

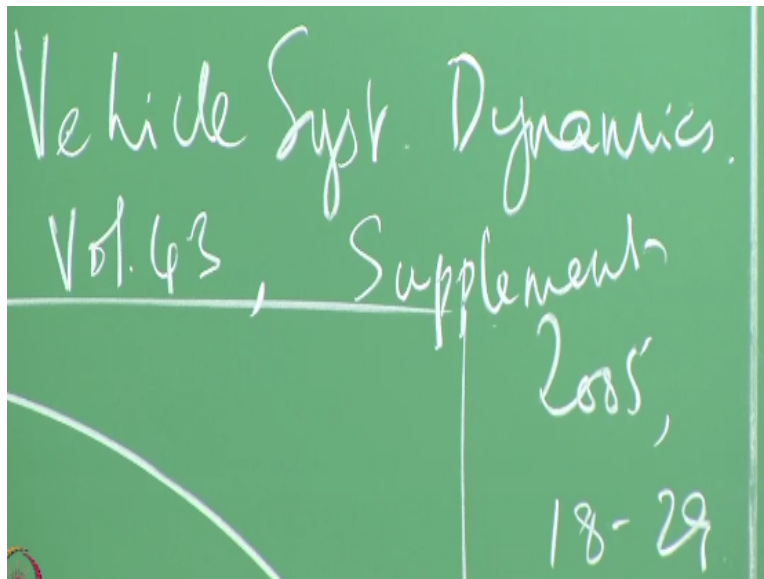
Vehicle Dynamics
Prof. R. Krishnakumar
Department of Engineering Design
Indian Institute of Technology – Madras

Lecture - 16
Classification of Tyre Models and Combined Slip

In the last class, we were looking at brush model, right and then we moved on to what we called us a magic formula model which is due to Pacejka, right and we saw that we saw that the basically the first model that was put forward in the Pacejka mode, tire model is to fit the experimental curves that were obtained during tire testing, right and we will now slightly go into the details of tire models before we nothing we have to quickly go to the lateral dynamics.

We will not have time to go into completely the tire models. There were some questions on in fact what happens during combined slip? We will spend a few minutes on combined slip before we go to the rest of the topics including the tire model, but there were lot of questions on tire models out of the class so let me explain this. This is from a paper in vehicle dynamics.

(Refer Slide Time: 01:30)



A vehicle system dynamics, this is the reference. This is a very nice figure. Volume 43 supplement 2005, 18 to 29, [Alex Eichberger](#) and Marcus. So, this is the, this is the paper just this is a reference for this. Let us just quickly run through this so that you understand what we mean

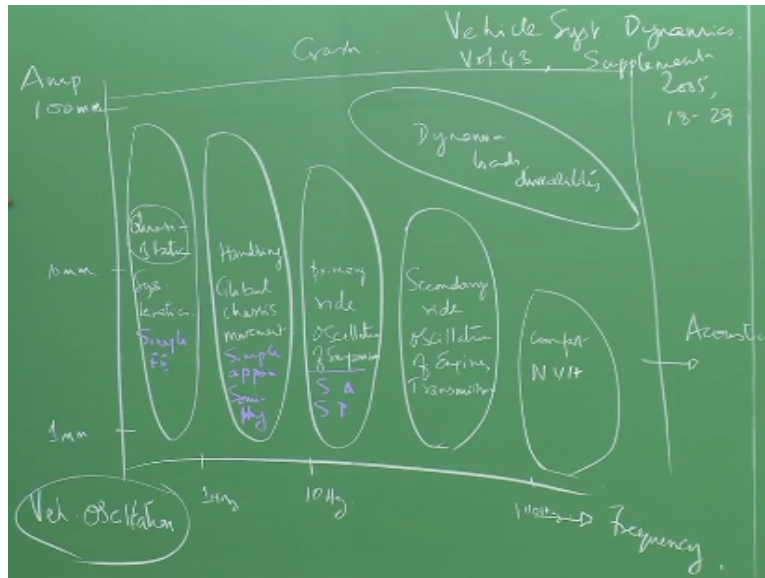
by tire models. As I told you earlier in the class to started that a lot of codes that are multi-body dynamics codes that are used and multi-body dynamics becomes extremely important.

Because of the fact that there have been a lot of companies use it and that the companies look at how to optimize the vehicles and so on right, so they use in number of softwares. The software that is used okay, depends upon the reason or why they use this or in other words it is not one comprehensive software and one comprehensive model, but there are a number of models and the models also depend upon the reason for which they are doing an analysis okay, and a part of it is the tire model.

Please understand that we are looking at the dynamics of the vehicle, we are looking at oscillations, be it cornering, or be it ride, or whatever it is we are looking at oscillations, and the oscillations are the frequencies of oscillations that we are looking at covers a whole huge range okay. And the models that you put in should be able to simulate okay, or get you the results in that frequency range, clear.

So, there is a frequency range and amplitude. **“Professor - student conversation starts”** What do we exactly mean by vehicle oscillation (()) (04:02). We will come to that a bit later, but before that okay, let us understand. Yes, I understand what you are asking let us look at this figure then you will see what we want to say okay. **“Professor - student conversation ends.”**

(Refer Slide Time: 04:21)



So the vehicle oscillations can be classified in terms of frequency of oscillations okay, and the amplitude of oscillations, right. So, yes when you when you talk about oscillations immediately you think that there is going to be a bounce when a vehicle goes over a rough road and you see that there is no bounce and so on and immediately you have a tendency to sort of map this word oscillation to a vertical oscillation right.

But it is not necessary that it has to be vertical that is what we are going to see now okay, and you will see that when we talk about oscillations it covers a gamut of things, right. It is like the whole electromagnetic spectrum which we talk about when we talk about light, we talk about sound and so on. Like that you have a number of you know the whole gamut of things that we were going to look at various frequencies and amplitude right okay.

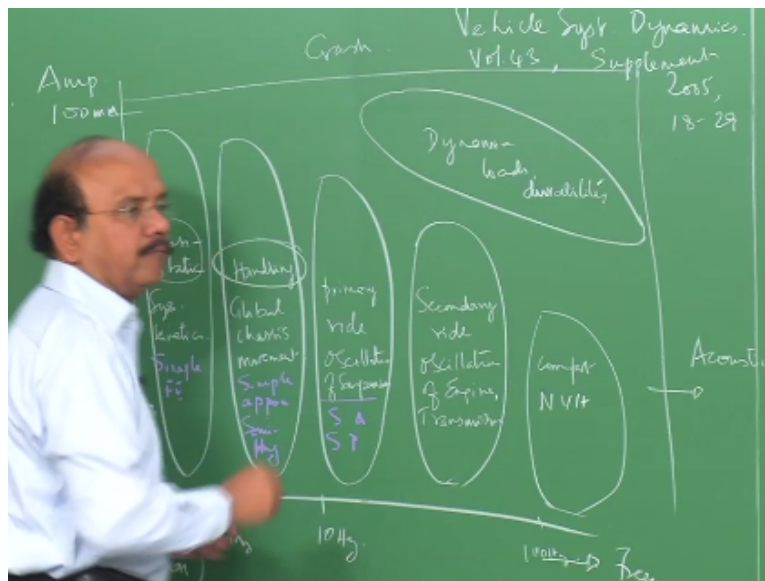
Now, at the lowest end we look at what are called as quasi-static models where we study the suspension kinematics okay. You had study say for example, four bar linkages and so on in your earlier class in mechanisms. So, you use that kind of concept to understand actually when you give a steering input how say for example, the steering linkages move okay. What is the arc that it covers and all those kind of things okay.

So, you use the fundamental concepts in theory of machines or kinematics of machinery which you have studied in order to understand that okay so that is the quasi-static models right. The

amplitudes may be very large because you are looking at when you turn a steering, how things move, and so on and many times the models that you use for a tire in this case is a finite element model and a simple static finite element model and so no right okay.

We will come to this tire models in a minute. Next, in the range look at this, you know this amplitude can be large. Next is what is called as the handling models.

(Refer Slide Time: 06:44)



Please note, model simply means equations. Next, we talk about handling models. We are going to do that from this class may be starting from next class right. We are looking going to look at the forces that are acting, the accelerations and so on. We had already seen this for the longitudinal case handling is more to do the lateral forces and the lateral dynamics and, that we are going to see as the next step and so, we are going to look at handling models.

In these cases, as we are going to see now you consider the chassis as one rigid body and look at how for example, the chassis is going to yaw or the body of the vehicle is going to yaw is going to roll, and so on right. So, we do not go into the details and usually this kind of models look at a picture where we look at the whole chassis as one group of components.

Yes, **“Professor - student conversation starts”**. What is the frequency change for that.? Yes, good question. So, what is that we are looking at. We are looking at various frequencies you will

understand that because, this depends upon see when we look at handling it depends upon the frequency of the whole vehicle, okay, of the vehicle at various modes or in other words in various directions in simple terms, okay. You would see that we will come to this handling when we take a turn okay. **“Professor - student conversation ends”**

You would see that certain frequencies okay, which are the result of the whole of the vehicle mass, moment of inertias and so on are going to have a role to play okay and those frequencies are in this range are in this range okay. The frequencies which are going to play okay when we look at lateral dynamics handling is about 4 to 5, 3 or may be 2, 3, 4 you know that is the range. **“Professor - student conversation starts”** Sir, but lower frequencies will also be there right?

Yes, of course see the point is this, is it that I am neglecting all this, no. What we are saying here is that we look at different frequencies to look at the effects okay, look at the effects. **“Professor - student conversation ends”**. For example, if you are looking at noise, yes you may be cornering maneuverability. If I am looking at maneuverability of the vehicle and so on the frequency range I am looking at is this, but at the same time I am looking at the NVH, the vibration characteristics and so on and then I would comfort.

For example, I would look at these frequencies. I am looking at actually acoustics, okay. I would look at higher frequencies. It is the vehicle is moving it is that you are artificially distributing it at these frequencies and studying the effect okay, at these frequencies. Go and study, when I say that go and study the frequency at this range which means that you are looking at handling and that is the result of characteristics of the vehicle when it takes a con right, and the right.

For example, the next set of frequencies that you are looking at is the right. Please note, why I am saying this please note that the model should be capable of representing the vehicle in these frequencies. It should be able to represent the vehicle in these frequencies okay. Suppose, you take a simple model what we are going to see for example bicycle model for handling okay. Then if you tell me that I want to study the acoustics, okay with this kind of model you will not be able to study okay.

Because it will not produce the kind of frequencies, the natural frequencies of modes, or whatever we call we are going to give some names to it okay. These things will not be represented by that mathematical model okay. So, there is no point in having a simple model which we were going to say I mean study and say that I am going to study this because that model will not represent the behavior of the vehicle at these frequencies.

Does that point make it clear? But, as a vehicle, yes, as a vehicle it has various spectrum of behavior as a vehicle, okay. But many times I will not be able to take everything together and start working and my interest also may not be on all these frequencies right. **“Professor - student conversation starts”** Sir, does this hold good for all vehicles or it holds good for every vehicle, it does not you say a heavy vehicle it is okay.

The frequency range may be this side that side, but it is around less than 10 okay. **“Professor - student conversation ends”**. Let us look at the next set of frequencies I will take the question after I finish this okay. Then we look at the next set of frequencies we are looking at ride okay looking at ride. Ride usually is divided into 2 categories okay, 1 is what is called as the primary ride and the other is what is called secondary ride purely, in terms of the frequencies.

The lower frequencies at 10, 12 Hz okay, you would call this as a primary ride okay. People vary when they define what is actually the primary ride, what exactly is the frequency and so on, but usually we will look at the primary ride as one due to the oscillation of the suspension system okay. This is because of road-induced oscillations and so on 10, 12, 14 Hz is the type of frequencies that you would call when there is when the vehicle is subjected to these kind of oscillations okay.

At those frequencies we call that as the primary ride, but then ride is a topic which is not only affected by note this carefully that is not only affected by the road okay. Today, ride encompasses a much larger things okay. So, for example, there can be oscillations of the engine transmission and so on which may be at a much higher frequency may be of a 30 Hz okay.

So, there may be oscillations of the engine okay which may be transmitted and your engine movements may not be very good, there may be transmission of oscillations due to engine and so on, okay. Then the frequency of excitation due to these factors will be at a higher level or higher frequency and that is what is called as the secondary ride right. Then comes harshness okay, what we call as loosely as NVH that is comfort okay that is at a higher frequency at a much higher frequency.

So, in fact if we look at the total ride how comfortable I am in a vehicle then it actually covers the whole thing okay. You can also see that as I go down the frequencies actually the amplitude comes down very well in our case right, the amplitudes comes down. Now, you move to a much higher frequency, okay which is above 100 Hz okay, strictly speaking going up to 20,000 Hz.

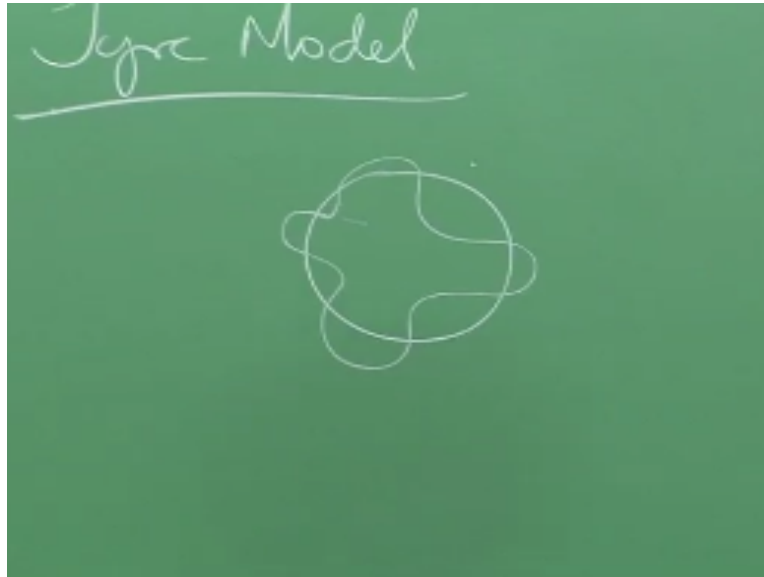
But we are not interested up to that because that is the acoustic range, but 20 to 20,000, but we are interested in that kind of large ranges, but the acoustic frequencies which is of interest to us may extend up to about 2000 Hz. **“Professor - student conversation starts”** yes, what are the quasi-static oscillations you interested in? The quasi-static is very I am looking at the suspension mechanics or mechanism okay sorry steering mechanism so, suspension kinematics, steering kinematics and so on.

Why does not that fall into primary ride then. Yes, so these are oscillations okay. These are then I take say for example when I give a steering input okay how does actually what is the range to which this steering has an effect on the wheel and that whether Ackermann steering is completely followed or there may be error what is Ackermann error, scrub radius all those.

Sir, how does it affect oscillations? No, they are not that is why we are saying that it is the much lower frequency that is why call this is as quasi-statics very near 0. It is though it is not actually static we call that as quasi-static okay right. What do you mean by (()) (16:09). We will come to that. **“Professor - student conversation ends”** You know these are frequencies when the vehicle is subjected to that kind of frequencies, okay the frequencies that whatever happens to that frequency range we call that as NVH.

It is just for us okay to understand and give a name and I am going to talk why we have done this as well. Now, I have to have these are the frequencies that which I am going to study. Okay, I have to have models as I said, which will depict these, the behavior in these frequencies, and one of the most important model here is what is called as the tire model, the tire model, okay.

(Refer Slide Time: 16:48)



Let us go to the extreme range yes. **“Professor - student conversation starts”** sir, you mean to say that in vehicle these all frequencies are available and at each frequency these (()) (17:08) available. Yes, these are the frequencies okay which will excite the vehicle, various components of the vehicle, vehicle as a whole various components of the vehicle. Okay, good question. **“Professor - student conversation ends”**.

Let me going to that with the tire model. Let us go the extreme case okay, now I want to study noise, say for example, I want to study tire noise okay, which means that I am looking at frequencies which are above 100 Hz okay that is where the usually that is a big topic usually the range of usually the range of hearing this from 20 to 20,000 Hz and a very interesting factors of come about life for example, your ear is very sensitive to frequencies of about 1000 Hz okay.

So, that is where we are very sensitive to this and so there are the magnitude of noise okay expressed in decibels or now adjusted and you have scales like A scale which adjust to the sensitivity of your noise and so on. So, your sensitive as a precipice you perceive vibration okay

due to various parts of your body, right. I mean so in fact we will see later when we go to ride dynamics okay, the last part of this course where we will look at right dynamics.

We will look at what is the amplitude which your body can withstand without fatigue, how long it can withstand, what is the frequency, and what is that corresponding amplitude for example, how much you can tolerate all these things we would look at it later in the course, but if you look at acoustics your what your ear is sensitive to those frequencies are above this.

Okay, your ear is very sensitive to frequency that is why you are able to enjoy music whose frequencies okay what we call in Indian classical music as swara what is that that is nothing, but the frequency okay. It comes from the Archimedean Principal of dividing the frequencies into different octave and so on. Let us not go get into those details, let us look at this model you know let us come back and look at the models okay. Now, to answer his question like we are looking at acoustics. Let us see actually physically that has been a practice in this course.

So, I have a tire okay, now the tire goes over some rough surfaces that is all of us know. Then the tire we saw in our earlier classes say for example, there are belts that are available in the tire okay. So, the belt starts now vibrating. It is something you know you have done some course in mechanics, it is something like a shell okay it is stiffened shell or an auto tropic shell okay, that is starts now vibrating because, it has gone over a rough surface, right.

So, when it starts vibrating Let us say that it vibrates like this, it vibrates like this, okay. Now, this vibration of the belt okay, which is actually the sum of various modes or mode shapes of the belt okay. Now, these vibrations of the belt what does it do, it displaces see it starts vibrating like this. So it takes inner and an outer shape goes up and down.

So when it goes up and down like this what does it do, it excites or passes on this to the air molecules which are near this okay that is one of the thing so it starts oscillating. So, the air molecules near it get I mean oscillations and there is pressure changes and this acts as a source okay for the pressure change and at certain distance you hear that as noise because these guys are

displacing the air above and below and then that is the pressure wave that starts so that is the source that is one thing.

The other thing is that when they vibrate okay, when they vibrate they also change the forces that are acting in another words if you now plot a force or acceleration at the axial level they also change. Okay, so in another words with time the accelerations change and these accelerations now start okay traveling inside the vehicle and if you are sitting in the driver location you know you are going to hear that as well.

So, in another words the noise that is produced okay passes on through the air and from the air it may be it is translated or transmitted to the door and it starts oscillating and we are going to hear all those noise. So, tire for example, tire noise can be classified into what is called as an air-borne noise and this structure-borne noise through the axial and gets it inside and then you hear this.

Now, if I have a model since you asked this question, now if I have a model which cannot actually capture this kind of belt vibration you know what we are looking at is also a brush model simple mode now I am not able to capture this then whatever I do obviously I will not be able to capture noise. Hence, that is why models become important so what model we use unless I have a model which I am able to capture this which is the source of vibration or excitation at that frequencies I am not going to produce that frequency in order that I hear this right okay.

Our simple brush model for example will be enough to do things here, but then I need to keep innovating and improving it in order that we do it further. Yes, **“Professor - student conversation starts”** why are we interested in only sound and noise characteristics? No, I am not saying that why are we interested in not I am saying that noise is one of the factors okay, it is not that I do not want to make too much noise right.

So, if we consider some other effects of vibrations then there may be 10 Hz and 1 Hz also ((
(23:44) yes, every frequency is of interest okay. So if you are looking, if you are going to make a car and sell it, okay you are going to be worried in this whole spectrum of frequencies.

“Professor - student conversation ends” Your car is not going to sell, if you say my handling is very good, but only thing is you cannot sit inside the noise is very high right.

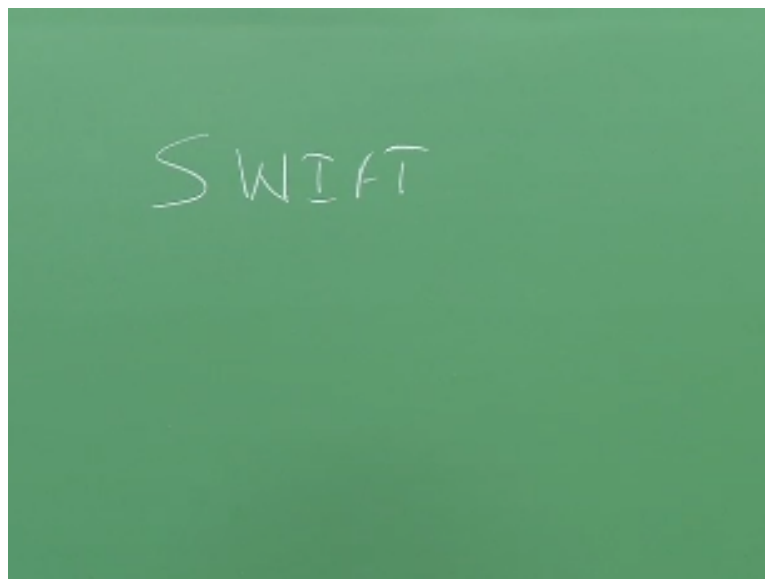
The engine noise is very high when you sit inside, but I can handle my car handling is very good.

“Professor - student conversation starts” So every component that is will not work. Only engine would have vibrations corresponding to exactly so at different frequencies, so that has to be now taken care of, isolated, so that you do not hear or you do not feel whatever is your sensory perception is of these frequencies. **“Professor - student conversation ends”**.

It is very important to understand this. Yes, this is a well know fact in the industry, but you are all students that is very important that you understand okay, what we are taking about and the macropicture is extremely important for you to appreciate the course on vehicle dynamics. Okay, I would have given this first, but you will not understand now that you know what is a tire model?

Now you know you are able to under that very well, okay. So, this is very important for us to look at right. So, the tire models that are used are now different at different in the different frequency ranges. You have today a set of tire models which can handle different frequencies okay. You have for example a model called swift model.

(Refer Slide Time: 25:25)



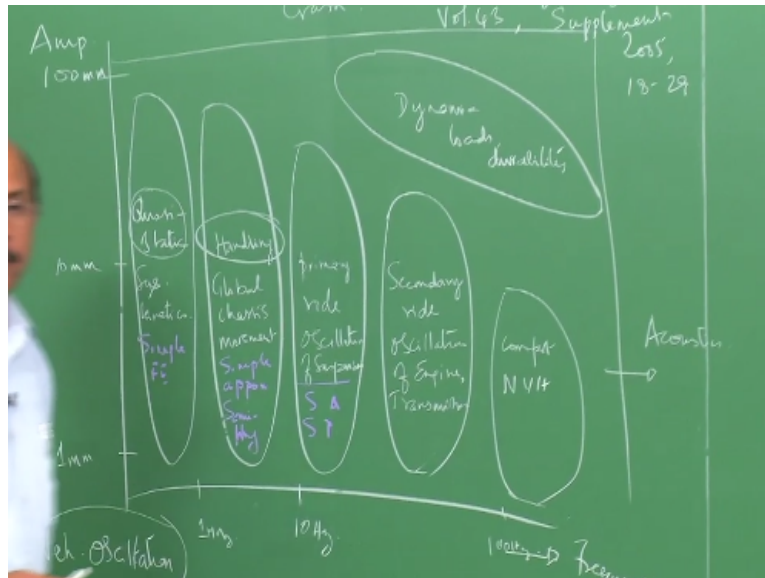
The name itself is like that. Short wavelength, intermediate frequency tire model right, so that may be about 70 to 80 Hz and that is the frequency which it can accommodate and that is what you see. So, most of these models are yes we saw an empirical model, but there are semi-physical models as they are called.

In other words, that tire is modeled as a consisting of a number of springs okay, mass systems and so on and that approximates the behavior of the tire. We will not be able to go into the details of it. We have another course actually on tire mechanics where we would talk a lot more about tire we are not going to do that in this course, but I want you to understand where we talk about and what we talk about and what we are looking at and so on right.

So this is what is the importance of tire model right and this what we call us multi-body dynamics. This is multi-body dynamics, the range of multi-body dynamics applications also looks at the dynamics loads, the effect of dynamic loads for example, when you come to automotive structures, you would look at this carefully, you will look at dynamics loads, what is the load that is transferred for example, to various parts of the vehicle for example to the chassis frame okay.

What is the effect of this dynamic load it is affecting, in other words what is the durability, and all those things also will be studied. What is beyond this is what is called us as crash okay, crash is a crash covers a gamut frequencies, a very high frequencies and amplitudes are also very large. So that is the crash part of it.

(Refer Slide Time: 27:20)



Okay, so nice picture gives you the whole thing and let us get back to what we were doing in the last class right. **“Professor - student conversation starts”** last class you told like it is since it is low frequency it does not affect the say for example if you take handling you said yaw. Yaw how is it affect here (()) (27:40). You have to wait for some time to get an answer so what is handling, what is yaw, how does it have an effect all those things will be studied in this course.

So in other words you will get into models here okay. I will not be able to get into these models beyond this, but I will get into models which cover this and this you know these parts are a part of this course so you have to wait for sometime in order to understand and what I mean by the second thing right. We will do that. But frequency, we mean at particular incidence different components has different frequencies yes.

So, different, it is different component of the vehicle you know this is we are talking about the vibration of the vehicle okay induced due to various components. **“Professor - student conversation ends”**. Okay, for example, for example when you are looking at the primary ride okay, you can treat this whole body as a rigid body and we are going to look at that as a rigid body right at those frequencies.

So in other words the frequency is dominated by the spring dashpot system was a suspension. Okay, so when I go to higher frequencies then there are contributions at high frequencies and that

may be due to bending of the chassis frame okay. So, in another words the source for these vibrations and the way they are transmitted and so on okay would be different right at different frequencies so for an example when I am here I am looking at the body as a whole controlled by the suspension characteristics, right.

Wait for some time you will understand more about this as we go long. **“Professor - student conversation starts”** Sir, how do you at the road at the low set of frequencies the suspension will be handled how would you know that suspension kinematics. We are looking at suspension kinematics, we know handling is affected by these frequencies, so wait for some time and we will come into those things okay.

We will look at these things why I am looking at this frequency, why is it because this frequency when I say for example when I take a turn and we are going to look at this from a very simple bicycle model and then if I take a turn then the frequencies which are excited okay, is in that range right. **“Professor - student conversation ends”**. Okay, we are going to look at the mathematics behind it and then we will understand why these frequencies are, why this frequency of 12 Hz okay.

You will see that there is what is called as a wheel hop frequency and the wheel hop frequency is 12 Hz you would understand that there is a body, you know the frequency of the whole body that would be a 2 Hz you have to wait. Please, be patience to understand these frequencies.

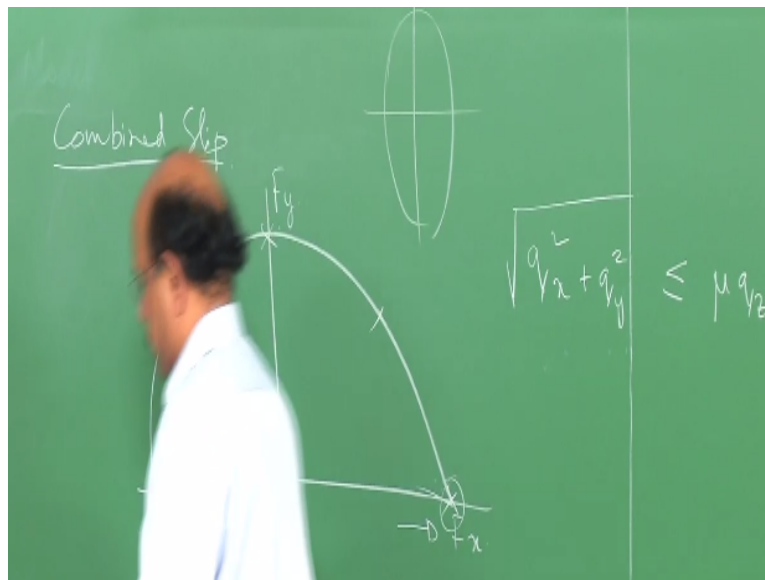
Understand this alone that vehicle as a whole as a gamut of frequencies which can be excited okay which can be excited and so you have to understand the behavior of the vehicle at all things. In other words just to complete this it is not that I have to excite all these frequencies. In other words I can isolate frequencies okay by different means at different levels. For example, I can isolate completely the noise due to the engine there are ways and means by which you can isolate this noise completely.

But that does not mean that the technique that are used in order to isolate this noise where to make by handling better. So, you have to look at what you have so the idea of studying this is

because, the solutions as a design procedure or a design problem is different at different levels. So when you see that this vehicle in other words when you say simply put it, when you say that this vehicle handling is fantastic, but my ride is not very good or vice versa okay.

What is that you are saying? your saying that the vehicle behavior or you are characterizing the vehicle behavior at different frequencies that is all okay. You are characterizing that is it? Okay, let us move on let us now look at combined slip.

(Refer Slide Time: 32:27)



I am not going into the details I thought I would do it, but I think we are running out of time we have to move to the next topic on lateral dynamics. I just want to point out that when we have combined the slip we are looking at longitudinal as well as lateral okay. Both of them are acting together okay. In other words the forces in the longitudinal direction and the lateral direction are going to now compete with each other in order to win over the $\mu \cdot f$ or f right.

So, if I now plot say for example the F_x , and the force F_y okay if I now plot this graph this would okay because of certain reasons of anisotropy and so on would become an ellipse something like this. In other words, what did simply means is that, this is due to longitudinal say breaking our acceleration and this is due to cornering okay. When I corner, when I do a cornering without any lateral acceleration.

I am going to get the full benefit of my μ this is the boundary after which I am going to it is going to stop and after which I have reached the $\mu \cdot F_n$ or I have cashed all that is that is in the bank called the friction forces. So, once I do not have any breaking on acceleration, okay that is the complete force that can be used in order to generate F_y and that is the end okay where I would generate the complete or I would use the complete frictional force in order to get this.

“Professor - student conversation starts” Why is this F_x , because this is nothing but the force okay. $q_x + q_y$ square okay that is the force say for example force per unit length should be less than equal to $\mu \cdot q_z$ per unit length is what you know q_z is a normal force per unit length okay and if you give it like this actually should be a circle, but q_x and q_y need not be the same you know like.

What happens in the longitudinal and lateral direction forces that need not be the same, because the forces are split in the ratio of the slip and so on and so the practically this would not be a circle, but it would be a sort of a distorted ellipse. What this implies that the boundary the μ and value is a kind different for F_x and F_y yes, so that is the because the way it is shared is different here. **“Professor - student conversation ends”**.

So, ultimately say for example if you are doing both cornering and braking that is the type of you know forces that you would generate. **“Professor - student conversation starts”**. ((
(36:00). This is the F_x and F_y . The major axis is F_x . This is an ellipse it looks. So you can say that that is an ellipse like this. Okay, you are putting only the one part of it. **“Professor - student conversation ends”**

(Refer Slide Time: 36:25)

$$q_x = -\frac{V_{sx}}{|V|} \mu q_z$$

$$q_y = -\frac{V_{sy}}{|V|} \mu q_z$$

$$F_x = \left[C_{px} \left(\frac{a^2}{2} - a x_s + \frac{x_s^2}{2} \right) + \frac{3 \mu F_z \left(\frac{2a^3}{3} + a^2 x_s - \frac{x_s^3}{3} \right)}{4a^3 \sqrt{a_x^2 + a_y^2}} \right]$$

Actually the q_x and q_y is given by I will just write this quickly when it is in the slipping region. **“Professor - student conversation starts”**. Sir what is q ? Per unit length I had already said that. Force per unit length okay **“Professor - student conversation ends”**. So and you can derive this and I am not going as I said going to the details you can look at Pacejka’s book so derive this similar same way as you had done it for the longitudinal force.

Now, include longitudinal and lateral force okay and then you can derive this and the effects now becomes look at that. I will quickly indicate after this how we got this that is F_x . **“Professor - student conversation starts”**. Sir, what is V_{sx} and V_{sy} ? Just a minute let me finish this I will explain it, let me write down this equation. **“Professor - student conversation ends”**.

(Refer Slide Time: 38:16)

$$p_y \left(\frac{a^2}{2} - a x_s + \frac{x_s^2}{2} \right) + \frac{3MF_z \left(\frac{2a^3}{3} + a^2 x_s - \frac{x_s^3}{3} \right)}{4a^3 \sqrt{\sigma_x^2 + \sigma_y^2}} \sigma_y$$

$$c_p y \left(\frac{a^3}{b} - a x_s^2 + \frac{x_s^3}{3} \right) + \frac{3MF_z \left(-\frac{a^4}{4} + \frac{a^2 x_s^2}{2} - \frac{x_s^4}{4} \right)}{4a^3 \sqrt{\sigma_x^2 + \sigma_y^2}} \sigma_y$$

I will explain this and this would be your assignment how we are going to get this though the expression looks formidable it is not very difficult to derive this it is quite simple. This is seen to sigma X sorry that is the whole thing multiplied by sigma x the whole thing multiplied by sigma y, and Mz. Okay essentially what is that what is that you do, essentially how did you get this. That is too much noise that is going to have an effect.

(Refer Slide Time: 41:08)

$$\sigma_x = -\frac{V_{sx}}{W} ; \sigma_y = \tan \alpha = \frac{V_{sy}}{W} ; \sigma = \sqrt{\sigma_x^2 + \sigma_y^2}$$

$$= -\frac{V_{sx}}{|V|} \mu q V_z$$

$$= -\frac{V_{sy}}{|V|} \mu q V_z$$

$$a x_s + \frac{x_s^2}{2} + \frac{3MF_z \left(\frac{2a^3}{3} + a^2 x_s - \frac{x_s^3}{3} \right)}{4a^3 \sqrt{\sigma_x^2 + \sigma_y^2}}$$

Essentially how did we get this? We got it in the similar fashion as we did before. Remember, how did we get this kind of expression for a longitudinal force? We looked at the deformation okay you remember that we looked at the deformation, we looked at how much the tread which

was a brush deform as you move into the contact patch and then at a particular point of time we said that the force is not good enough to sustain the frictional forces and it starts slipping.

This is exactly what you would do, you would now look at the brush movement both in the X direction and the Y direction.

(Refer Slide Time: 43:12)

The image shows handwritten mathematical expressions on a green chalkboard. At the top, the force component F_y is given as:

$$F_y = \left[C_{py} \left(\frac{a^2}{2} - a x_s + \frac{x_s^2}{2} \right) + \frac{3MF_z \left(\frac{2x_s^3}{3} + a^2 x_s - \frac{x_s^3}{3} \right)}{4a^3 \sqrt{\sigma_x^2 + \sigma_y^2}} \right] \sigma_y$$

Below this, the moment component M_z is given as:

$$M_z = \left[C_{pz} \left(\frac{a^3}{b} - a x_s^2 + \frac{x_s^3}{3} \right) + \frac{3MF_z \left(-\frac{a^4}{4} + \frac{a^2 x_s^2}{2} - \frac{x_s^4}{4} \right)}{4a^3 \sqrt{\sigma_x^2 + \sigma_y^2}} \right] \sigma_y$$

To the left of these equations is a small diagram of a brush, represented as an oval with several short lines extending from its bottom edge. Below the equations, the following definitions are written:

$$u = (a - \lambda) \sigma_x \rightarrow q_x = C_{px} (a - \lambda) \sigma_x$$

$$v = (a - \lambda) \sigma_y \rightarrow q_y = C_{py} (a - \lambda) \sigma_y$$

An NPTEL logo is visible in the bottom left corner of the chalkboard image.

So, the brush movement in the x direction, so you define sigma x and sigma y as given here and you would this is exactly what you did previously into sigma y. Now which now becomes the basis for the forces in other words these are the displacement of the brush remember that we looked at the brush okay in the brush models. In the brush models that is the we had those brushes so now please understand that there is going to be a longitudinal as well as a lateral right both are acting so the brush that the brush.

The bristle will actually be displaced in the x direction as well as in the y direction this is the displacement right go back and look at it this is what we did before. This displacement would result in a force which I would call as qx okay and the force is because of this displacement multiplied by the stiffness Cpx in the x direction, displacement multiplied by the stiffness per unit length you know gives you the force per unit length so that would be Cpx *a-x *sigma x and qy= Cpy *a-x*sigma y right.

(Refer Slide Time: 45:02)

$$q_x = -\frac{V_{ox}}{|V|} \mu q_z$$

$$q_y = -\frac{V_{oy}}{|V|} \mu q_z$$

$$F_z = \left[C_{px} \left(\frac{a^2}{2} - a x_s + \frac{x_s^2}{2} \right) + \frac{3\mu^2 q_z^2 \left(\frac{2a^3}{3} \right)}{4a^3} \right]$$

$$\left[C_{px} (a - x_s) x_s \right]^2 + \left[C_{py} (a - x_s) y_s \right]^2 = \mu^2 q_z^2$$

These are the forces. Now when it slides root of q_x square that is this whole sum of these forces okay should be equal to μq_z so in other words the sliding distance x_s is determined by root of $C_{px} * a - x_s * \sigma_x$ okay so this whole thing squared because q_x squared + $C_{py} * a - x_s * \sigma_y$ whole square = $\mu^2 * q_z^2$ remember q_z we got it from the parabolic distribution in our earlier class so you can substitute for q_z solving which you will get x_s .

Then what do you do you find out the total force okay because of deformation plus slip sliding rather. So rest of the region it is going to slide up to x_s it is going to stick, deform, after that it is going to slide. Try this out if there is any question I will answer that in the next class okay. Please try this derivation see whether you get this it is not very difficult, because you are just going to add then integrate it they are going to get this. Okay.

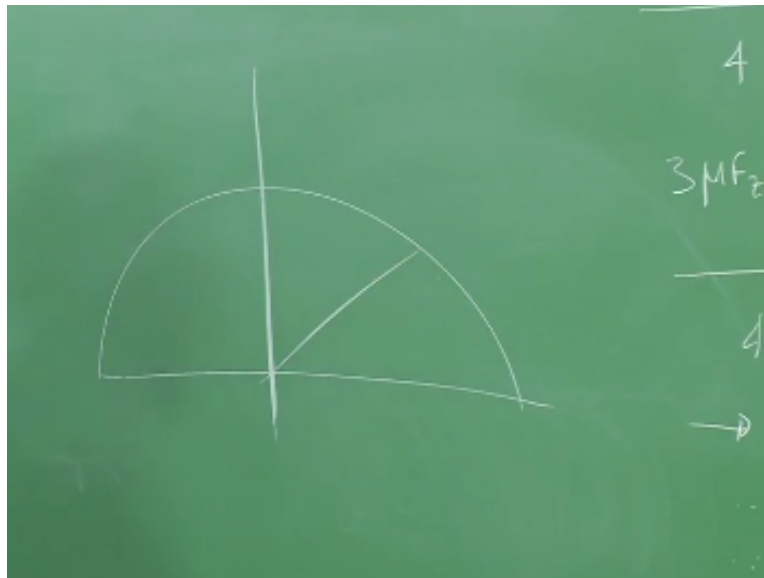
So we will not go further on this unless there is a question which I will answer in the next class. Try this out. Fine, we will we now know that a combined slip is more dangerous because whatever lateral force that is required is not given becomes traumatic in the case of motor cycle. q_z raise to what is called as high side fall. Okay it becomes really dramatic.

So, in other words when you take a turn in a motor cycle breaking the motor cycle and suddenly release the brake we will explain that may be in the next class starting that so that you will

understand lateral forces. Suppose you are taking a turn, braking, and suddenly release the brake what really happens now it is a very sharp turn, young blood going fast to take a sharp turn right.

Suddenly you see a kid coming crossing and just press the brake it is still taking the turn and then suddenly release it situation is very dangerous, why is it, because when I brake it.

(Refer Slide Time: 45:50)



Think about it we will continue that in the next class. Okay, when I brake it, I am at this position you know sudden braking, right. When I release it, when I release it, my F_x now comes to 0 okay immediately F_x comes to 0, but my F_y is not actually is not the same as what it was before. F_y now actually increases suddenly. So in other words whatever friction which the longitudinal force is holding now gets transmitted to the lateral force so lateral force becomes high.

Lateral force should be just enough to compensate for my centripetal acceleration. The lateral force becomes high whole vehicle okay turns over up, over turns. We will explain that in the next class before we go to the lateral dynamics.