Vehicle Dynamics
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Lecture - 19
Lateral Dynamics: Stability and Steering Conditions

In the last class we were looking at very simple model which we called us Bicycle model. We said that we will collapse the 2 types of the wheels into 1 in the front and the rear, right.

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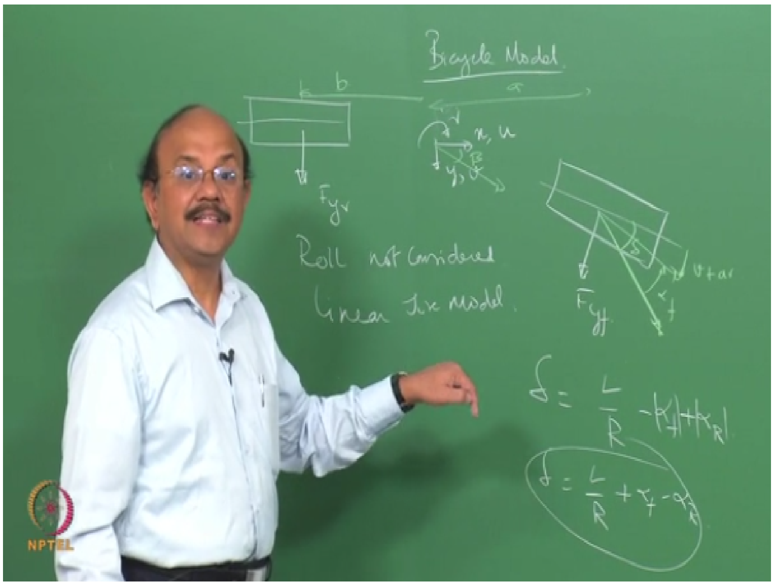

And define this model by means of 2 degrees of freedom is what we said. So that if this is the coordinate system x and y and z being so I thought sorry we did this like this x and y z being inside the ground. This is the coordinate system which we saw and we said that there 2 degrees of freedom. U is not a degree of freedom, right. We said that it is the U rate and V the velocity along the y direction.

We said that the vehicle is travelling at a velocity U which means that you are giving that velocity. So, let me call that as U that is travelling there. In other words, we are now looking at lateral dynamics when the vehicle is now taking a turn in simple terms or we are into a manual. A number of things I want to clarify because your questions at the end of the class. One is that what are the assumptions that we have made here, right.

It is quite obvious but let us put that very clearly. Now when the vehicle takes a turn obviously there is going to be a roll, okay. The vehicle rolls. Probably you would have heard about this term roll center, roll axis and all that, okay. So, there is a roll, right and that roll is not taken into account in this case, obviously. That is not a degree of freedom in this model. So, the roll is not accounted for.

So in other words all that load transfer that comes due to roll is not there in this model. I am not considering the wheels individually so that the stiffness of the wheel, rear wheel in this model is nothing but the addition of the stiffness's of the rear and the front wheel, clear? That is the second thing that I am going to do. The third thing I am going to do especially in this model, I can extend this definitely. In this model is to consider a linear tyre.

So, roll not considered. I am going to consider a linear tyre model. I can remove that later of course that I am not considering the compliances of various bushes and other things that come into picture all those things I am neglecting, okay and we will bring in all those things later. In other words, what we are going to essentially do is to look at this model derive the equation, understand the physics and then slowly include, okay.

The effect of those aspects which I have neglected right now that is the whole idea, clear? Obviously there is going to be a load or force rather that is acting which we call this as Fyr and Fyf and that there is a, that is the delta which is the steer angle given, okay and that is the alpha f. I mean these are all exaggerated so all angles are small, okay just to show you, okay from the center that is the delta that is given that is the delta f and that what was that?

V+ar where r is this distance remember that that is the distance and that distance is b, okay. Then we said that that is beta so tan beta = v/u which is approximately = beta and then okay we said that delta – alpha = V+ar in actual terms and as you look at it. But this alpha f is actually negative in order to bring that into fx we define that to be negative, right. So ultimately if you go back and look at the expression which we derive we said that delta = L/R–alpha f+alpha R.

Like this is what was defined. There is a small confusion in this because there are text books which give this equation for example if you look at (()) (6:30) vehicle stability he use this equation. If you look at Gillespie okay he gives another equation. The equation is slightly different, okay. So and there are other books many books which give the equation differently. So there write that as L/R+alpha f–alpha R, okay.

So they give it like this. Now, there is a confusion like which is correct? Both of them are correct, actually they are the same. The only thing is this is the mode the actual value. Go back and look at the way we had written okay.

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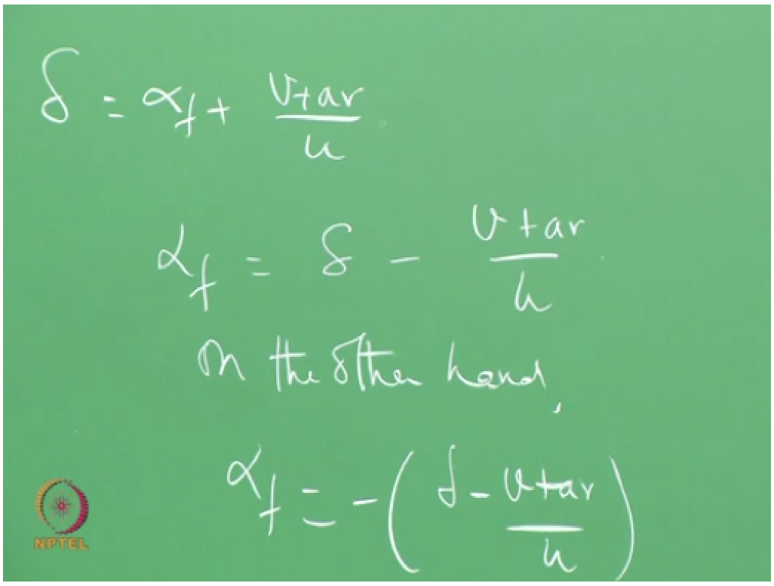

We now can write also I mean there is the variance of this is delta = alpha f+V+ar/u. I can write it like that, okay. So that now alpha f would become the actual value of this angle and sign is not included, right. But go and look at what are the way we had written or in other words in this case what happens is alpha f = delta – V+ar/u, right. This is the actual alpha f. Now how is that? How did we write it? Alpha = - of delta – V+ar/U.

That is all we wrote. But on the other hand we wrote alpha f = - delta–V+ar/U, okay. Simply because that – sign is given because if I now consider alpha f it is outside, okay it is like that which is actually negative and that is why this negative sign. So you can use both, it does not matter. But be clear about the angle and be clear about the force that results from that angle, okay.

So signs if you want to be consistent, signs are important follow the correct signs or you know intuitively and so follow only the values. So in other words you can write this as if you are going to write the task like that and then you say that it is just this value. In the same fashion if I see the force Fyf okay, it is alpha f is in this direction – okay Fyf is positive.

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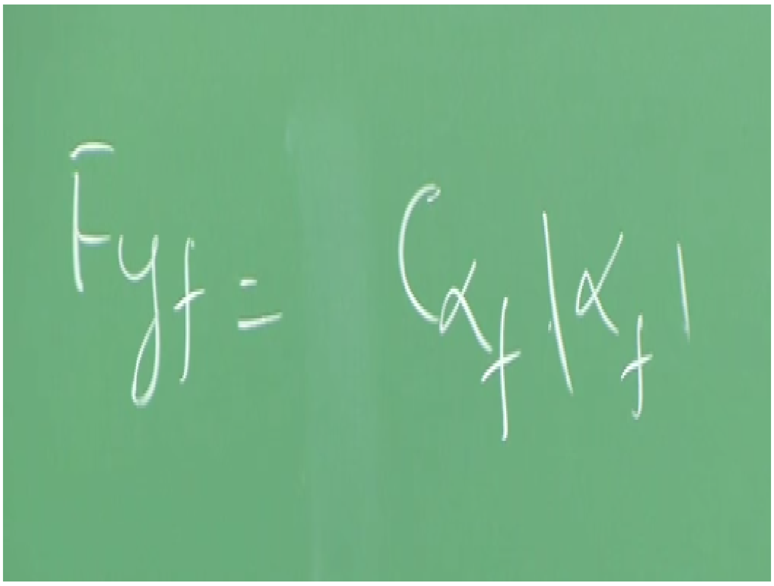

So if you want to follow the signs clearly then you can write that as Fyf =-okay the constant which we called as C alpha f*alpha f. That – sign is again because the physics brings that sign one is negative direction gives the positive Fy that is why it is, right. If you do not want to follow this this is confusion okay fine. Then you can write that as like this but then this is the mode then correspondingly you can substitute.

In fact if you put a – here notice that ultimately the force nothing happens. There was no difference nothing happens because when I put – alpha F actually that is the sign I am following sign conventions correctly when I put – alpha f obviously it becomes +. So Fy is in the+direction, okay. So this is what is, there is no confusion in this but many books follow, every books follows its on terminologies and the way it writes and so you have to be careful on that, clear? Any other questions? Fine.

Now there was one more question which I said, I will answer it later let us not be in a hurry. Let me, I am going to answer that but then before the class I know some of you may have this doubt what is this you are talking about stability and you said that I am right now not worried about Eigenvalues, okay.

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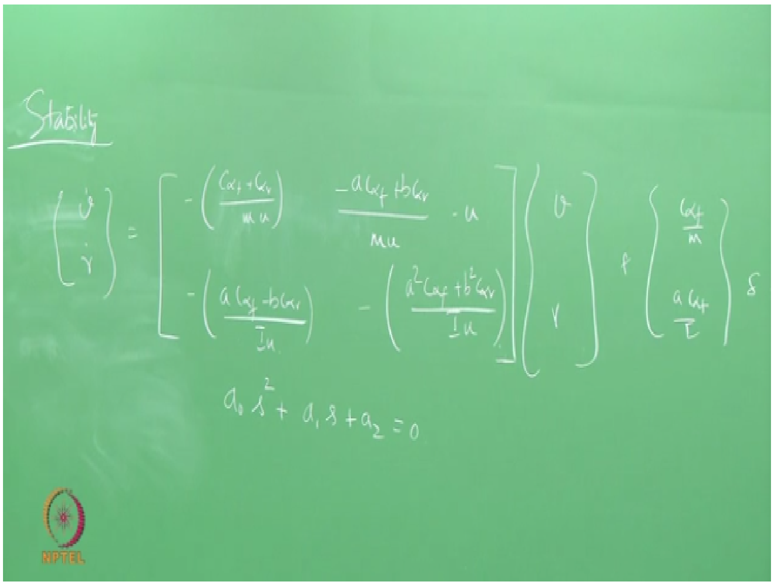

And I know that stability depends upon the Eigenvalues. I know that I have to calculate the Eigenvalues and that if the real part of the Eigenvalue is negative okay the system is stable. But these are the things which I have studied you suddenly say that do not worry about that let us first look at stability and then look at this, I thought there is a confusion. Though I clarified it but let me put that further sake of others.

Yes of course but usually pedagogically you are absolutely right. Usually what we do is if the values are given all the values are known okay and if I give this as an equation when I substitute all the vales then it is easy for me to calculate the Eigenvalues and easy for me to find out whether system is stable or not. But if I do not know the value I want to develop a condition for it. I need not calculate the Eigenvalues.

So I will follow a stability criteria like (()) (12:52) criteria and so on and develop a condition, okay which gives me stability. This is not in variance with what you are defining, okay and we will also see later a bit later why, okay this stability concern what you learned is also important and we will use that but right now I am not going into that because as I told you I do not have numbers. I now only, I now know the governing equation, okay.

So I have to only look at that from that perspective I have to develop the stability criteria, clear? So that is why I said that let us first look at stability from a different perspective, okay or in other words same principles are same from a different angle, right. So for that let me go back and see where I left. I had written this in the state space form right. I mean that is where we left and we wrote that as V. r..

So V and r is our state equations and I mean so that this would become – C alpha f+c alpha r divided by mu – ac alpha f+bc alpha r divided by mu – u – ac alpha f – bc alpha r divided by Iu. See I is nothing but the moment of initial term I z term, okay or I zz or I zee zee however you want to pronounce, okay. So that is what we had written in the last class, right?

From which we took off we wanted to define what is the stability criteria if any question is given that is where we went about right and then we wrote down you remember that we wrote down the differential equation for (()) (15:46) system, right. I think we did all that clear? And then we took the, what is that we did later? We took the Laplace transform, right and then we arrived at an equation Laplace in the Laplace domain and so on, clear?

So ultimately we wrote down an equation for this as a0 s squared+a1 s = 0. This is where we left, right? Clear? Now for this system for the second order system first and second order system you know (()) (16:39) stability criteria I have done that in controls I am just going to not going to details of it.

I am just going to state restate it that the necessary and the sufficient condition for this system to be stable I mean this is the system which we are dealing with now, okay. For system to be stable these quantities should be positive. a0 a1 and a2 should be positive, okay.

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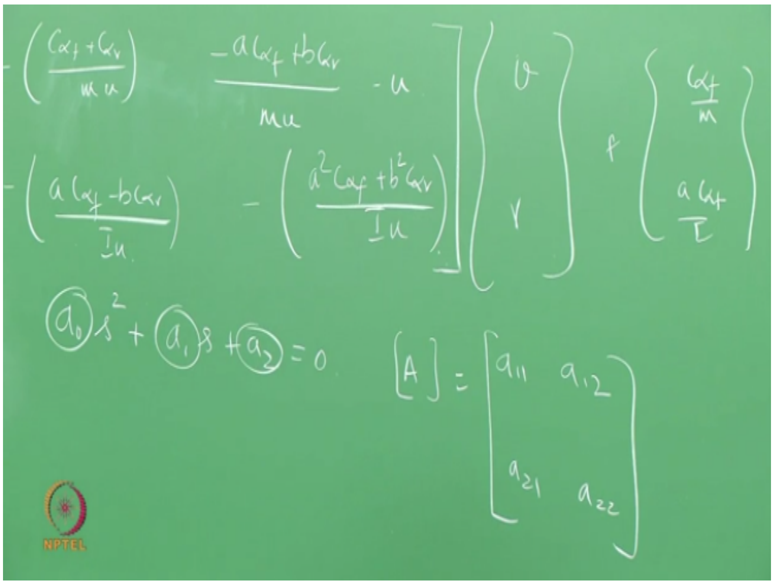

Now in other words if a matrix is a11 if a matrix is written as a11, a12, a21, a22 remember that I have to do s-si – a okay and so on right. It is all done. Any questions? So this is what we are going to use that is why we are not looking at right now whether Eigenvalue the real part of the Eigenvalue is negative or positive and so on. We are not looking at that, okay.

So in this system, I am going to write down so I am going to write down a0, a1 and a2 okay go back and look at it and where a0, we have already written that.

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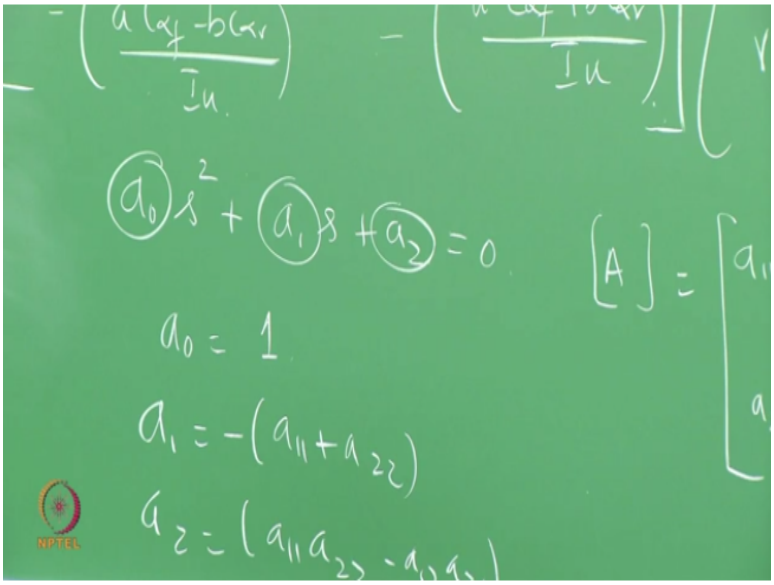

So a0 okay can be for example ultimately looked at like this a1 =-of a11+a22 and a2 = a11 a22 – a12, a21, okay. So, that is where it actually boils down to. Now I am going to write that down from here we will take off we will look at that. So a11 is my first term so that is this term and so on a1 to a21, a22 so on.

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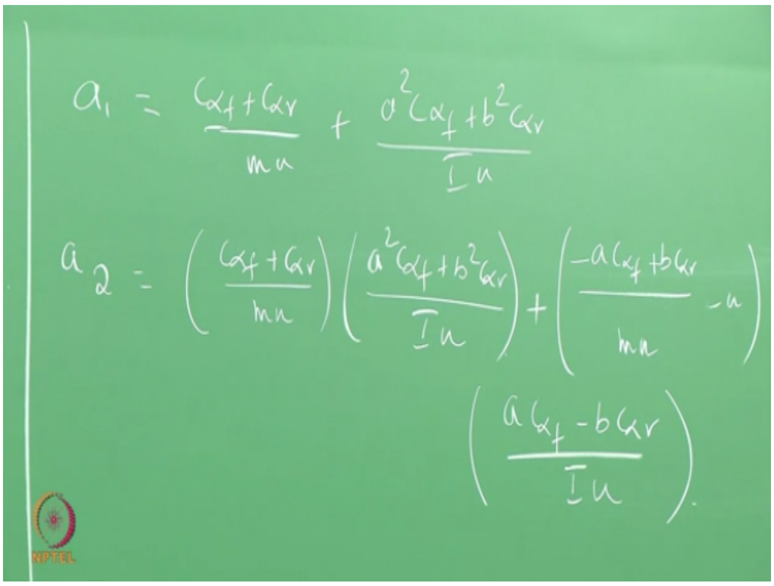

So, let me now write down a1. a1 is – a1 this is how I have normalizes that a0 = 1 and this is the equation, okay and go back and look at that. So a1 =-of a11+a22 that is = c alpha f+c alpha r divided by mu okay+a squared second term c alpha f+b square c alpha r divided by ru, right. Let us look at the second term a2 or the 3rd term rather so a11*a22 – and – so that becomes +.

So c alpha f+c alpha r divided by m*u multiplied by a squared c alpha f+b squared c alpha r divided by ru that is the first term – what is a12 – ac alpha f+bc alpha r divided by mu – u multiplied by which term? That is the term right. So I would put that as+because there is a – there and so multiplied by ac alpha f – bc alpha r divided by Iu, clear. Simplify this, look at that expression and simplify it let us see what you are going to get.

Take a minute simplify it.

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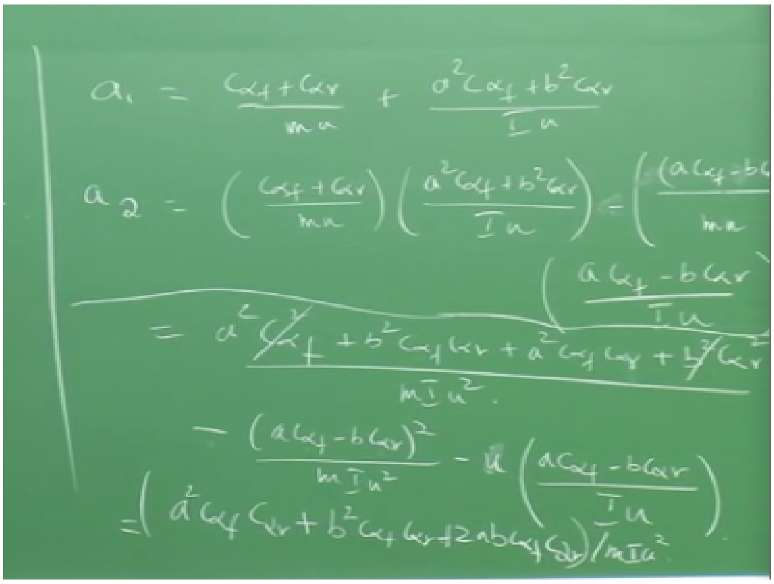

So, let us do that a squared c alpha f, first terms+b squared c alpha f c alpha r+a squared c alpha f c alpha r+b squared c alpha r squared, right. So that is the expression. C alpha r squared divided by mIu squared, okay. We will do a small jugglery here. Let me put that as- okay and write that us like this, the small jugglery I will do, okay.

Why I am doing this become simple the first term now becomes – ac alpha f – bc alpha r whole squared divided by mIu squared, okay that is because that is-I had taken that out, right. Then next term –u because-is there. –u*ac alpha f – bc alpha r divided by Iu, clear? Simplify this, expand this, so you will get what is that you will get? So, this will be a squared c alpha f squared+b squared c alpha r squared, okay – that is both of them are – outside, so what will happen?

This guy will go off and that guy will go off, okay. And you will get -2ab alpha f alpha r right? Clear? And – this-is there that will be +. So, you will have a squared so I will write that as a squared c alpha f c alpha r+the second term b squared c alpha f c alpha r and the third term comes from here which will be+2ab c alpha f c alpha r the whole thing divided by mIu squared, okay. That is the first term.

Then I have the second term let me rewrite that term.

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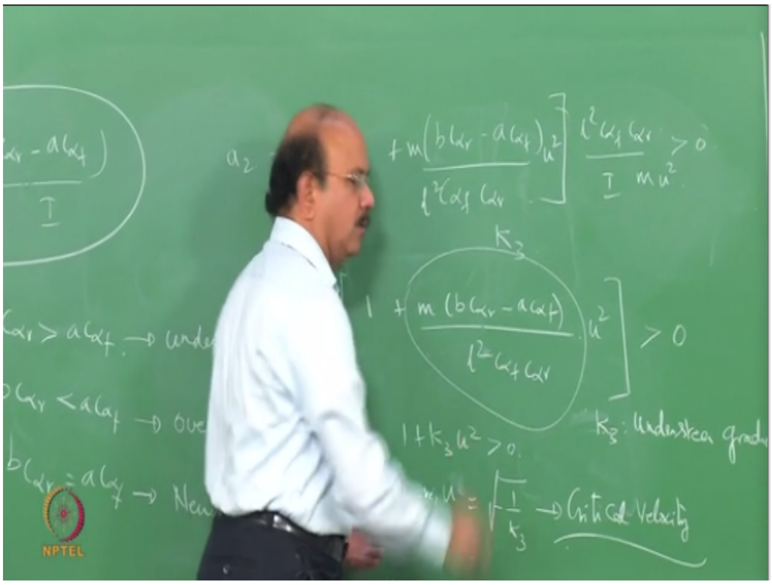

I will take care of all those -es and say that+I am adding that all that a squared+b squared and all that this+what is the second term? bc alpha r – second term ac alpha f upon I, okay. So, let me look at this carefully it is a squared+b squared+2ab, so c alpha f alpha r I can take that out. So, I will write that as c alpha f c alpha r*a+b whole squared which is l squared divided by mIu squared.

So, that is the third term which I called it as a2, right okay. Now let us go back and look at the stability that I hope I have written correctly look at mI rest of it I know it is correct, mIu squared I hope I have not left anything, okay. So, now look at these 3 terms. The roads conditions says that this 3 things should be positive, yes. The second condition or the second one, sorry not second condition second coefficient a1 look at that coefficient.

Can that be negative so that cannot be negative because we said that c alpha f alpha r are all positive mu everything is positive. So, that cannot be negative, right fine. But let us get back to a 2, okay. First term no worry that will be positive but second term it is possible that the second term is negative, okay. So, when ac alpha f is greater than bc alpha r obviously this is going to be negative, right.

So, the only possibility of the terms becoming negative is dictated by this guy here, right. So, as long as in other words as long as c alpha r is greater than ac alpha f bc alpha r is greater than ac alpha f, do, I worry? Nothing to worry, everything is positive. On the other hand if I now if bc alpha r is less than ac alpha f then this becomes negative. There is a cause of worry which immediately it is not that the total sum becomes negative but there is cause for worry.

That is all I would say, right. In other words this can be so high that the I can over take this guy and make a2 a negative term, alright. Not that just because this is negative it is already you have done and do not, no. Clear? So, this condition we will later call this as an under steer condition. Why how this term all those things we will define shortly what is called as under steer coefficient and we will define all those things.

When bc r alpha r is less than ac alpha f, okay then we have over steer condition. Why not bc alpha r = ac alpha f, yeah sure it is possible and that brings out the condition called neutral steer. Now why are we calling this as under steer, over steer you know what is this condition under which it will really become unstable. These are the issue which are very interesting, right and that is what we are going to see now.

Now let us rearrange this term and quickly get into a position where it will really become unstable, okay. Rearrange the term look at that and rearrange the term and let me know when it will become unstable. So, let me do that may be I will write that as a2 to be 1/mIu squared+bc alpha r – ac alpha f divided by c alpha f c alpha r l squared multiplied by l squared c alpha f c alpha r. Let me do something like that, okay.

Let me take that I also out, I, right. Let me bring another jugglery here. I am just only manipulating this solve this thing because I am zeroing in on terms which are going to be important. So, look at all these terms they all cannot be I want all this I mean this to be greater than 0. So, these terms are good guys they are all positive, okay. So, I am not very worried about it.

So, my condition is actually 1 let me do one more thing. I will put this u squared here, okay and then put u squared like that, right. So, because u squared is positive, so no problem. So, my term actually reduces to m*bc alpha r – ac alpha f divided by l squared c alpha f c alpha r*u squared, oaky. That is how it reduces to and that should be greater than 0, okay fine.

As long as this is greater than 0 I have no issues but if this is less than 0 then this term is becomes negative, right and at a point of time cross 1, okay and make this whole thing negative, right. So, let me call this us K3, why K3 I will tell you why I am calling that K3 a bit later. Let me call that as K3, okay. So that I will write this as 1+K3 u squared, okay to be greater than 0 that is the condition.

And ultimately this reduces to a condition where K3 becomes negative, okay. So, u squared, yes so, when u squared = root of 1-- of K3, okay that is where when K3 is negative this is the condition you would get, okay. When K3 is negative u squared = root of – 1/k3 is the condition beyond which not u squared sorry beyond which the vehicle is going to be unstable and that is a very interesting conclusion, clear?

So, in other words what happens is that entry of that term u here in that expression, right. So, what does it mean really let us physically understand it we will go into the equations a bit later, okay? So, there is a K3 look at the K3 carefully, okay. K3 consists of m characteristics of the vehicle, no problem it is going to be a constant, okay. a and b, what are a and b the distances from the center of gravity to the front and the rear, right.

Alpha f and c alpha f and c alpha r are the result of assumption that the tyre is linear, clear? Linear so they are constants of the tyre l squared c alpha f c alpha r and so on so this that makes this whole thing K3 to be a vehicle parameter, okay. So, the vehicle can be an under steer vehicle or an over steer vehicle. When the vehicle is an over steered vehicle K3 becomes negative.

Immediately does not become unstable K3 becomes negative, okay I am not yet defined why I am calling this is under steer, over steer that we will come in a minute, okay. So, that+into ku*u squared simply means that the over steered vehicle has a velocity at which this third term goes to negative or becomes negative and that velocity is called as the critical velocity. Above which the vehicle is going to be unstable, okay.

And what is this instability going to cause? That also we will see in a minute, okay. So, that is called as the critical velocity. Why I have put this as K3 because I am going to have 2 more definitions K1, K2, K3. There are books I am not sure I do not have the list but I know that some of the books, okay call this as an under steer gradient, okay. So that they would write this as 1+K3 u squared and K3 being an under steered gradient.

Why I am being careful and write K3 because this definition varies from one book to the other, okay. The concepts do not change but the definition changes. Another book would just simply state l will remove that l and say that is under steered gradient, okay. So, we will see why the other books say like that. So, this is a simple way of defining this under steer gradient, right okay.

So, that is one thing, the first thing that we have but why did we call this is under steer and over steer, okay. In order to understand that let us get back to our original expression, okay and treat a steady state condition for the body or for the vehicle, okay. Let us get back to our original governing equation. All of you remember the governing, original governing equation? How did we write?

Okay. Can we remove this, any questions? Clear? Good. So, remember that in the very first class I would have to get back I think we call it as 1 and 2. Iz r., what was the expression which we wrote for this?

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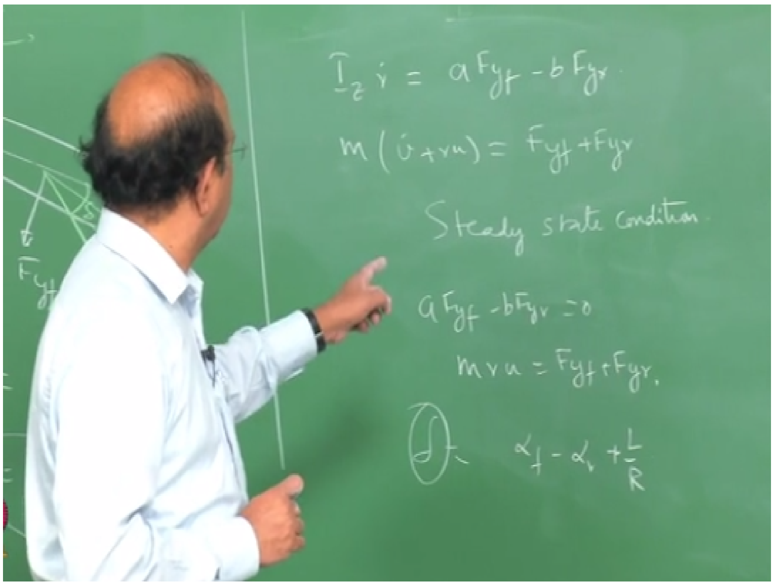

Iz r. you remember that is the first thing you wrote a*Fyf – b Fyr, right and what is the second expression we wrote m*v.+ru = Fyf+Fyr. This is what we wrote. Let us now consider what is called as the steady state condition. What is the steady state condition? What do we mean by steady state condition? What we mean by steady state condition is that there is no time derivative terms, okay.

So that I will write the first equation to be aFyf – bFyr = 0 and mru = Fyf+Fyr, right. These are the steady state terms sorry steady state equations. Let us now get back to my delta terms, you know delta what I had written here, okay. Write that down how we wrote, I hope there is no confusion on that. I can write this in 2 ways as I said I can write that as l alpha f – alpha r+L/R, right, okay.

Now what I am essentially going to what is that I am going to do before we go further let us understand what we can do. Now I am going to look at the effect of delta, effect on delta due to the new introduction or new term which we called as under steer gradient, okay. Now I am going to look at how that delta is going to be affected, okay. How in a very, in a steady state maneuver, okay where we have simplify the equations and we are going to look at how that is going to have an effect.

This is what I am going to do, right, okay.

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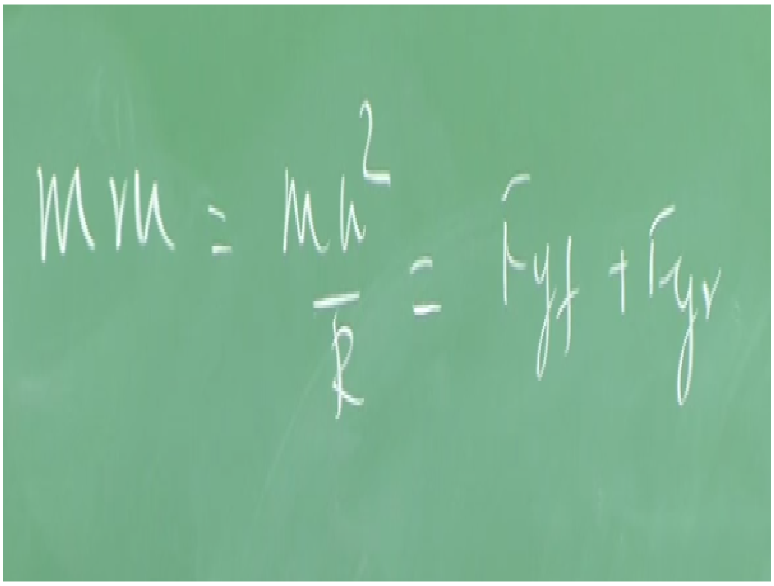

Let us look at the first this equation mru, what is m r what is r, remember that it is the o velocity and so it is = to u/capital R, okay and so mu squared/R and that is = Fyf+Fyr, right, okay. Note that I can also treat this as-alpha f+alpha r. So, I mean I have to be careful in that note that carefully, right. Now let us look at the geometry part of the vehicle motion, okay, here it is. So, we defined very clearly what is capital R and so on.

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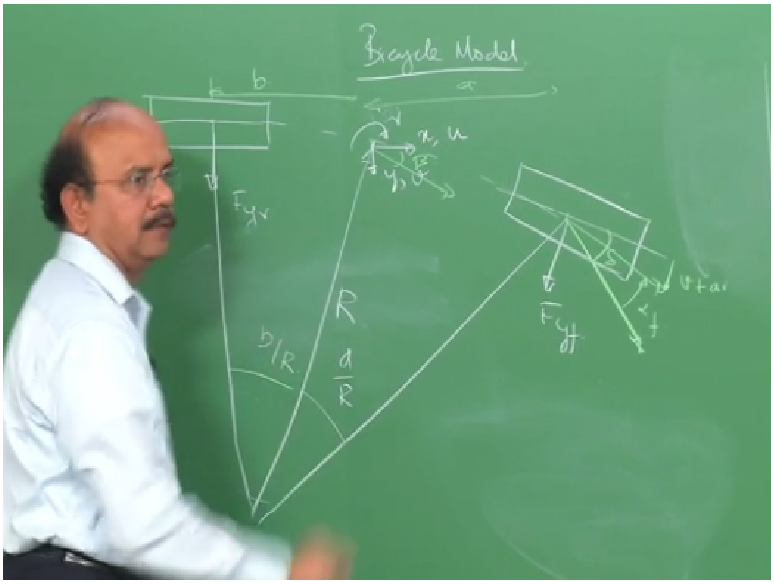

So, obviously that is the radius that is the radius of the turn 1/R is actually curvature, right we know that, okay. So, I have that angles what is this a and that is b approximately there is a lot of it are approximations because every angle is small and all those assumptions we have made. So, that angle is what, what is that angle? okay and that angle is b/R, clear? okay and l is actually the total length of the vehicle.

So, that is the geometry which is the vehicle takes when it takes a turn, right. We would go through a number of smaller things so that we understand it clearly. One thing we know from our earlier classes how my normal force is distributed, okay. Remember all this equations I am going to use them here and there, okay may be in the next class when you start we will write down all those things, okay.

So, have your notes ready so that we will when I put down an equation you will go back and refer to where from where we are getting.

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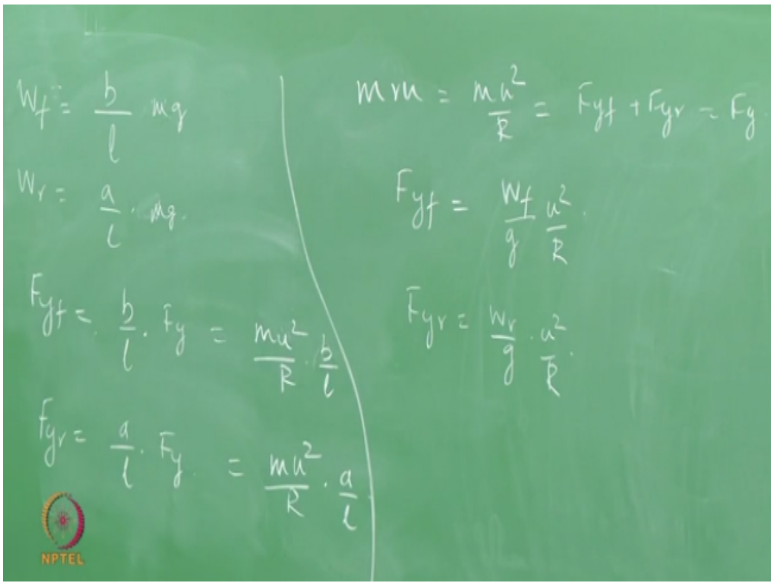

Now I know that W front which is the normal load that acts on the front = in this way what said b divided by l*W right and the W rear = a divided by l*W, okay. Now if Fy is the total load that is acting obviously the same or how did I get that by taking a moment, okay about the center or about one side and so on. The same thing I can do it here as well because this is a steady state I have no issues on taking the moments.

And hence Fyf is going to be distributed in the same fashion as b/l*Fy and Fyr = a/l*Fy where Fy is actually the total force that is Fy**.** Yeah, any questions? Okay, so it is first thing is interesting to know that the lateral forces are distributed in the same fashion as that of the (()) (48:16) which I can write it as m u squared/r to a that is Fy, okay sorry into b/l.

That is m u square/r is Fy the total Fy and that is = to m u square this is u square/r*a/l, right. Now I want to use this. Can I use that? Not very difficult. This is nothing but mg and that is mg, so I can substitute that into that expression and rearrange the terms in order to I mean and get Fyf. Why I am doing it that will become clear in a minute, okay. So, in other words the first thing is that the lateral forces are distributed in the same fashion as that of the normal modes, okay?

Rearrange it and tell me how I can write that in terms of Wf m*b/l, so W/g*u squared/R, is that right? Same way Fyr = Wr/g*u square/R, okay. I took some time to get back to these 2 things, okay. Now we will go fast. We will start from here and then we will substitute that into delta we will see what happens and so on, right, okay. There are the only thing is that there are number of equations please go through them, okay.

If you have any doubts, ask me in the next class. I am going to us them now, okay to bring out the physics. So, you should be clear with every equation that we had written, okay. We stop here and we will continue in the next class. Yeah, right. So that is why I said that as an exaggerated figure and that it is all those angles are neglected and that the forces are distributed something like this and like this, okay, that is all.

Oh yes, of course u is what, no we are not looking at longitudinal forces this is the lateral force alone is what we are looking at, Okay, so we are not looking at longitudinal forces. The 3 equations if you remember we wrote and fx equation we are not considering, okay.