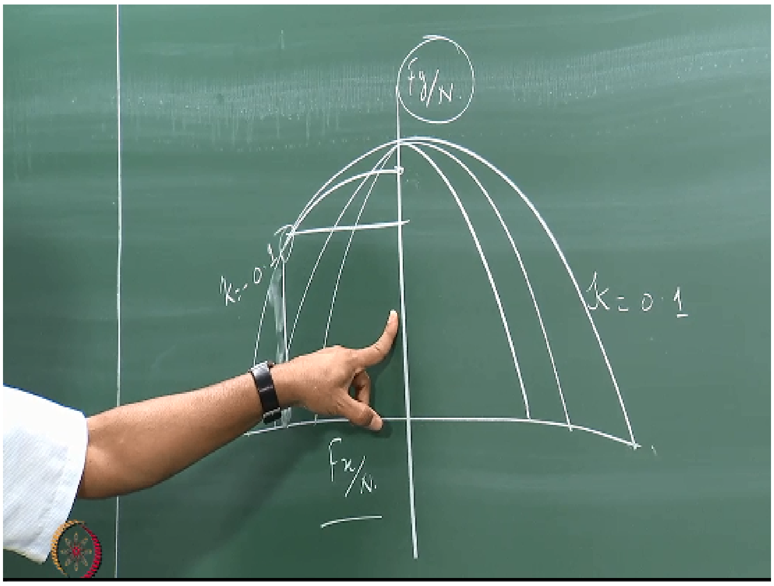
Vehicle Dynamics
Prof. R. Krishna Kumar
Department of Engineering Design
Indian Institute of Technology – Madras

Lecture - 17
Lateral Dynamics-An Introduction

We said that we will start the lecture on lateral dynamics. And we are talking about as a precursor when we started this in last class. We were talking about what we called as high side fall.

(Refer Slide Time: 00:40)



We said that there is the graph and both of them are acting I hope you are able to derive the expressions which we got we wrote in the last class. If there are any questions we will answer that in the tutorial class. And please derive those expressions which we said or which we gave or which we saw in terms of combined slip. The lateral and the longitudinal slip for the brush model.

Try it out or else we will do that sometimes in the next week in the tutorial class. So we were just talking about this as a precursor to the lateral dynamics. We said that suppose motorcycle is taking a turn. You are taking a turn in the motorcycle and that you are applying a break. Okay when you apply the break actually what are you doing you are getting both the longitudinal forces.

You require a longitudinal forces for breaking as well as due to cornering you also require a lateral force. Assume that you are the edge of the friction circle or ellipse as we can call it or in other words you are say at this position. Let say that you are at that position. In other words, you are making use of all the friction that is available. Suppose you let go the break that you have applied and you keep cornering what would happen that is the question.

Suppose you let go the break and then you continue your cornering. In other words, what would happen now? The longitudinal force goes to 0. The longitudinal force goes to 0 and suddenly you are now shifting from this corner where you are at the boundary to the vertical axis with the same total force. You are now shifting from here to here. In other words, the lateral force is more than what is required for the equilibrium of the vehicle.

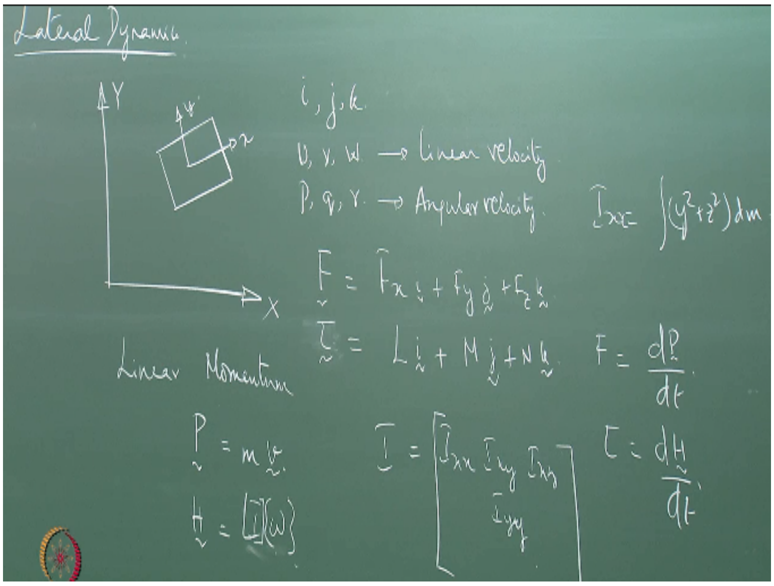
(Refer Slide Time: 03:35)

Because what happens is something like this. I have plotted the slip curves. See suppose you are at this point then if you now apply a break the slip force remains the same. In other words, you are moving from here these are the slip curve forces that are generated due to the slip angle. The slip angle remains the same, but you are shifting from this corner on to the Y axis. So in other words you are shifting from B to C position.

So because look at this curves how they are. This is the lateral force curve maybe this is= alpha = 3 degrees and so on. So you are shifting now to the Y axis. In other words, the forces the Y axis force which was this much has suddenly increase from this point to that point. So when the force has suddenly increased and is more than what need to be compensated for the centripetal force then there is no equilibrium there is loss of equilibrium.

So in other words you overturn. We will come back to this may be towards the end of this lateral dynamics where you understand it better. But I am only telling you that there are practical applications of what we are going to study. Now we will get into the complete lateral dynamics derivations so that when I come back you would understand this better.

(Refer Slide Time: 05:25)



Let us look at how we are going to deal with lateral dynamics. Let us go into some fundamentals of dynamics. Now I am going to deal with what is called as a body-centered coordinate of a vehicle. I am going to deal with what is called as the body-centered coordinate of the vehicle.

In other words, you can study the vehicle motion with respect to what is called as the inertial frame of reference which I would put in terms of capital X and capital Y or you can study this motion with a body-centered coordinates which I would put as X and Y. It is a usual practice to study the motion of a vehicle in terms of a body-centered coordinate rather than the inertial frame of reference.

You all have done engineering mechanics so you would understand what would happen when I have a body-centered coordinate. You would see that you have to have additional terms to take care of the realignment of the body centered coordinate in order to take care of realignment of the body-centered coordinates. In other words the body-centered coordinates.

What are body-centered coordinate? Suppose this is the vehicle so I have a body-centered coordinate in this vehicle. So the vehicle takes a turn the body-centered coordinate also takes a turn. So they also move along with the vehicle. In other words, I have to take into account this motion of the vehicle and the body-centered coordinates. How do we take this into account? First thing is that is what we are going to see and then we are going to apply Newton–Euler equation in order to understand the motion.

Let us say that the usual I, j, k is what defines the unit vectors along x, y and z. And let say that the velocity of the vehicle in the 3 direction are u, v and w. These are the velocity of vehicles in the 3 directions and let say that the rotation which we would call as p, q and r denotes the rotation of the vehicle or in other words the angular rotation velocity or angular velocity of the vehicle.

So this is the angular velocity or rotational velocity and u, v, w is the linear velocity of the vehicle. In simple terms what is P? P is nothing but the roll. Q is the pitch and R is the yaw of the vehicle. So we call this as p, q and r. Now what are the peculiarities if I am going to deal with this kind of systems that is what we are going to see, you know, like what are the additional terms that come into picture.

Let us say that the force acting on the vehicles. Let me write down the forces as Fxi+Fyj+Fzk as a 3 forces that are acting on the vehicle. And the moment M or let me call that as tau. Okay the moment that is acting at when they are all vector quantities I am putting this figure at the bottom to indicate that they are vector quantities. So let me call that as Li+ I will maintain the same thing so Mj+Nk**.**

What am I going to do? I am going to first write down the linear momentum and the angular momentum and I am going to look at the rate of change of this momentum and the result is the Newton–Euler equation that is all I am going to do. So let me write down first the linear momentum P=m, v and the angular momentum let me call that as H what is angular momentum i omega.

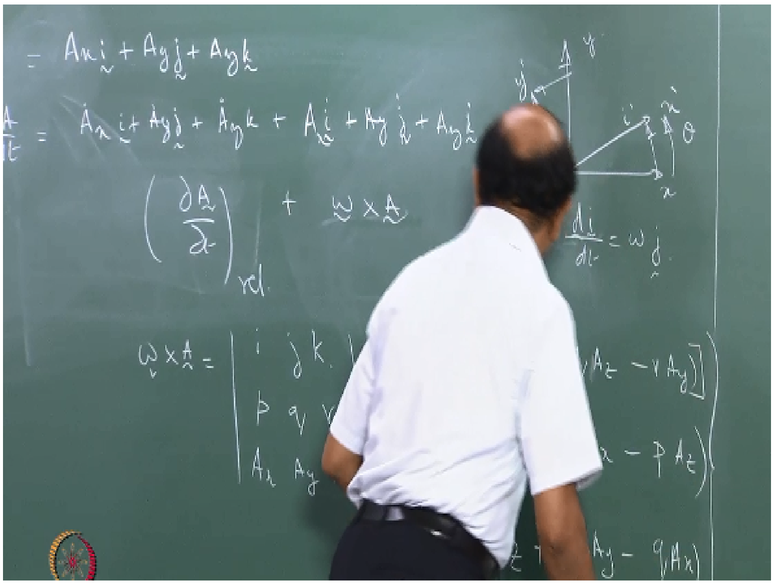
Now what is i now in this case the moment of inertia it is actually a matrix. So you actually have this as i Omega what are omegas angular velocities. How are we define this p, q, r. So I omega and what is i? i is if you want we can replace that with the matrix notation. You would have studied this as one single quantity along xx and so on. Now we have just extending this to 3d. So I am writing this a matrix and so Ixx. So this is Ixx, Ixy, Ixz, Iyy and so on.

So you all of you know the definition for Ixx integral Y square+z square dm. So you know the definition for Ixx throughout the volume of the body and you know also what is Ixy-integral xy dm. From this we are going to have some peculiarities we will come to that in a minute. We are going to write the Newton–Euler equations. So what does the Newton–Euler equations say?

So how do I if it is just F=Ma. We call this as a Newton equation. So if it is a Newton–Euler equation we also include the moment that is acting on the body. So we write F=dp/dt where P is the linear momentum of the body. Similarly, we write tau=rate of change of angular momentum. So far is very simple dynamics. Now we have to be careful in writing these terms.

We are going to write it with respect to the body-centered coordinates i, j, k I have introduced that for the body-centered coordinates. Now I have introduced them with respect to a body centered coordinates. Let us see how for a body-centered coordinates a particular vector changes that is what our goal now.

(Refer Slide Time: 14:27)



Let us consider a vector A. We are following Karnopp the reference for this is vehicle stability by Karnopp vehicle stability is the reference for this. So let us say that I have a vector A which I am going to express in terms of i, j, k which are the unit vectors in the body-centered coordinates. I want to find out now the variation or rate of change of A. So in other words if I now write da/dt. I write this as A dot Xi+A dotYj.

A can be any vector. It can be velocity we will see that. A is z dot K+ because I is not the inertial frame of reference and I also changes with time. So I have to rewrite that part so in other words clear. All of you from fundamental dynamics know how to write I dot. How do I write I dot. I dot, J dot and K dot. In order words Da/Dt. You remember that in our earlier classes you would have done this. Suppose I have x and y along with I and J are defined and if it is rotating we would say for example omega.

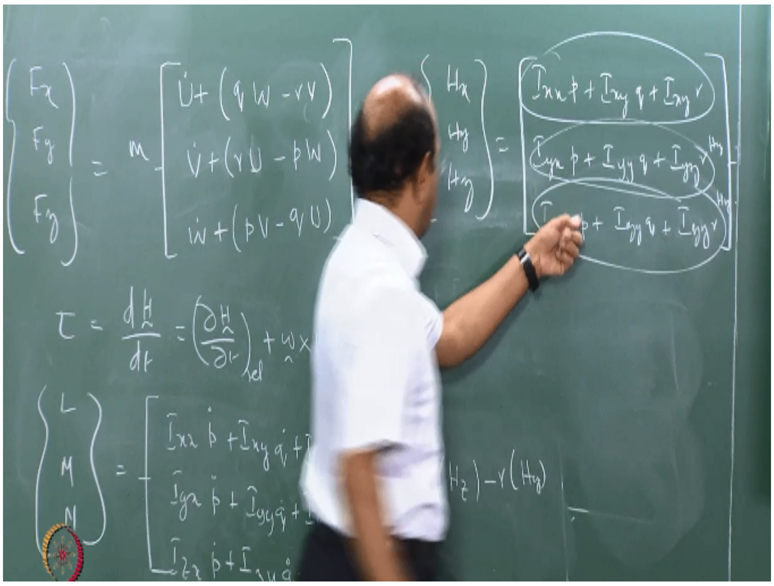
So the new coordinates due to or rotation of an angle theta gives you x prime and y prime. So the new value of I which let say that I prime which is=I+Di/Dt is given by the changes. Here which is along j same way along i and so what is di/dt? D theta/dt*j because this is unit vector I is unit vector R*theta gives me this. So this is R*theta. It is along the direction of J. So di/dt. Di is this di/dt is nothing, but d theta/dt. D theta/dt is omega* J. So this you can do it for all 3 and write down this part. This we will keep that +omega* A that what happens.

Omega in our case is p, q and r. So we will call this part we will give a new name to this and we will call this as dou A/dou t call this as relative. So what is the physical meaning of the first term? The physical meaning of the first term is simply that is what an observer who is sitting in the body-centered coordinate would see as a rate of change of A. So because I have to apply this my formula in terms of inertial frame of reference. Now I know that when the body-centered coordinates changes with time.

I take into account that as well by putting that omega*A. What is omega* A all of you know this where omega*A. How do I write that i, j, k. What is omega? As I said before p, q, r and A is Ax, Ay, Az. So now Da/Dt if you write that in terms of this write it in terms of that so what would happen for I A dot x+ pick that up from there +I*qAz-r Ay the whole thing is I right. If I want to express this as a vector then I would remove that I and just write that as a vector plus after all (()) (21:52) that is a vector.

Write down A dot Y+ what are the terms that are there that is it that is for the j term. rAx-p*Az. And A dot z + p Ay-qAx. So that is Da/Dt. Now I can apply my Newton–Euler equation by assuming that A is or substituting for A, the velocity I can write down the first of the equations which is Dp/that is here that equation I can write it down. Of course M is a constant so you can take that out.

(Refer Slide Time: 23:30)



And so write down what is the first of the equations simple Fx,Fy, Fz= the first term becomes so m*what is it U dot+q*Uvw is what we said q*W-r*v. Any questions? No questions everything is crystal clear. Okay write down the rest of it V dot+r*U-pW and w dot +pV-qU. Okay so that is the first set of equations. The second set is my tau is=dH/dt which is here. Tau is given by LM and N so that becomes my left hand side.

So let us write down that so tau=dH/dt again that is=dou H/dou t relative this is the first term that is H dot term +omega xH. Remember H=I*PQR. So omega is p, q and r. Omega is not p dot, q dot and r dot. It is not p, q and r. This is a very standard terminology for example small r is used in the literature in vehicle dynamics to indicate yaw. Okay this is a very standard terminology. It is not usually written as omega x, omega y and omega z.

Write down that that is going to be an interesting expression. It is going to be a long expression. (()) (27:01) i with respect of course T. So what is the first thing. Okay let me remove this because you know that already L, M, N. Ixx, p dot +Ixy q dot+Ixz r dot that is the first term in the vector + write down omega*H. I will write down the final things this is just nothing it is not very difficult.

I will quickly write down that I hope I do not make any mistakes in this if there is anything just point out Iyy q dot+ Iyz r dot and Izx p dot+Izy q dot+ Izz r dot. So that are the dou H/dou T term. So then I have to put the same way Q*Az-r*Ay. Okay what is Az. Az is Hz. Hz is what remember that I had to multiply I have written already what is H. So Ixx, Iyy, Iyz multiply it and put that there and so there will be + q*Hz. How many terms would be there in Hz how many terms there will be 3 terms. So write down those 3 terms. So you can write down that that as q*Hz-r*Hy.

Remember that Hx, Hy, Hz is what? Ixx p+Ixy q+Ixz r. So what is this term is the Hx term and that term Hy term. Hy and that term is the Hz term. So that is what is going to come here and so on. Though the expression looks very formidable and long and is very simple to understand. There is nothing much it is very simple to understand this simple expression. Do not worry I am not going to deal with this kind of huge expression I am going to make as usual some assumptions.

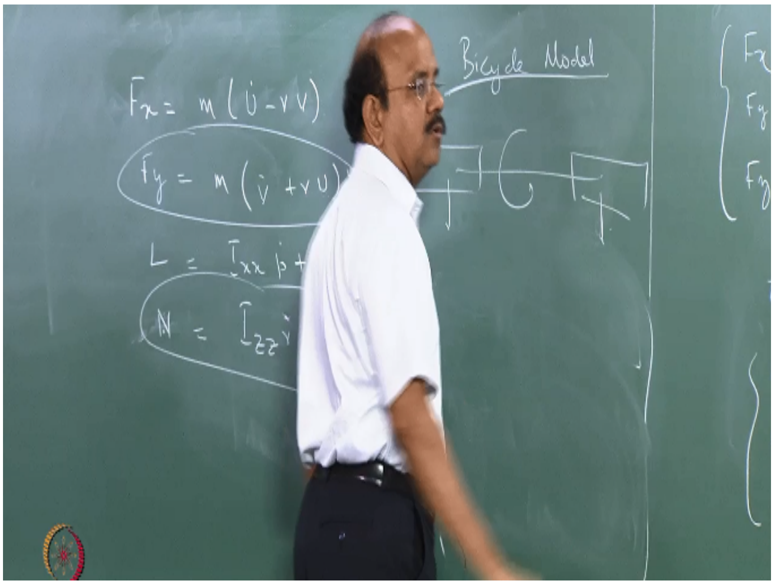
What are the assumptions you think we can make? What are the terms I do not like, but I want all that, you know, U, v, w terms I need. The terms usually I do not like are this Ixy and so on. So how do I eliminate that? Imagine that you have a car I mean up to this is mundane there is nothing great about it, you know, just the relationship between the inertial frame of reference and body-centered coordinates.

Now what are the assumptions you think we can make? Symmetry fantastic. So symmetry in a car is about what are the axis is that you can consider the symmetry. What do you think we can consider. So one is the longitudinal axis you can consider this as symmetry. The other is that we are dealing with principle axis. Let say that the Ixx, Iyy and Izz and so on what you have chosen is Xy and z for the car.

Let us say that it is the principle axis are they may not, but then in that case if we chose them to be principle axis all others go out. So we make this kind of assumptions and we also say that the Q the pitch is not of interest to us neglected and ultimately we will bring it down in a very simple handle-able form. We are going to make all these assumptions and then we will bring it to handle before.

So what is my first expression? I am going to now write down I mean this is clear any questions. Now with that assumptions I am going to rewrite my expression 1 say here 1 and my expression 2 substitute that and write it going to be a huge expression. I am going to rewrite them in simple form handle-able form which I am going to use. So let see - let us write down what is the expression which I can use Fx and Fy. Let us write down Fx and Fy.

(Refer Slide Time: 34:46)



So Fx that is the longitudinal force in the longitudinal direction which we had already seen, but with respect to the body-centered coordinate is m*U dot –r v right. So that is the body-centered coordinate expression for longitudinal forces. Now write down for the lateral force Fy=m*V dot okay that is the first term our relative dou A/ dou T so because of which I get a V dot term what is the next term I would get I mean neglecting q and so on+ru.

And what is my L and M. L and M from my expressions here so it will be the first term will be there Ixx*P dot if you are not considering you just consider some material not considering that you are in the principle axis then Ixz will be there. So Ixz*r and so on. So you can write down for M=Izz r dot again +the last is this term what we have is here the other terms because of symmetry goes off +Izx*P.

So I am going to remove these 2 later and look at I would say the expressions for the lateral dynamics. So this in other words this is the most general expression you can look at for the 3 dimensional case, but from here I am going to make geometric assumptions so that ultimately the equations I am going to use become handle-able that is what I am going to do.

And from this handle-able equation. Handle-able in sense that you know I want to work it out in an analytical fashion I want to look at the results I want to interpret it and so on. So from this I am going to extract out the equations for lateral dynamics that is all I am going to do. Clear. Any questions? Okay that this term and you can write this down I think you should have Izx*P yeah you are right, good.

Okay now we will move and be more specific and we will get into the expressions for the longitudinal dynamics. So what are the 2 expressions of interest to us in longitudinal dynamics? What are the 2 expressions which are of interest to us. When we are talking about lateral dynamics. We are talking about lateral dynamics what is the force talk about Fy and what is the other thing that we are talking about is the moment due to lateral forces.

So the moment due to lateral forces will be along Z direction. The moment due to the lateral forces will be along Z direction so that is my interest. So in other words just to quickly summarize so I have forces that are acting that is the vehicle then that forces create a moment which is perpendicular to the plane of the board which the Z. So the 2 equations of interest to us are the lateral forces and the moment that is acting due to the lateral force which is this

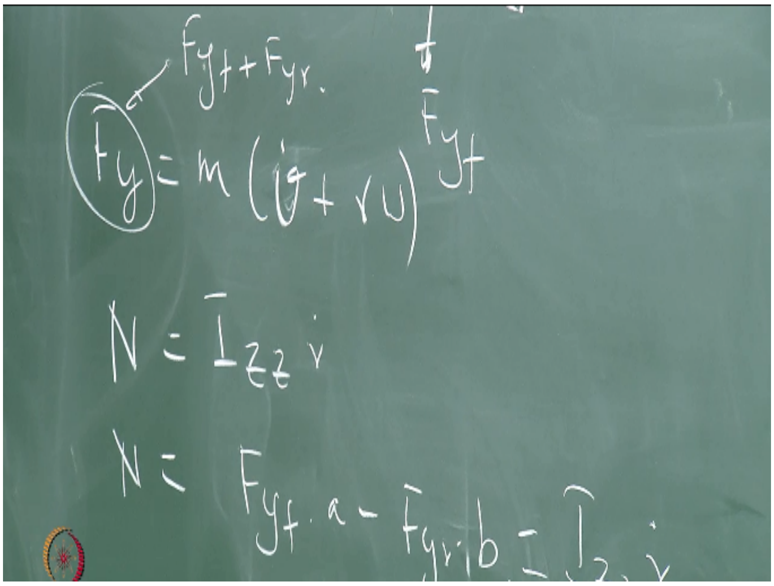
We will come to that. So we are now going to look at a simple model which is called as the bicycle model. So when I consider that is why I have drawn it like this. I am not going to consider the roll. So you are absolutely right I have to roll is an important phenomena that happens during cornering, but right now I am going to neglect the roll. I was expecting that question I am happy that you ask this.

So let us now look at these 2 expressions. In other words, what I am going to do is I am going to shrink this 2 wheels okay into 1 wheel like this the front and the rear wheels and look at the vehicle by means of these 2 wheels the front and rear and call this as a bicycle model. This is one of the most famous models in vehicle dynamics. As he very correctly said we are not talking into account the roll right now.

But then after we study this we are going to introduce complexities one by one in order to understand what happens because of other factors which we have neglected what happens due to the other factors that is what we are going to study. So we are into one of the most important models in the whole of vehicle dynamics which is called as the bicycle model. So I will remove this as I promised I will make it much easier so that is the one equation.

I will remove this and I will remove this and these are the 2 equations that I am going to look at. Let me now expand this bicycle model. Now we will introduce after crossing all these equations. Now I will introduce lot more again Physics.

(Refer Slide Time: 43:15)



So let us say that this is the rear wheel which I have collapsed and let say that this is the front wheel which is I am giving a turn to the front wheel. Let us say that I am just exaggerating this. I am giving a steering input to the wheel of course that is the vehicles that is the center of gravity location of the vehicle. So that is the lambda which I had given which is the steering input. Now so I am taking a turn when I take a turn I will of course need this Fy which we have been talking about and how is the Fy created and how is it generated.

We have already seen that this is due to the slip angle. So a slip angle so if this is the force that is acting okay a small angle differences which I am going to neglect. So that is the Fy in the front and this is the Fy at the rear. So how is it generated by slip angle and what is the slip angle that is the slip angle. Slip angle is alpha no doubt my coordinate system that is xyz is perpendicular to the plane of the board.

Now the very first point I want you to notice. We will continue this in the next class is that the slip angle which we have seen it before, but I am going to put some sign to it. The slip angle is look at that is in this direction which is in the negative direction and the force is created in the positive direction of Y. So this negative and positive directions we have to though we know this physically we know how Fy is created, but this negative and the positive directions we will be careful when we write down the equations.

So there will be 2 alpha that are created because I want the front and the rear and let me call that as rear alpha or alpha r the rear slip angle and let me call this as the front slip angle. So note that there is a confusion I have said that again and again note that what is slip angle. This alpha is nothing to do with this delta which is the steering angle. And we need a slip angle to generate the force.

Now I have these 2 equations and the N=r, but r that r which is the yaw velocity and R dot is the rate of change of R. Let me call this is again usually done like that which we call this as L1 and L2 let me call that as A and B because in the lateral dynamics literature the distance of the CG location from the center of the tire is A and that is B. So that A*Fyf-b*Fyr is the moment that is acting.

So N=Fyf*A-FyR*B which is=Izz r dot. Fy we will come to this in a minute Fy is Fyf+Fyr. We will continue this in the next class. We will stop here and we will continue the bicycle model this is probably the most important central piece of the whole lateral dynamics. So we will look at this very carefully and then understand the vehicle behavior during cornering. So we will continue in the next class.