Vehicle Dynamics
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Lecture - 02
Longitudinal Dynamics

So we will continue with what we were doing in the last class. We were looking at a broad picture or perspective of vehicle dynamics. So we were looking at how we are going to approach the subject of vehicle dynamics. We said that for us though there is a vehicle, it has its components and so on.

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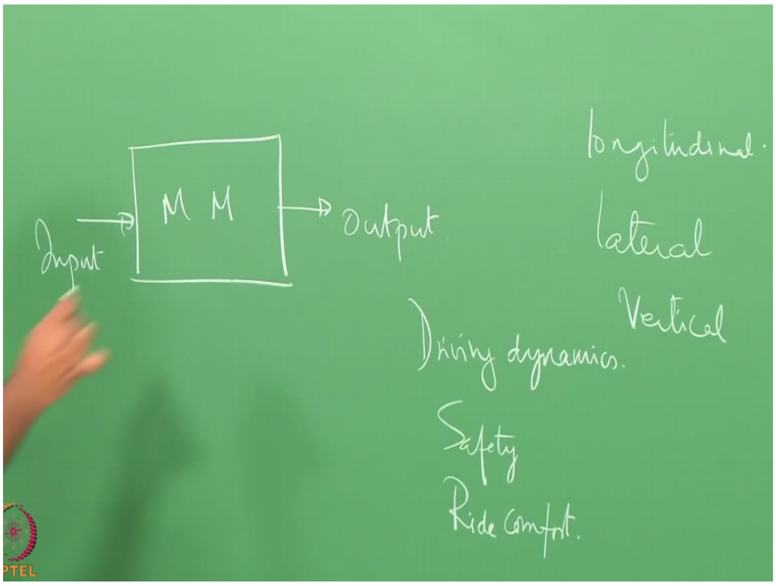

When we are studying vehicle dynamics, we said the center of this whole thing is the mathematical model. So mathematical model comes from our good old Euler-Newton equations and this has an input and an output. Remember that when we looked at the dynamics okay, which is defined by using these mathematical equations, they are classified into what we called as longitudinal dynamics, lateral dynamics and vertical dynamics okay.

We said that we classify the dynamics what we are going to study using this mathematical model into a longitudinal, lateral and vertical dynamics. We also said that for these of understanding its effect we may most of the times delineate or decouple them and study them in isolation. Is that correct? Does it not have an effect or one has an effect on the other? Yes it is possible there is an effect.

But in order to understand the subject most of the time we will be decoupling the effects of all these 3 right. There are very important things that will happen like low transfer and all that. We will understand it as we go along. We also said that the input for this model is the driver's input through the steering or the acceleration and braking okay. In other words, how the driver interacts with the vehicle?

Right that is what we are going to use as an input. We said that we cannot look at every scenario by the driver and we will have some test conditions okay in order to understand the behavior of the vehicle. Of course, we said that the vehicle itself, which goes into this mathematical model will be defined by means of certain parameters okay what we call as kinematic and compliance parameters, mass, moment of inertias, compliances, stiffnesses and so on.

The output from this model as we said yesterday or in terms of displacements, accelerations okay velocities and so on. We said that what comes out as an output has an effect on the occupants okay and we are going to study that not to a great extent, but at least as an introduction we are going to study how this is going to have an effect on the occupants okay. So this is the broad I would say basis under which we are going to study the subject.

We also said that we can again look at it from a different perspective and call this as driving dynamics okay, safety and ride comfort. When we look at this from a different perspective, same problem going to look at it from a different perspective okay so it is not organized like this. It is not that longitudinal dynamics is driving dynamics, lateral dynamics is safety, vertical dynamics is ride comfort.

It is to a certain extent it can be looked at it like that, but they are not necessarily a clear demarcation like what we have here. So we would look at the safety during driving okay both in the longitudinal as well as in the lateral dynamics right okay. We will continue now with this short introduction and we are now going to look at this mathematical model okay and already we know or we had a very, very, very simple model in the last class.

We are now going to extend this simple model into not a very complex model, but just the same model simple model making it more elaborate. So we are not going to make it very complex, we are going to use that same equations F=ma okay. So in other words, we directly plunge into what is called as the longitudinal dynamics and look at a very simple mathematical model that we will be using in order to understand the longitudinal dynamics.

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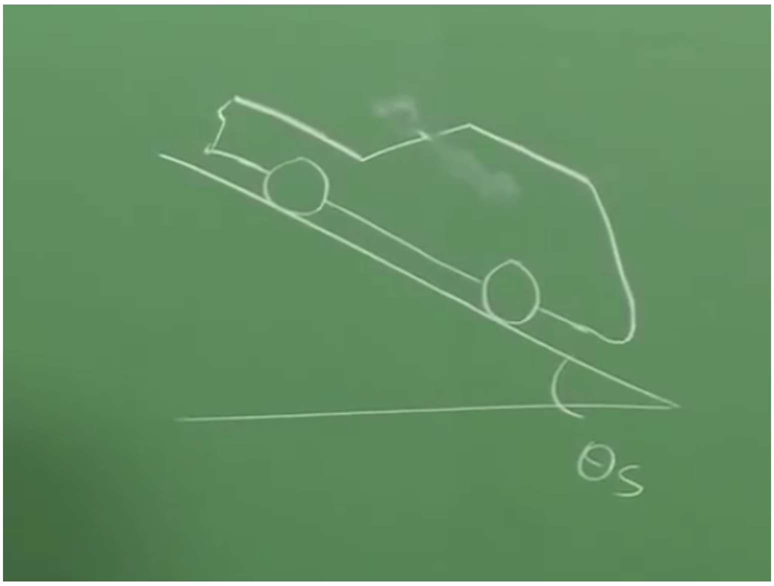

Whenever we talk about dynamics, 2 things that comes to our mind, the very first thing are the forces that are going to act on this vehicle and the other is of course the acceleration or deceleration and so on and our good friend F=ma from Newton is going to be of great help okay in writing down this mathematical model okay. So the first thing is first, so what are the forces that are acting on the vehicle okay?

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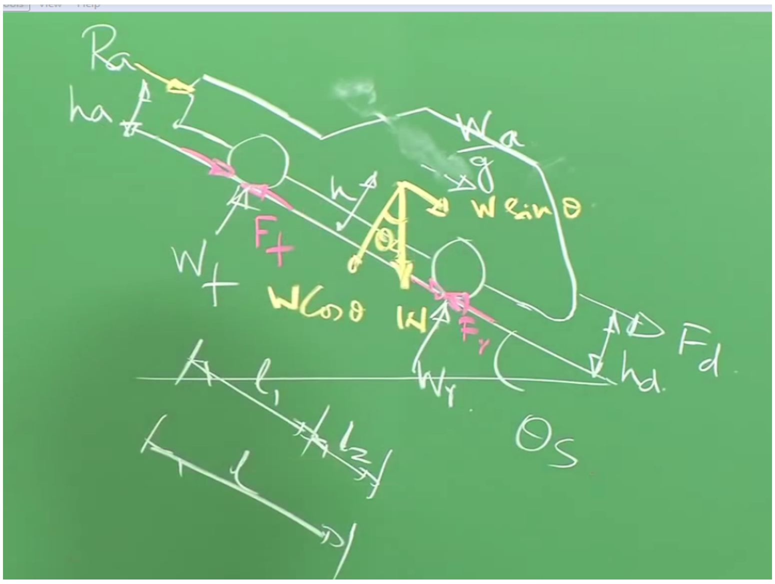

All of us have experienced this, the first okay we are looking at the external forces. External force that acts on the vehicle okay is what is called to say aerodynamic force, which we would call as Ra okay. The other force next force which is important to us, which is acting on this body is the gravitational pull or gravitational force okay and that write it as Mg or w and that can of course be resolved into 2 directions like this and of course this is equal to theta s.

So if I call this as w then we have what is this? W cos theta and w sin theta. You may have a trailer you know towing with this vehicle, which is called as a drawbar. So if you have a vehicle or if you have another trailer with you then there will be a drawbar pull, which we would call as Fd clear. Apart from this, the vehicle itself is going to give us some force okay, some nice guys, some not so nice guys.

So if I am going to accelerate, we need what is called as the traction force okay. So the traction force is now going to act in that direction. So let us say let us put the traction force in the front and the rear okay. Let us look at this tractive force and call this as Ff and Fr. Of course, in every vehicle Ff and Fr will not act together or it may be a front wheel drive or it may be a rear wheel drive and so on.

So depending upon the front and the rear wheel drive okay you will have either of the one or if the 4 wheel drive then you will have all these things. Apart from this, what are the very important forces okay which consumes our fuel okay is what is called as rolling resistance of the tire. Rolling resistance of the tire okay acts opposite to. We will understand this rolling resistance in a minute, it acts opposite okay to this tractive force okay.

And in fact, it is something like a braking force that acts on the vehicle right. Now let us first understand, see up to this it is not very difficult to understand, all the forces that are going to act okay. What I am going to do is very simple. I am going to find out the reactions of the front and the rear okay. Let us say that we are accelerating a force, we are playing a tractive force, so we can say that I have a d'Alembert’s force.

Actually it is not a force it is a pseudo force okay, which can be written as W/g*a okay. It is not a good practice actually to put this d'Alembert’s force. It is nice to write F=ma, but then when I take some moments then it becomes easier for me to have a force there and that is the reason why I have a force and call as the d'Alembert’s force okay. Now all these forces are familiar to you.

In order to take the moments of course you need some dimensions right. So let us call the dimensions something like this. Let us say that length=l1 and that length=l2, l1 is the distance from the front wheel to the CG location, l2 is the distance from the CG location to the rear axle, rear wheel and let the total length of the vehicle l1+l2 let it be called as L. The other thing that is important to us is the heights.

So let us call this height as ha and let us call the CG location height=h and let me call that height to be hd. You know how to determine the 2 W's or the reactions of the wheels, Wf and Wr. If I want to find out Wf, I take a moment about Wr okay with proper signs I can determine Wf. In other words, Wf*l=whatever the moments that are due to the other things okay.

But before we go further there are 2 comments that are important to us, 1 is the system that we are going to use in this course, the x, y and the z direction that we are going to use in this course okay.

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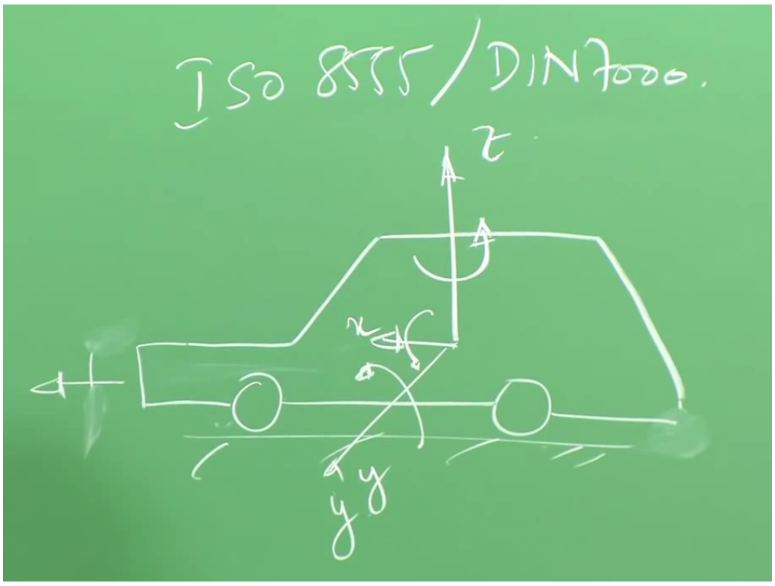

This comes out of an ISO standard and we call the direction, which is along the direction of travel okay as x perpendicular like that as y and the other direction normal to the ground as z okay. So longitudinal, lateral, lateral and vertical directions. Of course, you know that there are motion okay the angular motion along these directions. For example, the angular motion in the direction of x okay.

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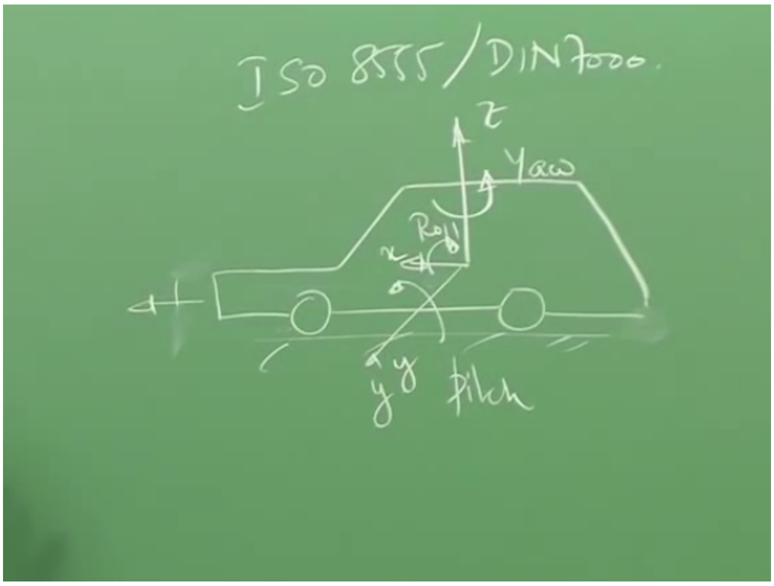

In other words, that angular motion okay along the direction of x okay let me take that as to be a correct one let us say positive okay that is positive is called as what is that angular motion called as? Roll, so this is the roll and the angular motion okay here which is in the y direction that is that angular motion is called as the pitch and that is what we call as yaw right. So let us say colloquially pitching okay that is moving in that direction right okay.

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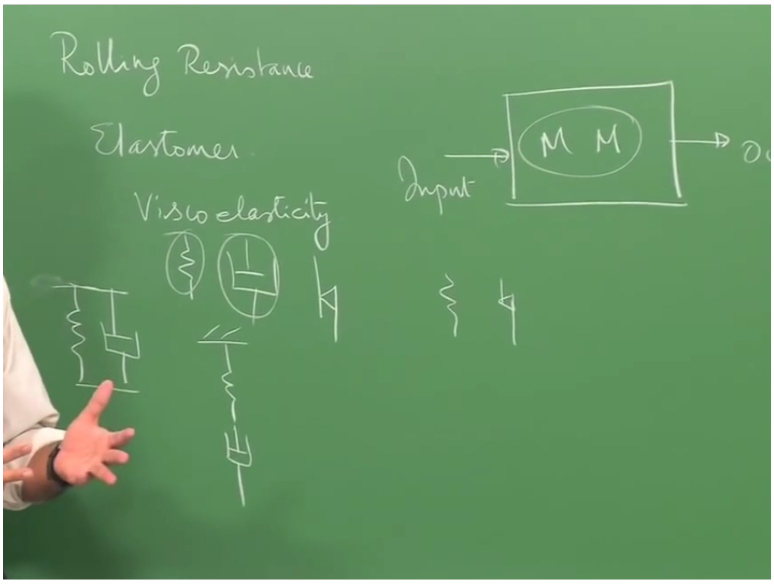

So that is the first thing. The second is let us go into the details of what is called as rolling resistance. Rolling resistance is today very, very important for fuel consumption especially in trucks can you imagine that the rolling resistance whose origin are the tires consumes nearly 30% of the fuel of the vehicle. We are going to do quite a bit of tire dynamics in this course, but let us understand what is rolling resistance? And how do we get?

Quickly we will go into details later just to understand because I am putting a force there so you have to understand what this rolling resistance is okay. Now there is a misnomer, many students assume that the rolling resistance is just the frictional resistance of the tire, absolutely not. It is not the frictional resistance. Rolling resistance comes from the property of the elastomer or rubber which is the material of the tire.

Elastomer or rubber as it is called that is what goes into the manufacture of the tire. Elastomers have a property called viscoelasticity usually depicted by a dashpot in order to understand the effects clear. Now what is this viscoelasticity and how does that going to have an effect? Will see that in a minute as I said we will elaborate it later. Any material can be looked at, it is a small (()) (16:55) here.

Any material can be looked at from 3 simple models, one a spring, other a dashpot and third one is what I would call as a friction. Suppose I say that a material is purely elastic okay then you can say that the material can be represented by means of a spring okay. This is not a very correct representation. We are not going into too much of details. We can say that okay spring a linear spring especially okay is good enough to model say a linear plastic material okay.

So it is something like an understanding of the material behavior. A linear spring where the force is proportional to the displacement with the stiffness K can be looked as if it is a material and K is something like E okay. So when I leave the force the spring comes back to its original position and that is what we usually call as elastic okay. Now to this we can add other material behaviors.

For example, if you look at elastomers, elastomers are of course elastic and then viscous behavior okay. So in other words, I can model elastomer or I can understand elastomer as if it is made up of a spring and a dashpot okay. This dashpot can either be attached in parallel to look at it okay or we can understand the behavior by attaching it like this and so on. There are names to these models okay.

Kelvin and Maxwell models, but we are not going into details of this models okay. We are putting this in order to understand okay the behavior. For example, if you have a metal, which you are taking into the plastic region. Then I can model this metal using that spring and the friction element okay. So you can you know join together in parallel or series and so on you know these elements you can join them.

And then write a mathematical equation, which can form the basis of the constitutive equation or stress-strain behavior okay of the material clear. Now we are not going into this as I told you into the characteristics and I am not going to write down equations here. We will understand only the elastomer part in this case. Maybe pass a comment afterwards about this friction and why friction is used to model what we call as plasticity?

Now what is the difference between an elastic material and viscoelastic material? Sometimes people call this as hyper elastic viscoelastic material and so on.

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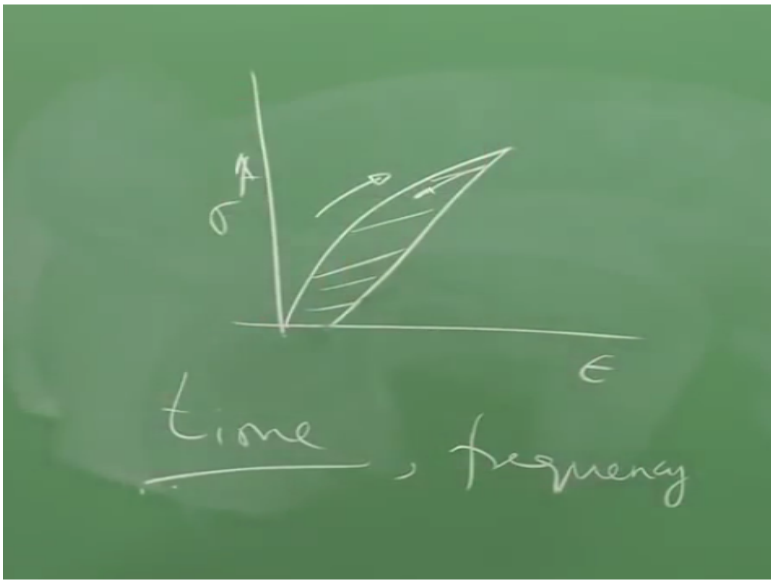

First of all, let us understand that elastic material is not necessarily linear, it can be nonlinear elastic as well. So if I now have a load deflection of the stress-strain curve okay, when I load a material which means that I am applying forces, I keep increasing the force because of which the stress is increased and there is increase in strain and so on. So when I load the material let us say that the path taken by the stress-strain curve is something like that, it goes like this.

When I unload an elastic material, it would actually all of you know that it would follow the same path. On the other hand, a viscoelastic material does not follow the path when it is unloaded and would now follow a different path okay and that amount of energy is lost and usually called as hysteresis loss clear. So there is an amount of energy that is lost. Is this same as plastic?

There is a subtle difference, good difference that though at the end of loading there is a residual strain here. The strain will come back to 0 with time. So time is an important factor in viscoelasticity. Time and frequency are important factors in viscoelasticity right. So in other words, what I mean by time and frequency are important is that the material behavior is affected by the rate at which you loaded or in other words, the frequency at which you loaded and so on okay.

So time and frequency are important factors. So the first thing is that to conclude whatever we have been saying that there is a loss of energy when the material is loaded and unloaded. How is it going to affect us? Why is it that the tire should develop a rolling resistance okay? And that is what we are coming now.

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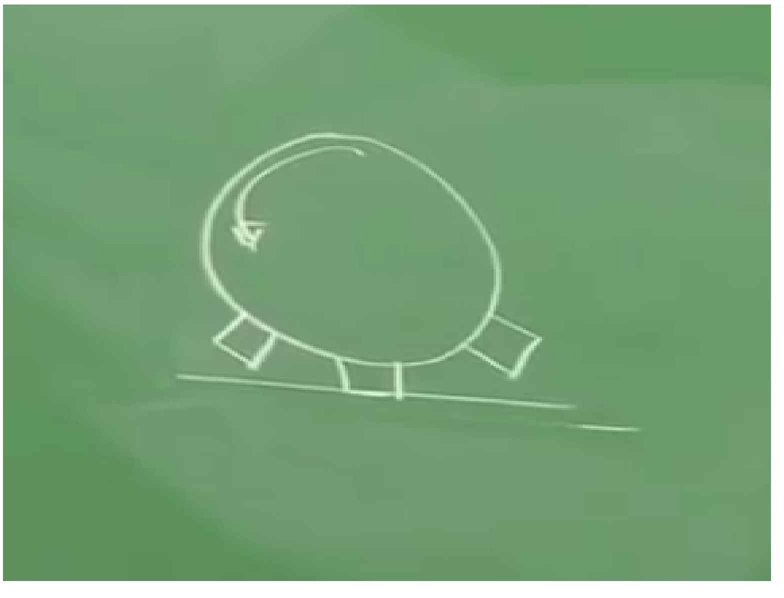

Now let us say that obviously all of you know it, but I am just reiterating what is well known. Let us say that I have a tire, we have what are called as treads, let us say that tread okay and that is the ground, so this tread material as it approaches is going to get let say that it gets compressed and then again gets released okay. Why tread the material inside the tire which we are going to see what they are okay?

Also gets compressed and released or in other words, there is a loading unloading cycle as the tire rolls similar to what you see in this stress-strain curve. So in other words, if I go and sit here in this tread okay and go through the cycle of rolling right. I will go through a compression and then whole compression as I come near I get completely compressed. So go out you know the load on me gets released.

So because of this cycle okay I lose energy okay or there is hysteresis loss right. Now who is going to compensate for this hysteresis loss? Because your vehicle has this tire and tire is losing energy. So who is going to compensate? The vehicle has to compensate okay. The vehicle has to compensate. So the first thing is that because of the material of the tire, there is lot of advantages why then rubber you know let us not talk about that.

We have lot of advantages we will see that okay. So because of the material with which this tire is made of, we have hysteresis loss and the loss has to be compensated by the engine ultimately and so this opposes the motion.

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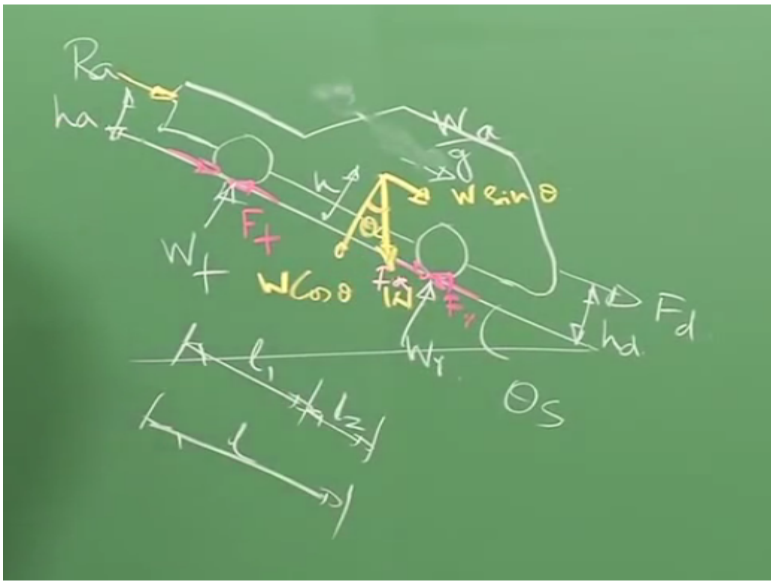

Now let us understand how did I get this force? I said that there is a rolling resistance force, which let us call this as Fr okay rolling resistance force, it can be the front and the rear okay so how did I get this as a force? Okay so in order to understand this we have to look at what is called as the contact patch of the tire. What is a contact patch?

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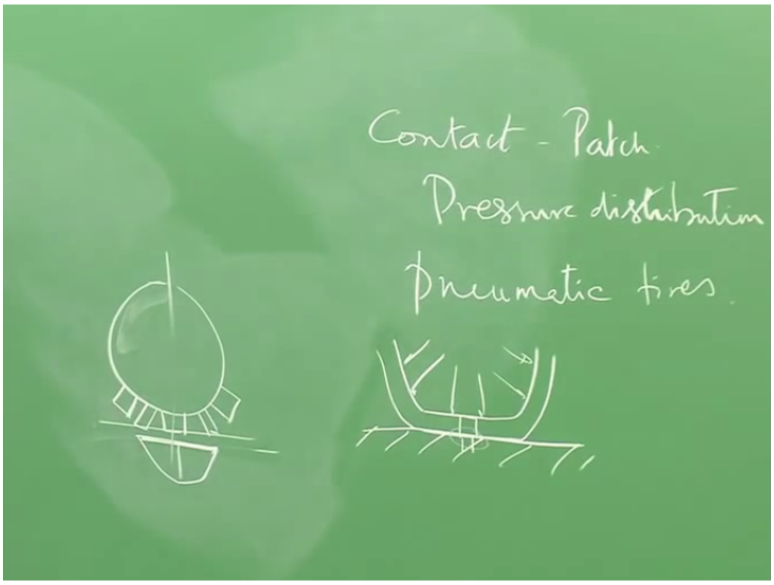

A patch that is formed obviously by contact of the tire with the road. In other words, more precisely, it is the pressure distribution at the contact right. So we will see the 3-dimensional pressure distribution later or rather 2-dimensional pressure distribution later. Now let us understand a section of this pressure distribution okay. So let us say that I come into contact at that point and leave contact at that point.

In other words, that is where my contact is. It is not necessary that the contact pressure exist only when the tire rolls. When the vehicle is stationery also you have contact pressure. Let us for a moment stop the vehicle okay and look at this contact patch. So the contact patch now is not that of a one tread okay, but there are number of treads so the contact patch would look something like this.

So in other words, rubber is symmetrically compressed about the center. This is the vehicle that is standing okay symmetrically compressed about the center. So whatever is the force that is compressed okay by this and it has to be, it has to come out in the other side okay whatever is compressed has to come out the other side okay. Now let us understand one or two more things about tires before we go into the details.

The first is that the tires that we use or what is called as pneumatic tires okay. In other words, we inflate the tire to a particular pressure right. So many of you might have driven a car even now when you go to a gas station to fill or inflate your tire still you talk in pounds per square inch units okay 32 psi, if you are driving a huge vehicle truck it is 120 psi and so on right. So let us go into some details and look at the section okay from this angle.

So let us say that the tire okay that is how it is deformed okay let us say that the tire is deformed like this right. So when you look at it from this section or whatever be the section that is how the tire is deformed okay. For a moment, I am taking out the tread and I am saying that the tire has a thickness something like that right. That is what the inflation pressure okay, which we have used in order to inflate the tire okay.

When we inflate the tire we get what is called as inflation pressure. So now we know very well equilibrium equations, we know very well that whatever infinitesimal element you take should be under equilibrium between the forces that are acting on these infinitesimal elements okay. So obviously when I take an infinitesimal element here okay I said contact pressure is what is acting in that region right.

So if I want this to be under equilibrium or if I want it to be at equilibrium then the pressure that is acting the contact pressure that is acting should equilibrate the inflation pressure okay when it is in full contact okay. So the contact pressure should be equal to the inflation pressure okay. Contact pressure should be equal to the inflation pressure so strictly speaking the contact pressure should have been uniform okay.

But contact pressures are never uniform. We will see more about it a bit later. Because of the local bending because of the bending of the side walls, these are called side walls and that is why the contact pressure is never uniform okay, it has a particular shape. We will study this okay after 2 or 3 classes right. So now let me come back. So in other words, there is a lot of theory as to how contact pressure develops?

How contact pressure is distributed? Whether it is uniform? Whether it is not uniform? and all those things okay. Now here when I talk about this I am only talking about the pressure okay because of the tread as it travels along or around the circumference okay. So here I am looking at the pressure on the tread okay. So the pressure on the tread compresses okay, goes to a maximum and then gets released okay.

So let us not right now confuse between this and this. We will come to that later. So the contact pressure what we are talking about is because of the tread getting compressed right. When the tire is stationary okay, then we have a contact pressure something like this because there are a number of treads, there is one tread that is getting compressed, another tread getting compressed a bit more, another tread much more, another tread slightly less and so on.

So number of treads are involved at various compressive positions okay and hence we have a contact pressure like that of the treads that are formed okay. On the other hand, let us now roll the tire okay.

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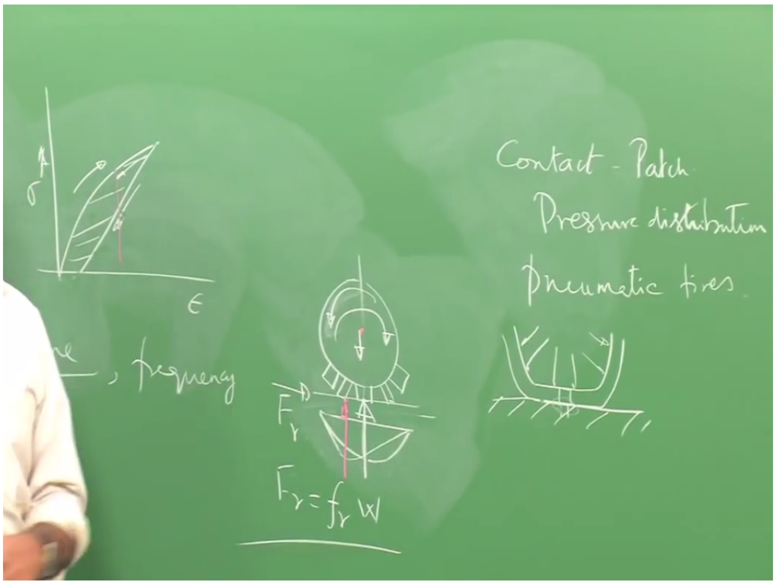

Now when I roll the tire let me follow a tread so that is for a static so I am just removing that. Let us now roll the tire, when I roll the tire, one tread or one block that block is what we are going to follow, that block gets compressed okay, goes to a maximum compression and then gets released. So one block here okay that block as I rotate as the tire revolves goes into this position so same block goes into this position maximum compression goes out and gets completely released.

So in other words, a block gets loaded like that okay and then gets unloaded. Now how is that it is going to be unloaded? It is going to be unloaded like this. So unloaded like this. So as the blocks gets loaded and unloaded, these blocks loose energy or hysteresis develops in these blocks clear. It is not only the blocks that gets compressed or loaded and unloaded, but the sides of the tire they also go through the same thing.

So in other words, the sides of the tire gets also loaded and unloaded and so on okay. So in other words, this loading and unloading cycle gives rise to this energy loss and that has to be accounted for by the vehicle. I am just repeating that so that you understand it and that is quite clear. Now how does this loading unloading cycle affects that contact patch okay in a very simple sitting as we had seen.

How does that gets affected? Because the loading cycle or loading path is different from the unloading path? How does that get affected? So it was symmetric when it was stationary that is fine, but when it gets loaded and unloaded, look at this carefully for the same strain in the unloading path, the stress is less okay. So now we are talking about the pressure okay that is acting on the treads since for loading and unloading they are different.

This curve cannot be symmetric because both of them are not the same so they cannot, this guy is due to loading and this is due to unloading okay. So they cannot be symmetric because I am following the same tread okay, which is going through the cycle so it cannot be the same. So how it should be? This has to be a different curve, this has to be a different curve. So the curve actually shifts and becomes something like this.

Because the loading curves are different from unloading curve, the curve of the symmetry is lost becomes something like this. If I now say that the reaction force in order to support. This is not a very correct picture that is why I introduced this inflation pressure, keep that in mind, we will come back to this topic again okay, not a very correct picture. We are going to see very interesting things how inflation pressure is going to act? and how actually the tire carries a load?

You know we are going to get to details there okay. So we will come to that a bit later, but let us now understand this from a different angle okay and give an explanation only to the rolling resistance. We will refine it as we go along. So if this is the load that is acting on the tire then the load is now equilibrated from the ground or in other words that is the load that is going to act okay which opposes the weight.

Now since this symmetric distribution is affected what will be my resultant force due to this contact with the ground? The resultant force which is developed due to this compression, which opposes the load that is on the tire would now get displaced and hence actually instead of acting right at the center, the load now acts okay away from the center right and that is how the load acts.

When it acts away from the center then if I now look at that load with respect to this center not only I am going to equilibrate this load with this force, but I am also creating an additional effect correct. So what is that additional effect? That will be a torque. That will acting or a movement that will be acting like that right. Watch carefully that the moment is now going to oppose the motion of the tire okay.

So there is an opposing force or opposing moment that is acting okay. Now I do not want to put that moment here. I know that the moment opposes the motion so I just want to replace this moment by means of a force that is acting here okay because that will oppose the motion of the vehicle. So I replace this moment, which in reality exist because of viscoelasticity by a force here okay.

And call this as rolling resistance force and say that this force rolling resistance force creates the same moment which opposes. In other words, Fr*r= this into this, we will give names to that in a minute. So first let us understand the philosophy of development of a rolling resistance force.

So the philosophy of this rolling resistance force to summarize is the viscoelastic behavior of the elastomer, which means that there is a loss of energy, which means that the symmetric contact pressure distribution when its stationary gets affected or in other words it becomes skewed okay and this skewed distribution produces a normal force okay, which not only opposes or not only supports the vehicle or the tire.

But also creates a moment, which opposes the motion and the opposing moment or motion or torque is now also depicted as a force, which opposes the motion of the vehicle or the tire and we call that as the rolling resistance force clear okay. So that is why we have a rolling resistance force. The rolling resistance force of course you can see this very clearly. Rolling resistance force, since it comes out of a moment which supports the weight W okay.

So this force has to be proportional to W right. So we usually write the rolling resistance force to be a rolling resistance coefficient multiplied by W. Obviously, the rolling resistance opposes the vehicle motion and hence is not “a good force” okay, it is not hitting us to travel. Actually it is opposing you, since it is opposing you or opposing the motion of the vehicle, we consume energy. We have to overcome that like you have the aerodynamic forces, we have rolling resistance forces, which opposes the motion.

Interestingly note that when the vehicle brakes, this rolling resistance force would act in the same direction as that of the braking force, which is now going to flip and act from the other direction. So rolling resistance force aids in braking and opposes traction clear okay. So the first thing you would tell that why not I completely reduce rolling resistance go to 0? Is it possible? How low you can go?

There are lot of issues. We will come to that later when we talk about tire mechanics. So first things first. So that is the rolling resistance force, which is written in terms of rolling resistance coefficient and W okay.

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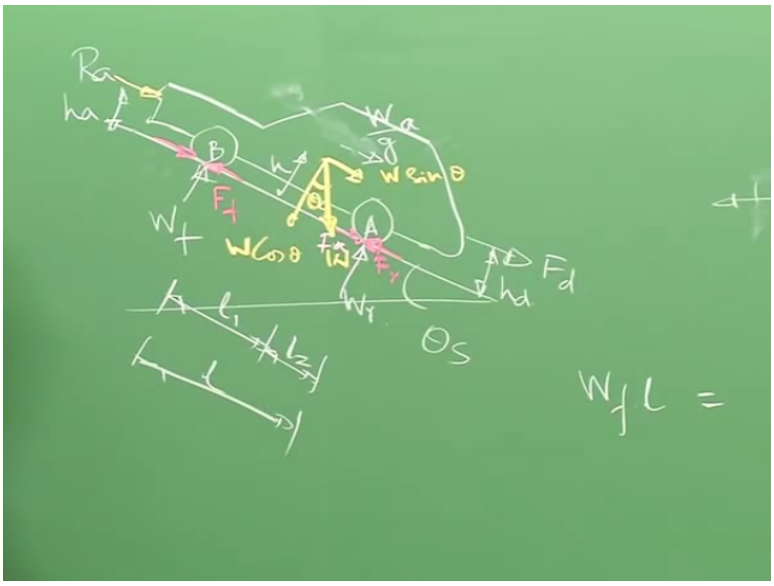

Our next step is to find out Wf and Wr okay and Wf determined by taking a moment about the point A and Wr determined by taking a moment about the point B okay. So on one hand, we have Wf*l=on the other hand you are going to write down the moment due to the forces okay. So you know this very well. So Wf*l is in the clockwise direction so accordingly put the forces and the moment okay.

Rather the moment due to the forces, put the signs properly and we will see how we end up with this equation in the next class. We are going to make some assumptions okay with respect to these heights. We would see that usually in a passenger car these heights are almost the same and when you make an assumption that ha=h=hd okay that makes our life simple.

One of the things which is obvious which all of us experience, which you would immediately notice is that Wf and Wr is going to get affected when a vehicle is accelerating okay.

Or in other words, that is what is called as a load transfer. You would have noticed this when you go in a vehicle in a car obviously, all of us know that very simple mechanics that when you accelerate, you tend to fall back and when you brake, you tend to fall forward or in other words there is a load transfer to the axles as well. Another very interesting effect, so we will write down this equation.

We will find out Wf and Wr. Then we will look at traction and braking and so on okay. We will stop here and we will continue in the next class.