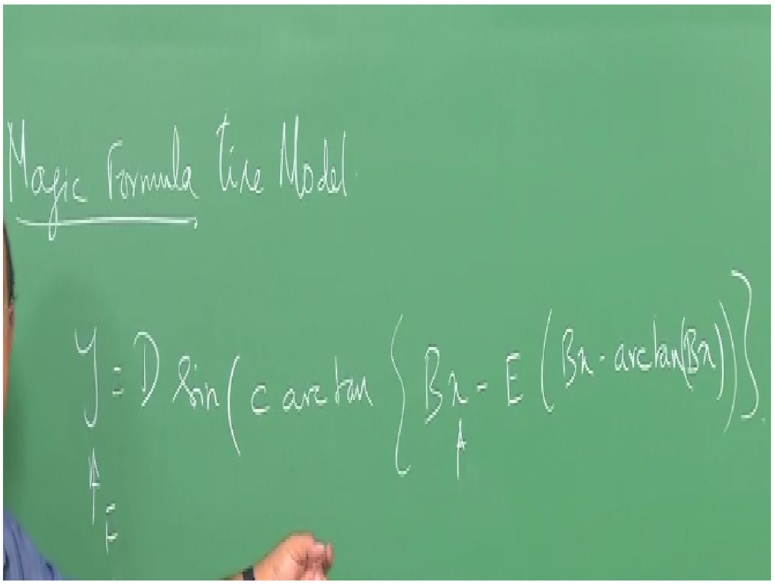

You use Ti or Ty whatever you want but the magic formula tire model is one of the most important models used today by vehicle dynamics community. There are other models we will see that very short introduction to other tire models later. Let us look at magic formula model, and let us look at how for longitudinal force magic formula model is used. You will get a feel of it, then we will look at the combined slip

First of all what is magic formula. You might be wondering what is magic formula. That is the first question. A bit of a history, this is one of the greatest contribution of Professor Pacejka who is a foreigner in the (()) (04:33) and he has a very interesting book which I have already pointed it out, and he with Volvo company Mr. Bakker of Volvo company they together in around 1987 they put up this formula, initially called Pacejka tire model and sometime later it was called as magic formula tire model and so on.

87 to 89, there are 2 2 or 3 papers in SAE which actually formed the basis or laid the foundation. Later in 1993, this model was adopted by Michelin Tire Company and they came up with a modified version of it. And they called this as magic formula model using an empirical technique. After this, this tire model has undergone so many changes, there are so many versions up to 2006 version.

We are not going to look at every version, you know it is a huge topic I refer to the book of Pacejka on this. We will only go back to the original paper and just look at the philosophy of how this was done, that is what we are going to do. Before we go further, what is this magic formula, why is it called magic formula. Magic formula is because this formula which I am going to write down now can be used for all the 3 curves.

In other words, the frame or the form of the equation is the same, which I can write down this as $Y=D \sin C$, do not worry about how you know why this is so complex that we will see it in a minute how this came about $Bx-E*Bx-\arctan BX$. This has undergone lot of changes let us first stick to this. Now this formula, this fundamental form with some modifications we will see that is used for all the 3 cases.

If I substitute now X for kappa, slip in this form, then I would get F_x. If I now substitute this for alpha here, slip, then I would get F_y, the same way I will get, if I substitute for X the slip angle then I would get M and so on. So these 3 cases are the ones which are, these are the 3 cases which are important, there are other thing there like turn slip, let us now concentrate on these 3.

So you will get you have the same form, does not mean that you will have the same values. These vales of D, C, B and E what they are, we will see in a minute, how did they come here, why is that this so complex, we will see that. But these values or these are values which vary with of course F_z, the normal force and they take different forms. So in a very simple term you can say that I put down a formula and I have 3 different curves.

For example, I have a curve like that for moment and then this curve is symmetric with respect to this and then I have another curve which may be like that for F_y so this can be F_y, suppose like kappa or alpha. This can be the F_x curve, this can be M curve and so on. The reference which I am going to follow in this class is from the 1987, very first paper I said, 870421, SAE paper 870421 is a paper which I am going to follow for this work.

Of course there are after that there are a number of papers which I have come about, but this paper, this is a very fundamental paper and this actually gives the basis for obtaining the ship. So that is what we are going to follow. Fundamentally we have 4 of them, 4 factors D, C, B and E these are the 4 things that are there. I would say 4 constants they depend upon Fz, these are the 4 things. Now the whole thing started with a very simple equation.

Suppose now I want to represent this equation form. Equation form is very easy or important if I want to do a multi body dynamics analysis. We will talk about that a bit later, but just have that in mind, in other words equations are important in order that I combine the dynamics of the vehicle with the tire road interaction. If I have to combine them, then equations are important.

Remember when we talked about longitudinal, we have to talk about lateral dynamics a bit later, talked about longitudinal dynamics, we were interested in the traction force loop, fundamentally a physical to MAV road. Now we just wrote that as F. If I now have to go to the next step, then that force F has to be replaced by this curve, where did that force come from, from the tire. So how did that force developed we saw all the mechanics.

If I now want to go back to that equation, then I have to put down this. So in other words the force developed would now be a function of my kappa. So in that equation, go back to the equation, in that equation you have a traction force, that traction force will be replaced by an equation which combines kappa and Fx. That is why mathematical form of this equation becomes important.

I can find out, in fact if you want me to get this kind of acceleration, whatever you see, you have seen lot of advertisements, where they would claim that 0 to 60 6 seconds or 0 to 80 7 seconds whatever it is. Then I need that kind of traction force and whether I can develop that force depends upon this equation. I have maximum force that can be developed, say for example this would of course depend upon the road and so on, Fz and so on.

Of course depend upon the friction, characteristics. So this becomes very important where am I going to develop this. In other words it also tells me whether I am going to have the wheel spinning, locking and all those things. So in other words that equation has more meaning, when I now combine that with the interaction with the road. You would definitely know, suppose to make the point very easy to understand.

We have been lot of questions before as to what is this formula, why am I using it and so on, very experienced user of sometime this would be nothing for a new person who get in this is going to be difficult.

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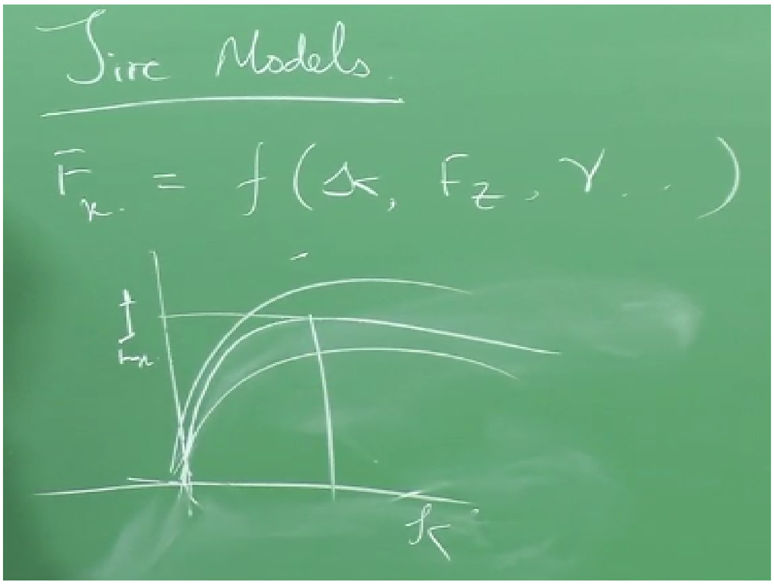

So suppose I have a curve like this, so what is the maximum force, I know this is the maximum force and that is the kappa, Fx was this kappa. Kappa is a slip. We have defined 2 slips, remember theoretical slip called sigma, the practical slip called kappa. So that is what I am plotting here kappa, yes I have plotted before sigma, so kappa is the slip, percentage slip if you want to call it. We have already defined that.

Remember V-omega and so on, go back and look at that if you have questions. Now the point is this, if I now want to develop a force, suppose you are saying that I have a vehicle it has this kind of rolling resistance, this kind of tire, and if this is the kind of aerodynamic forces that are acting and this is the mass of the vehicle and all other things, and then now you say that I want to have this kind of acceleration, I want to have 0 to 60 6 seconds.

So you get an acceleration. First of all that acceleration would result in a requirement for a force, and that has to be realisable, whether it is realisable or not this graph would say. Suppose I require a force like that obviously that force will not be realised, so that would be a problem. While plotting this curve, all the other parameters and their effects will be depending on one point. That is a good question.

In other words, this parameter, is this curve a constant what are the other parameters, that is what I wrote first. Suppose I change Fz, this curve would change. This becomes very important, for example when I break or when I, remember we had that redistribution, when I break or when I accelerate then Fz changes, so that becomes an important question. So can I have only one curve is, does it not affect, is this curve not affected by Fz, even if you say that mu is independent of pressure.

We know this is F and that would depend upon mu and so it should depend upon Fz and so on, precisely. So this curve cannot be one curve, so it has to depend upon Fz. Now then there can be a series of curves, so function of Fz.

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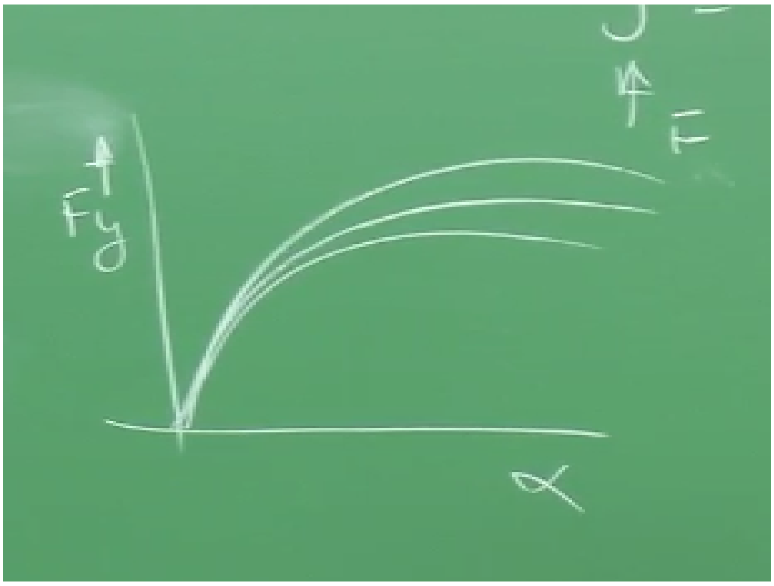

For example, if you look at cornering you go to see later, may be from the next topic, that there is my cornering forces Fy becomes important and its effect on the vehicle dynamics we will see it. Now here again there is going to be a load transfer due to the role and again there is going to be a change of this forces with respect to Fy and so on. So when I now calculate it, for example when I do a calculation I have to take into account this kind of transfer of load.

In other words, if I have to calculate Fx or Fy, then I should have an equation which would now give me a new Fy or a new Fx, because of the load transfer. I would shift from one curve to the other curve when you have Fz, so in other words this equation D, C, B, E and so on should be a function of Fz, and that is what we are going to see. Hold on to your questions let me finish this, then we will look at your questions.

I know that is why I am going bit slow, people who know this, experienced users bear with me, I will develop this slowly. Now let us now get into this equation how did you get this equation, it looks very complex, some arc, tan, sin, this, that, B, C, D, E, that is the genius of these people who worked in it. So what they did initially was to put, see whether this whole thing can be given by a very simple formula.

Now I am going to modify this, please wait for a minute. So they wanted to know whether they can just put sin of Bx, say D sin Bx, they first wanted to this, this did not work. So actually because let us get back to my all the 3 curves and we will see why not going to be easy to work. One curve looks like this, looks very much like a sin picture, the other curve looks like this and the third curve, let us say that it looks like that.

Accordingly, you have kappa and alpha that you know already Fx, Fy. So this was not working, the sine curve is not working, does not work. So I had to now accommodate now that what is there in the sin, or in other words I would adjust what is there inside the brackets in the side. So if you now look at this curve, for example this looks as if it is an elongated sine.

How do I adjust that and I want some function, which is elongated in the x direction. So what is the function, which is elongated, arctan is a function, which is elongated.

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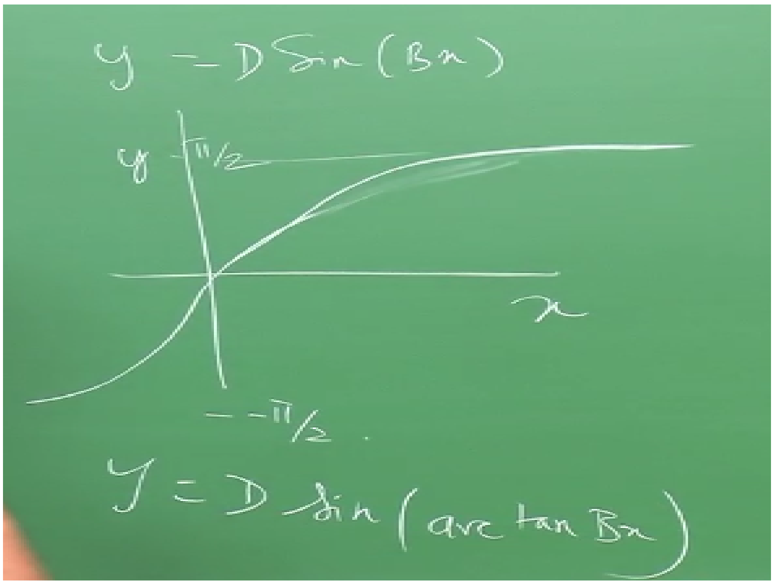

In other words, if you now plot an arctan curve, extensively used in so many applications in mathematics, x versus y, the curve looks something like this. It asymptotically converges to a value, what is that value, pi/2. Now that is the first thing. So this asymptotically converges to a value of pi/2. The same thing here, it is –pi/2 and so on. So that is an elongated curve. So I will now replace. If I replace, what is there instead of x, I would replace that value.

So that I would now put this as Y=D sin arctan Bx. Let me introduce one more term here. Then I would elongate it. But the amount of elongation I have to know control. Here it is not very much elongated. Here it is elongated more and here it is elongated less and so on. This looks like a sinusoid with a lesser elongation, larger elongation, very short elongation. So I have to now adjust this elongation. How do I adjust that elongation.

By multiplying it with a constant called C. So what am I doing. I am looking at this graph and looking at mathematical expressions, which can actually model this graph and adjusting this equation. That is all I am doing. Obviously, right now, you can immediately tell me what should be the maximum value of y. This is purely a graph. This is I am just fitting that. So what is the maximum value of Y=D obviously.

So this should be the value of D. So C in fact gives me that kind of the shape, how elongated it is, how sharp it is and so on, because it sort of has an ability to shrink the frequency if I can call it. Do not get confused with that word frequency of this sine. Now I am not still happy with it. Look at these 2 graphs. There are some peculiarity as to how actually it rises up. Apart from the slope, there it has a curve.

I now introduce into this equation or change this form of the equation in order to introduce that variation. Let me call that as the factor called E.

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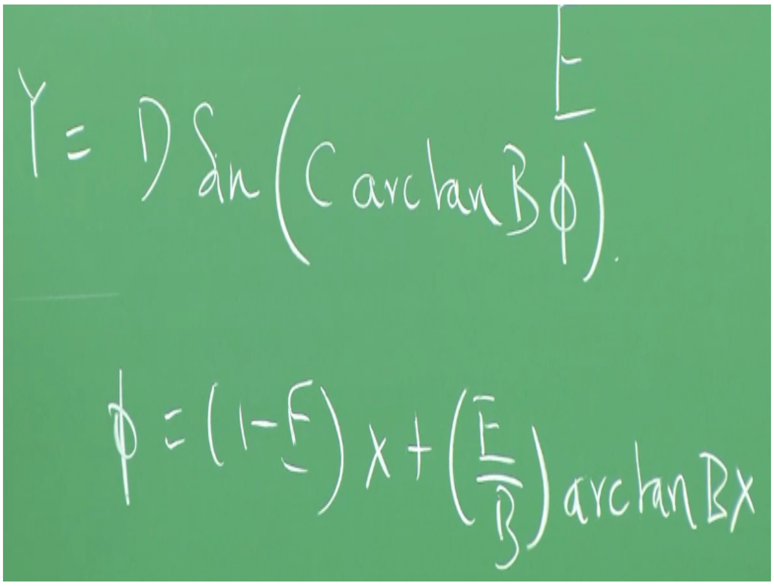

So that I will now write down this expression to be slightly variate and write down this equation to be Y=D sin arctan B. Instead of x, I am going to introduce that phi, a quantity called phi and phi=1-E*x+E/B*arctan Bx. In other words, what I have done is I have introduced this E which actually changes the curvature at this place. Now without confusing C and D, let me call the C as a shape factor, because this gives the shape and E as a curvature factor.

Now substitute that into this expression for phi. Rewrite this expression, what do you get, you get an expression, which is written there. In other words, what I do is to use the property of arctan, use the property of sine, and use the property of combination and then get a shape, which can be now adjusted. I have now 4 values, which can be adjusted. Let me give some names to it.

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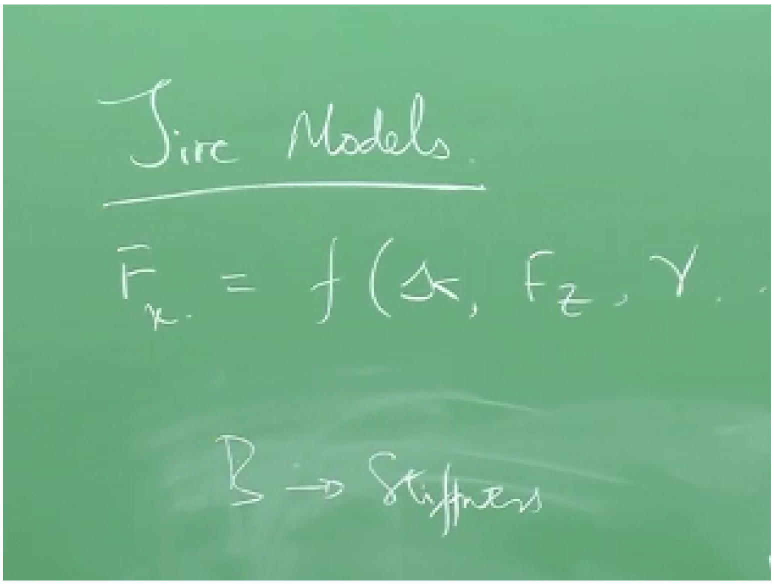

Let me call B as a stiffness factor. We see the stiffness factor is not actually the stiffness. C as the shape factor and D is the peak force. This we had already seen. D sine of something the maximum sine value = 1, so peak force D and E is the curvature factor. Why is it called curvature factor because if I now vary the value of E.

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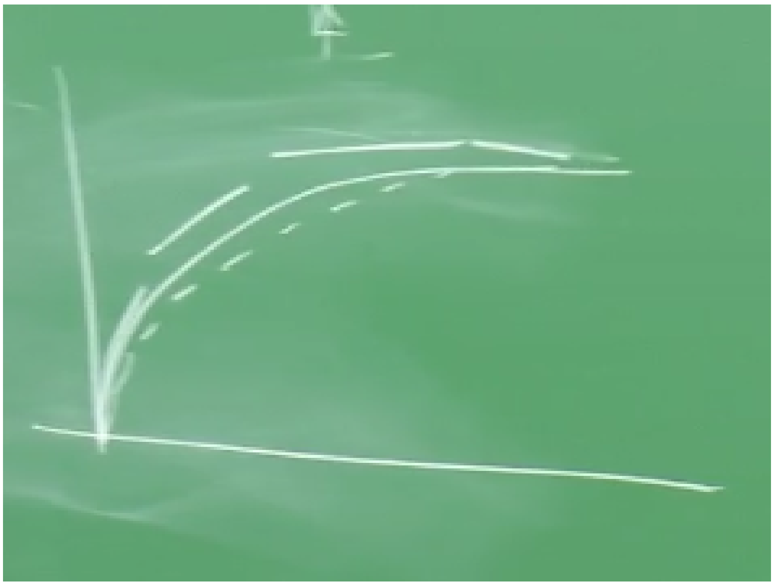

Suppose this is the curve which is E=0, then depending upon the value with if whether E is between 0-1 or E=-, there is a curvature here would vary. Sir, when we are using this magic formula, the structure of the formula remains the same, so our real job would be in relating all the coefficients as a function of slips, say kappa, gamma, that is what you would feel to your part.

Yes. So I am first establishing this formula, which by tweaking this values, I can get whatever be the shape. Now I have to introduce some further modifications. Is this clear, any questions. Now BCD are going to take care of all the other factors that contribute to Fx rate including fA energy. Yes, I am coming to that. How is that Fz introduced, there are a lot of things that are introduced. In other words, I understand the question.

First Fy, already I have told you that Fy is affected by plastier. I have told you that Fy is affected by conicity. I have told you that Fy is affected by camber. Now the question is what happened to all that. Now what do I do. What can I do? Look at the question carefully. Now what is the question that I have to introduce a force when slip angle = 0. So in other words, it simply means that let us take Fy, it simply means that this curve should not pass through 0, origin, fantastic.

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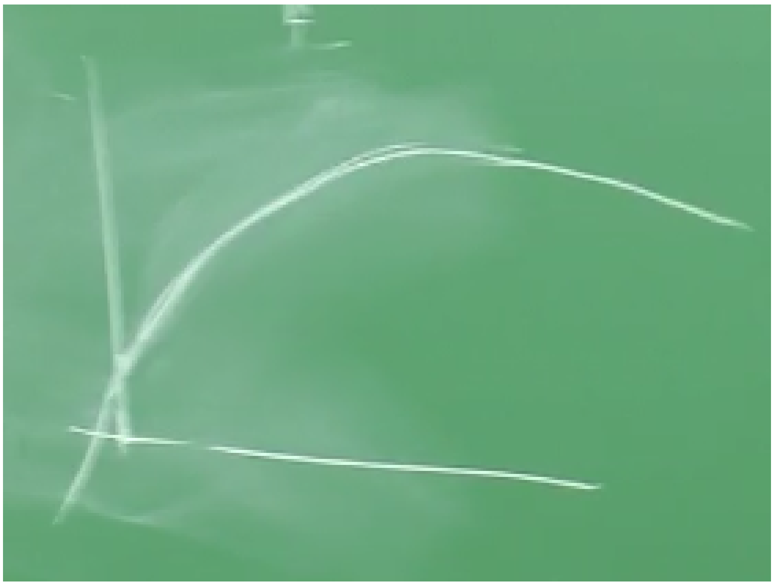

So it has to pass through something like that. It is an exaggerated one. It would not be so high, but just. So in other words, I have to shift y. Conicity the other way so I have to shift x and so on. So I am going to introduce a horizontal and a vertical shift in order to take care of the conicity and plastier. So I am going to shift that curve, so let me call that as SV and SH. This is only for FY. When we talk about conicity and plastier, we are looking at the values of.

This formula is unique so you can adjust this SV and SH. You can adjust this formula for SV and SH. When I said FY, obviously we are talking about conicity and plastier. Let me redraw that graph very cleanly.

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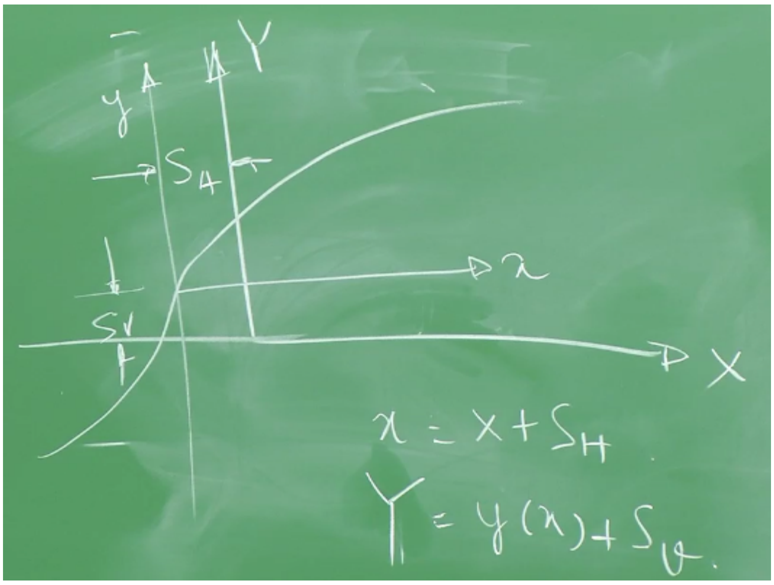

Let me call that as Y. Let me call that as X. Let us introduce this x and y and let me call that as SV, let me call that as SH and let me say that this equation is actually that is the curve and the whole curve as well as the X axis, I am shifting so that I would write X to X+SH and Y to be Y of X+SV. So I have introduced 2 factors. Camber will wait for a minute with our friend here is in a hurry, what happens to FZ.

So what I am going to do is, I am going to introduce all these factors as a function of FZ. Before that, there is one question. What is actually the slope. The slope of this curve is very important, the initial slope of the curve. So all safe drivers drive in this linear range, what is the slope of this curve. Look at that, what is the slope dy/dx, differentiate it dy/dx B cos whatever is inside, then put arctan as 1/1+x square and so on and then put x=0.

So you will get that = BCD, so dy/dx at x=0 would not become BCD. So BCD is the initial slope of the curve. Again, do not get confused. I am not getting this curve from these equations. I have got already this curve. For example, I can get this from an analytical formula. I can get this from finite element and more importantly I can get this curve from experiments. So I have got the curve with me. I am only fitting an equation.

My next job is to find out how I am going to express these factors BCD and so on. You just said that what I expressed BCD as a function of Fz, but Fz itself depends on these factors alpha. No, no, Fz is a normal force. It varies depending upon how or what is the way you do a maneuver or how severe is your cornering. How they would manifest in terms of the slip angles. Absolutely, so that is why I am now expressing this.

In other words, as I told you I have a number of graphs here and I want to express all these graphs in terms of one equation. Fi and Fx are also depends on each other, right. No, no we are not like. That is a good question. So we are right now looking at independent cornering this equation now. That is what I said right in the beginning. We are now looking at cornering separately and breaking separately. I have not come yet to combine cornering and breaking.

We will develop first a simple mathematical model in order to say that, what it is or in order to enumerate what are the things that act when I have combined cornering and breaking and then we will indicate how it is done. But even today most people they do not do combined cornering and breaking and all these formulas that I have used even today are only for a decoupled longitudinal force and a decoupled cornering forces.

Why are we interested only in kappa and slip angle, because they are the only parameters? Of course, these are the parameters, ultimately I am finding out. Why are we interested ultimately, we are finding out what is kappa, what is a slip and so on.

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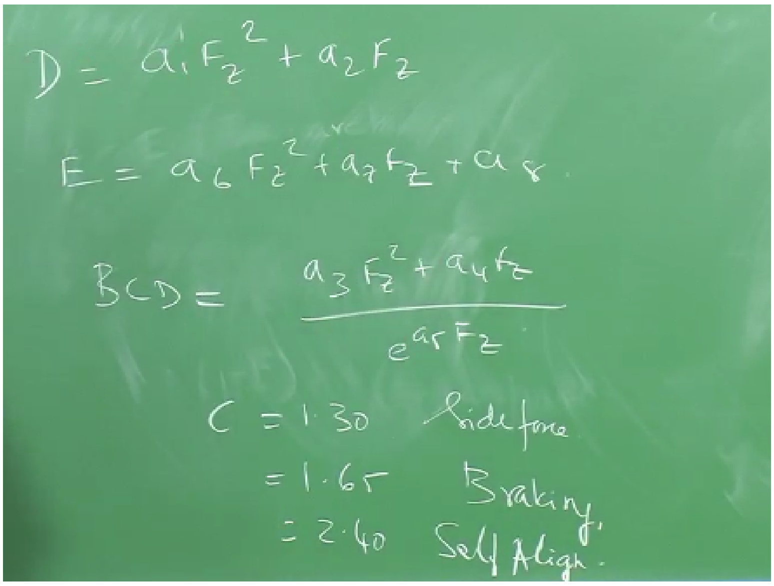

So what am I doing? I am going to write this as quadratic equation A1Fz square + A2Fz. How can you say BC are independent of the x. They might also be dependent on this. No, no, BCD, when I say B*C*D that is the initial slope. That is all I am saying. It is the initial slope dy/dx at X=0 is BCD. They do not depend upon x, y, z. That is the initial slope. Now these are the parameters. Let me reiterate these are the parameters which gives the slope of the curve.

That is the most important point. This curve depends upon Fz. I have repeatedly said this and I want to write that. So D=A1Fz square + A2Fz and E=A6Fz square+A7Fz+A8, BCD the initial curve is A3 Fz square+A4 Fz/E power A5Fz and C. These are initial curves than later models have changed to C. Let us stick to this first model. C=1.3 for the site force and =1.65 for breaking and acceleration and are symmetric = 2.4 for self aligning time.

So now what essentially I have done is I have shifted the owners of this curve from BCD and so on to A1, A2, A3, A4, A5, A6, A7, A8 and so on. So in other words I have now a set of parameters A1, A2, A3, which would capture these curves. All these A1 to A8 are determined from experiments. They are all determined from experiments. These are only coefficients that you would feed. Yes, these are the coefficients.

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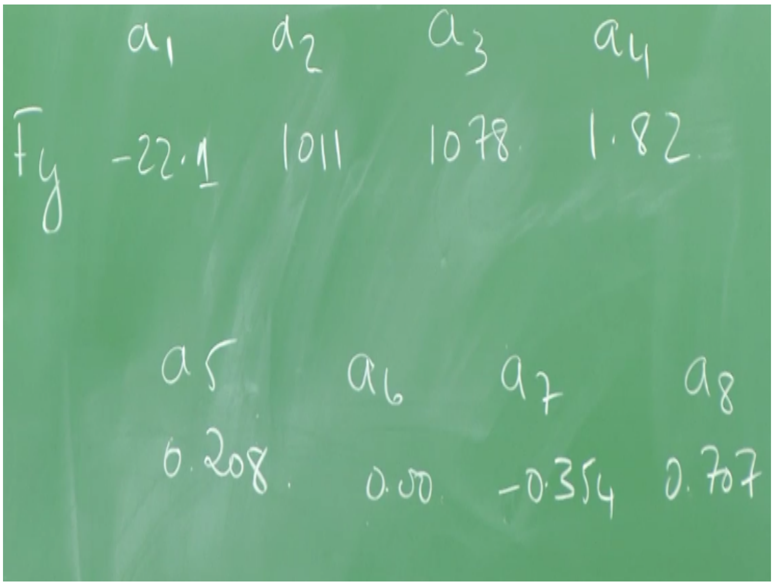

If you want the type of coefficients are how it would be, so you can say that for example FyA1 would be -22.1. Please note that it is not necessarily positive. It may be positive or negative. A2 will be 1011, this is in terms of kilonewtons. This is the tendency for many people in the tire industry even today to use pound force. Then these coefficients would be accordingly adjusted. For A9, we will see how this comes. Then we will have properties for A9, A10 and so on.

So A9 and A10 and other things which we are going to see now in a minute, we will then give the values. Take care of the camber aspects for Fy. We said that the camber gives you a camber thrust in other words, that gives you an Fy. So that would again be a shift factor and that is the shift factor is given by, due to camber.

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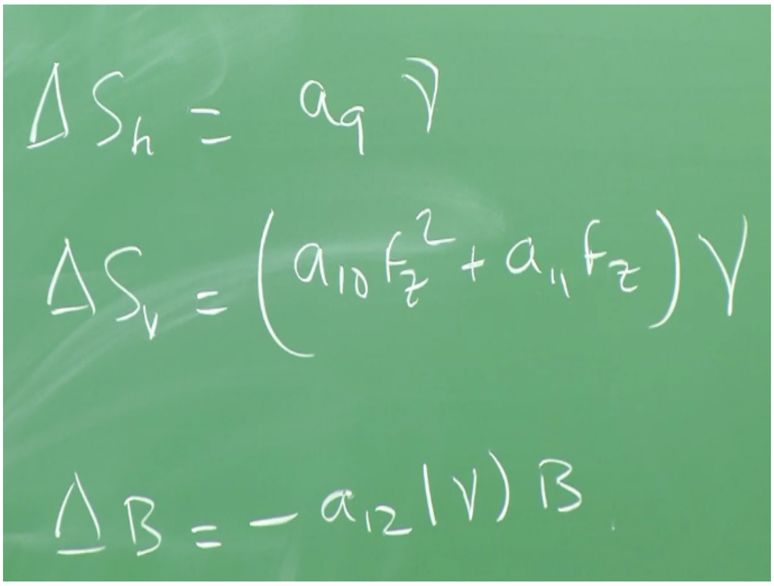

Horizontal shift factor for the camber is given by A9 gamma and gamma is the camber angle and the delta B change in stiffness that is also obtained as delta B gamma. Again there are number of parameters. I will not confuse you. I can again put on A9, A10, 11, 12, there is another 13, which again factor for E and so on. Let us not worry about it. So I do not confuse you. Let me summarize what all we said.

In other words, what we said is that we have an experimental curve and the experimental curve has to be input into my, say for example, into my mathematical model of the whole vehicle. Because I am interested there to determine the forces. I want to know whether my tire would develop that forces. If it develops the force, what would be the slip angle or what would be the slip, at which this forces will be developed, where do I sit actually in that curve and so on.

And that I have to meaningfully take into account what is the change in the Fz values or the normal forces that are acting due to the dynamics. So in order to take into account all that, I have an experimental curve and then these experimental curve is fitted by means of an equation, which has so many values A1 to A13. So this is not (()) (42:05). This is for one tire. If I have another tire, these values will be different. In fact, I do not have time, but if you are interested, we will discuss it later.

As to how to fit this, it is very important that I get some unique parameters for this formula. So fitting this becomes very important that I had a student who worked on this and so there is a way of fitting this. So what do you get, I get a curve. Then how do I get from this, I get a curve, not one curve. I have to change Fz. I will get a series of curves. I have to change gamma. I will get a series of curves. With all these curves, now I have to fit A1 to A13.

There are special softwares available for it and it is quite an amount of research tool, it is an optimization problem that can be used in order to fit A1 to A13. Definition dE, dCD changes will show effects. No, no, definitions do not change. They are all the form of this thing. The values of A1 to A12 would change. Camber the same thing, this is shift and I call this as delta SH. This is the shift. The same way I am going to shift. That is the shift.

Why you are not considering coefficient of friction. Yes coefficient of friction automatically comes in. Because I am not interested in mu. That is exactly what I said right in the beginning of the class. If I now replace Fx for example by a normalized Fx/Fz curve, people tend to call this as a mu curve. Here I am plotting Fx directly. So it is also important that brings out an important topic as to what is the role of friction. How do people determine this graph.

There are 2 ways in which they determine this curve. One is by doing an experiment in the road by a tractor trailer. For example TNO in Netherlands, they have a facility to determine this by taking the tire to the road and finding out this value, but many, many people what they do is to do an indoor test. They do an indoor test and find out, they have a machine on which they have a surface, on which this tire is mounted.

And they have facility to measure these forces and the slip, slip angle and so on and they determine this in the laboratory. There your question of surface becomes important because the question is how do you characterize mu. So the surface becomes important. So people have various surfaces, if they have to do this in the laboratory. This is a big topic and these surfaces are actually do have a roughness factor, which would simulate or hold good even if you have to go or these equations would hold good even if you have to do it in the road.

Next class we will see this. They mount a tire on a spindle and they have a flat surface, which is the surface may be moving and then they measure forces, slip and so on in the laboratory scale. There are many tire companies who have this kind of facility in order to determine Fx, Fy, and other factors. How these effects with mu with the coefficient. No, because it would depend on Fx. To a certain extent, I would say that this is a good question.

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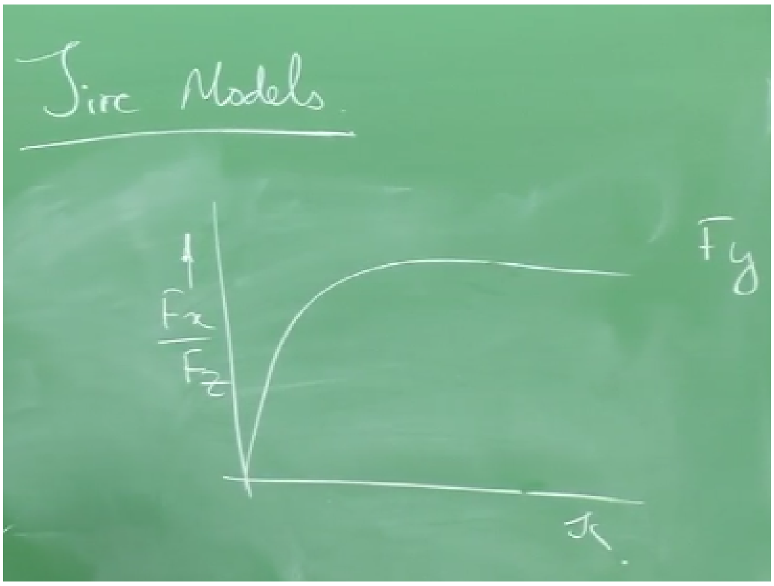

That is why people in those days, they used to do, what they used to do, is just normalize this curve and then plot Fx by Fz and then have the kappa value and then have this curve and they say that this normalized curve is fine, but then this brings out a very important topic on friction itself. The friction coefficient itself, can this curve be like this. So there being a lot of work as I told you before to how actually friction works between the road and the tires, big topic by itself.

So this depends upon as I said, not only the contact pressure, but what is called as V, you know slip and so on. If you want to model in a very detailed fashion, the tire behaviour, then you have to go into that friction models. When all the systems that we have are based on this, we try to get the slip, so that the loss of force is done. Now if the mu in the road and the test conditions are totally different, right. Absolutely that is exactly what he was asking.

What happens when there is a friction coefficient is different. So you have to be careful. The question is if I do it indoor, would not I get a different curve than if I do it outdoor in the road. Because the road characteristics are different, absolutely, there will be a difference between the 2 , whether you do it indoor or whether you do it outdoor and test. In fact, there is a paper in tire sense and technology by continental tires, which actually bring out and tell you what are all the differences on friction coefficients and so on.

So there will be, but the theory is such that the differences are not very high. It also depends upon the enveloping characteristics of the tire. So we would say that there would be a difference, but for all practical purposes, people use the data which is generated on this kind of rough paper or sand paper or whatever you want to call it, that is the type of thing that is used in order to get this state. Any other questions?

Where are the characteristics of tires incorporated in BCD. Yes, I am not looking at the tire design here, tire characteristics. I am looking at the final result. In other words, the question is how does the tire characteristics affect this. For example, if I change the compound or if I change the side wall profile, if I change anything in the belt angles, where is it reflected here. That is the question. It is not reflected here. This is for a tire, a given tire you are doing an experiment.

But there will be some formula that link A1 and all those. Absolutely go and look at the paper which we published, that is a very interesting question in fact. I and my student, we published a paper on how to link, very good question, I am very happy because it gives importance to the paper, which we had written. So how A1 for example is affected by design. In the next class, I will give you the reference of the paper where we have linked.

It is a very important question, because ultimately the tire manufacturer wants to know, how he has to change the design in order that he would vary A1 and so on. So there is a link between this and the characteristics. In fact, you will be interested to know that these are all affected by inflation pressure as a role to play as well and lot of design parameters. We will postpone that till the next class.