Vehicle Dynamics

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Lecture - 21
Handling Response of a Vehicle

Let us quickly summarize what we did in the last class. We had looked at what is called as the state space approach.

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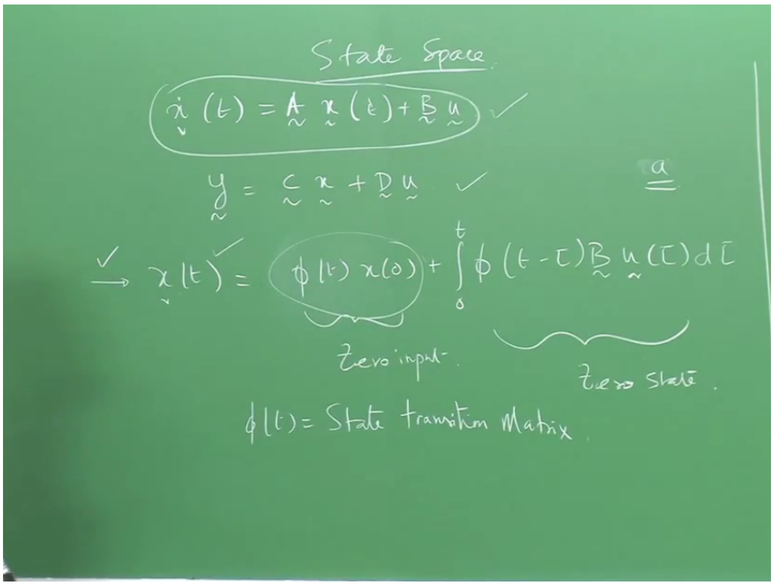

The importance of state space approach today comes from the fact that it is extensively used in the control algorithms in understanding automotive controls and so on okay. So we have to digress a bit and look at state space. It is also very easy to understand. How? For example, a vehicle behaves for a particular maneuver. We will see that in a minute. Let us finish this. We will see that how this can be used.

Now what we have looked at, this particular case is a simple bicycle model where we had the 2 degrees of freedom called as v and r. It can be replaced of course by say for example beta. You can replace beta and r and so on. You can also extend this. It is not necessary that you have to have only 2 degrees of freedom. You can extend it. You can go to 4 degrees of freedom okay.

Or you can go even up to 11 degrees of freedom okay where you would introduce I mean the tire characteristics and so on. So in other words, it is difficult in a class to derive all those things. That is the reason why we have taken 2 degrees of freedom. We will indicate how it looks like maybe 2 classes from now. How it would look like if there are say 4 degrees of freedom?

I will just give you the final expression. You can always derive it, it is not very difficult okay. So what we are now doing is that whatever be the number of degrees of freedom of the fundamental model, which results in what we call as the state space equation from that okay. The form of that equation can be written like this okay. So the form of the equation can be written like that.

So the general approach that we have now can be extended to whatever be the degrees of freedom. That is the whole idea okay. So the state variables that is the vector x can be any of those things. We will see that a bit later right. So the approach is general that is why we are putting this down okay, but in a class if you want to work out we will stick to 2 dimensions or 2 degrees of freedom and that we will see it later.

So that is the fundamental equation and that is the output all of you know. We went through this x t v, we derived this remember that we had Laplace transform and so on. So we have 2 terms ultimately for x, you know the x consists of 2 terms, one is what is called is the 0 input term okay. In other words, this is purely because of the initial condition 0 input term and this is the 0 state term. In other words, this is due to the input u.

So the response consists of the initial as well as the input conditions okay. Now all of you know, I need not repeat it, maybe for the sake of some of you remember that we had I mean if there is a differential equation, we had solution e power At or alpha t rather At I think we put A right, e power At and that if a happens to be negative okay then the solution does not become unbounded.

And so you had that stability criteria in your earlier classes that the real part of the Eigen value should be in the left and so on and so forth. We will come to that left of the z plane and so on, you know all those things, just to remind you that they all can be now factored into these equations right. That is the state variables and then we ultimately looked at the linear time invariance system, which is our system okay.

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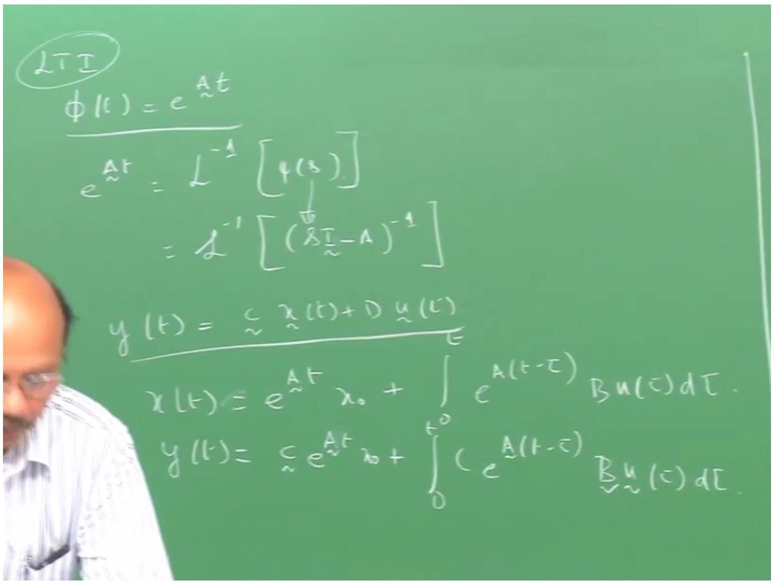

And identified that e power phi of t=e power At right and wrote down and determined also what we called as state transition. This is the state transition matrix as I had indicated here this is called as state transition matrix okay because of the fact that how it is participating in the expressions and we remember that we said phi of t is the inverse Laplace transform of psi of s remember that, that is what we did.

And remember that in which case that expression now boils down to this expression okay. This all we did before and that y of t can be written as this standard form in the state space form and so the x of t is this, you know that is specifically to this from which I can write down y of t okay. Y of t is C e power At x0+0 t C e power At-tau B u tau d tau right okay. Now one of the interests in any of the dynamic system is the impulse response okay.

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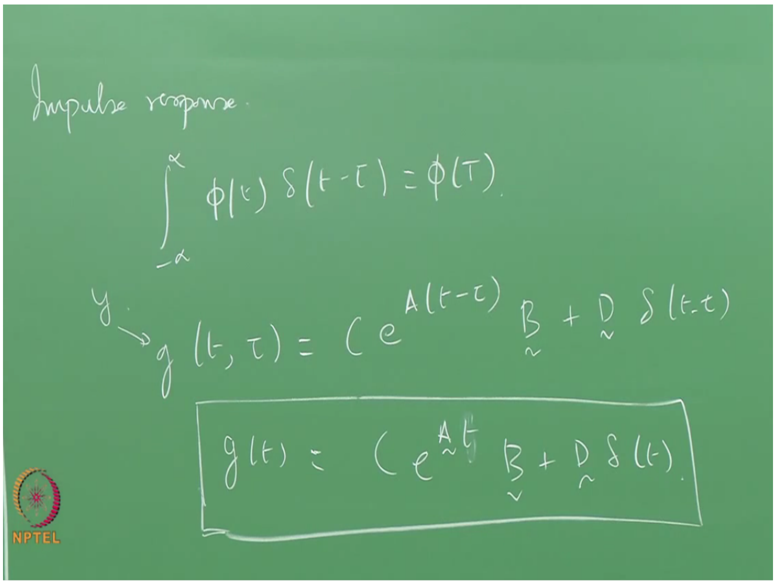

"Professor - student conversation starts." Why is that we are interested in impulse response? That is one thing, but more important also is that we can find out the transfer function, frequency response function okay because the Laplace transform of an impulse is what? Unit okay so directly we can get the transfer function from you know if the Laplace transform of the impulse=1 then you can easily find out the transfer function and so on. **"Professor - student conversation ends."**

Okay let us not repeat that you know those things already okay. Now I want to find out what is the impulse response of the system right. So how do I find out the impulse response of the system? So what is the input now? Input is the impulse okay. I can write down that and I am not going to derive completely, I am just going to write down only the sifting property, which all of you know.

Phi of t delta of t-tau=phi of T this is because of the what is called as a sifting property okay. Now substitute that and then into this expression and ultimately you can write down C e power At-tau B+D delta t-tau. For a minute, we will remove this D okay. D usually is not there anyway if you want to retain it you can retain it. How did I get this g from? This is nothing but y okay.

So from y I determined that expression right and then I write down that as g. Now by change of variable I can write down that okay. So that is the impulse response of the system okay right. We will come back to this because we let us not get lost in further equations on this, just keep that in mind okay. We will again go back to hard core vehicle dynamics okay and then expand this later right.

We will develop a simple technique to understand. Let us go back to physics okay. Too much of maths, too much of this and you will lose track of what we have been doing right okay. Now let us look at important quantities, let us look at the physics of the dynamics and then keep that in mind I will come back to that equation a bit later right okay. Now get back to the expressions that we had used before okay.

Now let me write down that expression. I am going back to the A okay remember that we had v dot r dot x dot=Ax you know I am going back to that expression with a Laplace you know after Laplace transform.

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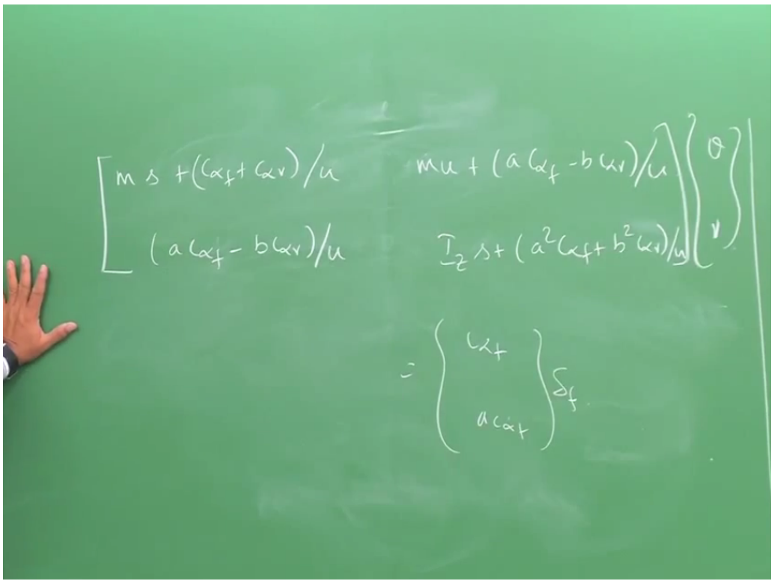

So in other words, let me get back to this expression. Okay clear what did I do? I just took a Laplace transform recognized that x dot of t Laplace transform is s*x of s okay, substituted it in that expression, I have just rearranged it right nothing very difficult. Now I am going to straightly digress from here and I am going to get the steady state gains okay and what is steady state gains and why am I interested in this?

So let us get back to the physics of that expressions. Now we are looking at maneuverability okay lateral dynamics. Suppose you are driving a car okay. There are 2 situations, there may be others, in 2 situations where the maneuverability of a car becomes important. For example, you are doing a lane change okay. You are doing a lane change.

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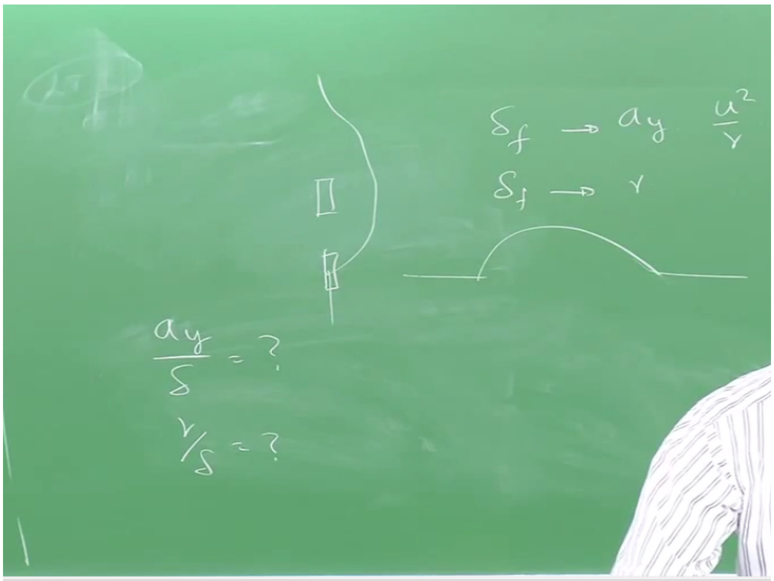

Say for example, you are here and there is a car before, then I have to do over take the car okay and I am doing what is called as the double lane change okay. This is one situation where you want the car to respond to your input, the driver's input okay, so that you will have no issues in overtaking the car. The other one is that when you take say for example a sharp turn, we take a sharp turn okay.

Why am I putting these 2 as if they are different cases. They are different cases where you as a driver would like the response of the vehicle to have 2 different aspects. Your input is the same. You are going to give the steering input okay. Your input is the same, but you are expecting the vehicle to respond in a slightly different fashion. From a dynamics point of view, going to look at 2 different things.

Let us take this case it is easier first then come to that case. What are they trying to do here? You are reorienting the vehicle as you do that clever okay, you are reorienting the vehicle, which means that what is important here it is the yaw, the performance of the vehicle to your input as far as the yaw is concerned becomes important right. So that is what the yawing of the vehicle.

Or what becomes important is the rate at which it goes or the yaw of the vehicle okay for your delta input. What is delta? Steering input becomes important. So in other words, here you expect the yaw to help the vehicle to get reoriented in order to take this path okay. You as a driver you are looking at that, not that lateral acceleration is completely removed. Of course, lateral acceleration is also important.

But for you the response is am I taking the turn correctly? Is the vehicle following that? That is what becomes important. Here more than the yaw, it is the lateral acceleration becomes important. Here it is the lateral acceleration because you do not want to go off the path because this may be done at a higher speeds and so the lateral acceleration as you take that curve okay that becomes important.

Yaw is not that important, it is not that they are not important okay, both of them are important, but the relative importance will be different in each of these cases. So in other words, what is that you are interested in? Given a delta, in both cases you are giving delta, you are giving only the steering input. Given the delta on the front okay, what is the response of the vehicle as far as the lateral acceleration is concerned?

Either is you can write it as u squared/rg or u squared/r, it does not matter okay. This is u squared/rg or u squared/r okay. The response of the lateral acceleration and what is this response of this to yaw? Right so these are the 2 things that become very important for us. So that is r right I mean what is the response to this? Now how am I going to determine that from this expression?

In other words, we call this in our control system language as the gain okay. What is this? That is the lateral acceleration gain okay and what is this? These are the 2 things that become important for us okay from the vehicle response. You can do exactly the same thing with these expressions. This is not a problem, but I will make it slightly simple so that we will not lose the physics.

I will come back to a more elaborate state space a bit later right. Why do I keep writing only delta f, you know does it mean that delta r okay is also there? Yes, I had already told you that delta r can be for a very long vehicle, but interestingly there is another condition, your own vehicle I mean what cars you drive they do not have a rear you know steering or the rear, but that becomes important when?

When you backup, when you reversing your car backup then actually what are you doing? You go okay and u direction is now in you know in the negative of u that is in other words you back up and then how do you steer? The rear steer okay. So in other words this is a condition, which is also common in everyday use of a vehicle okay. We always find it difficult to backup.

You know it is very easy to drive in the front not that it is difficult to see. Even if you have a camera before you, sometimes it is difficult to backup why is it? Keep that in mind and I would like to answer that later. Right now, we are looking at only delta f okay input in the front. We will then see what happens to delta r clear okay. How do I now determine this transfer function or in other words this gain with respect to s and so on?

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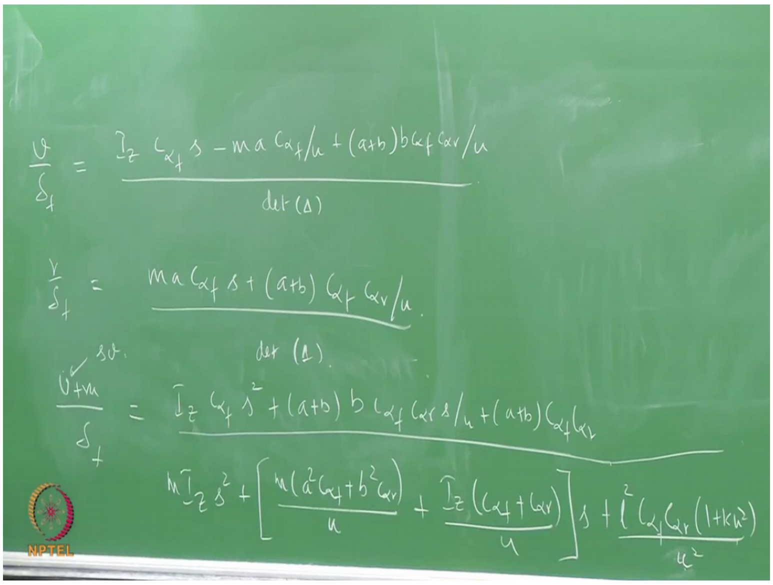

What I am going to first do is to find out what is v/delta lateral velocity because once I know that I can always find out A later v dot+ru v dot is sv and so on so it is not going to be very difficult. So I am going to find out what is v/delta and r/delta. This is what I am going to find out okay. So how do I do that, simple delta f, I am going to use the Cramer's rule, I am going to put let me say that let me call this as delta okay.

This whole thing as delta right this matrix and if I want the first expression v/delta f, I put this (()) (22:28) into the first column and use that as the numerator and use the determinant of the delta as the denominator. This is what we are going to do okay. So if I want now r/delta then I am going to put this here for that column, retain this column and determinant of that I will use that as a numerator and determinant of delta I will use that as the denominator okay.

So that is what we are going to do. Later to look at a very interesting physics, once we know this we will add delta r, we will do that separately and then we will use this technique in order to find out what happens to v/delta r and r/delta r that we will do a bit later right. Any questions? Do that. So v/delta f=substitute that multiply it okay. I am not going to do all those multiplication.

You do that so that I will write down the final expression C alpha f s that is the first term okay so what do I do?

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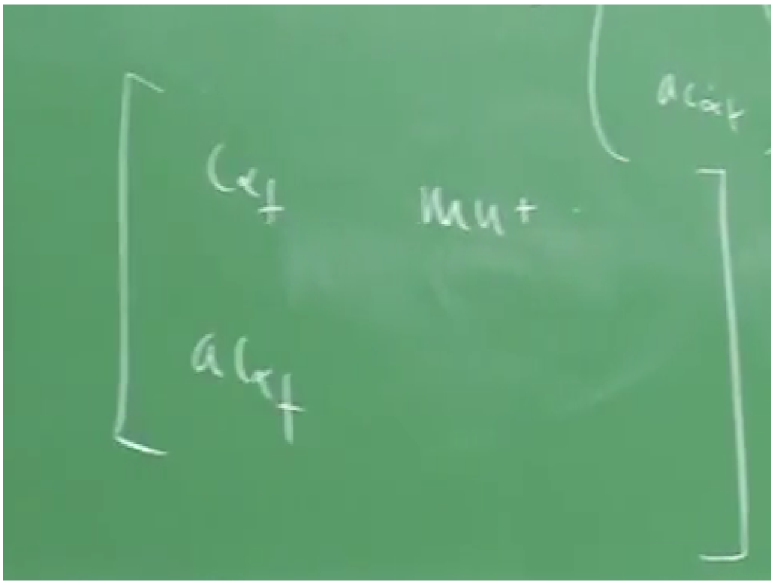

Obviously, C alpha f ac alpha f then mu+ so that is the determinant of that is what is my numerator okay. Determinant of this find out I hope my signs are correct just check this. Whole thing divided by the determinant of delta or let me call that as determinant of delta to the delta matrix that is it right okay.

R/delta f I did the same thing, put this here and I am going to write down the final expression, check this as well. If there is any questions I will answer that because of lack of time I am not going to substitute it, derive it and so on, that you can do it very easily, divided by determinant of delta okay. Now if I want the lateral acceleration then v dot+ru/ remember that is what we had in our expression divided by delta and v dot is nothing, but sv right.

And +ru, r I know, u so combining these 2 I will get v dot +ru as the lateral acceleration gain okay. I can find out that from these 2 expressions. Can I? Right. Write down what is that expression. Look at this sv, so Iz c alpha f s squared okay-m*a, what is a? A is not acceleration remember that a is the distance from the cg to the front axle okay, ma c alpha f/u*s right+a+b bc alpha f c alpha r*s/u.

Here r*u right so you would ultimately get an expression of this form that Iz c alpha f s squared+a+b*bc alpha f c alpha r s/u+a+b c alpha f c alpha r the whole thing divided by the determinant of delta is that is going to be a huge expression anyway this is not a very difficult, nothing very difficult to get. Okay right that is the determinant, this is the numerator and that is the denominator.

For r/u, just substitute here for this, substitute that. **“Professor - student conversation starts.”** Just check that or let us check this that is why I said put that here okay c alpha f so Iz s+a squared c alpha f+b squared c alpha r/u okay, substitute it and check what is the expression? So now multiplying this Iz so the first term is fine. So the second term is a squared so let us do that. **“Professor - student conversation ends.”**

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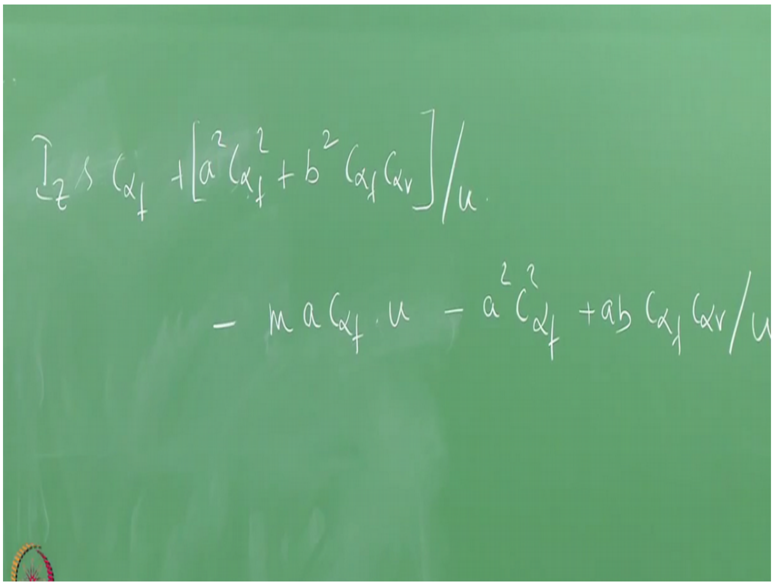

The first term is Iz*s*c alpha f right. The second term +a squared c alpha f squared+b squared right c alpha f c alpha r right/u that is the second term-what is the first term ac alpha f*ma c alpha f*u right-a squared c squared alpha f so +ab c alpha f c alpha r/u they are correct right. So the first term that is correct okay.

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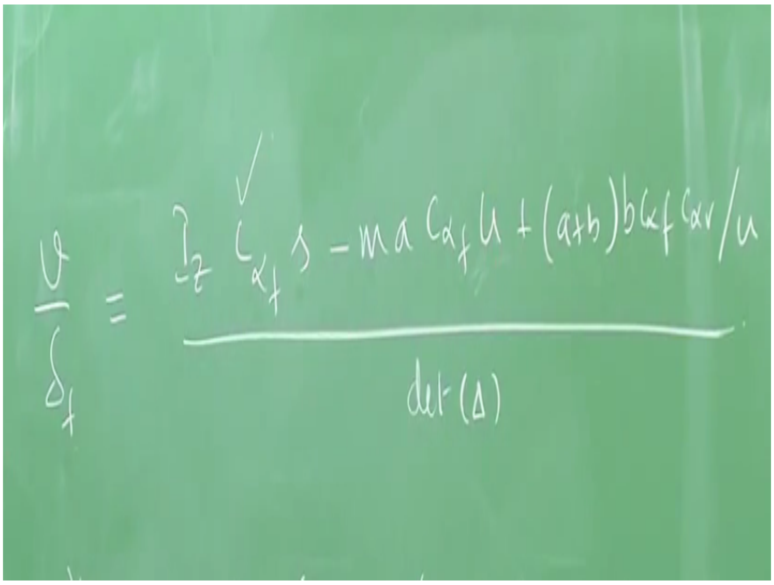

Second term should be u not divided by u right that is what right that is multiplied by u.

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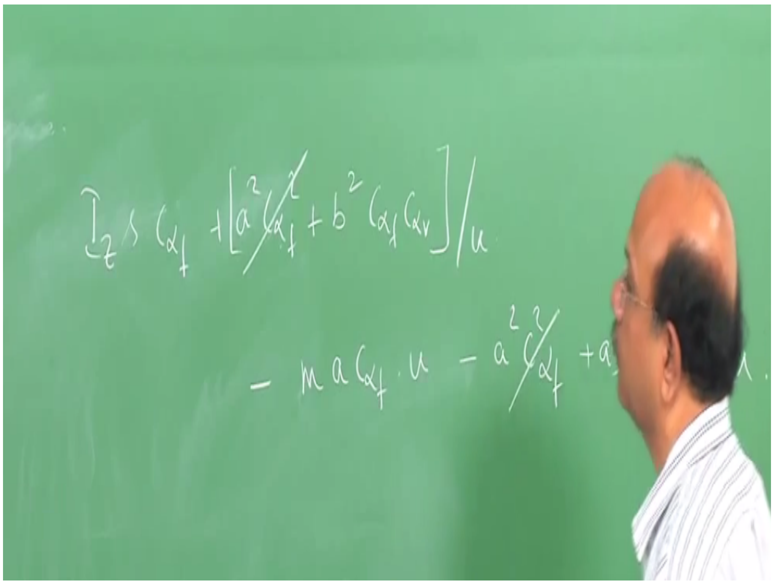

Then this term goes off this term goes off so you have that right okay good. All of you are awake and kicking good great. So now you add this I think now v dot s so Iz c alpha f s squared that is there okay. Now this becomes –a s c alpha f u okay right because here I have same thing ma c alpha f s*ru there will be a u here.

So this guy and that guy will go off right. So I will have absolutely so that is the important thing. So now you see that this term is multiplied by s and that is the term I have here and this term is multiplied by u and that is the term I have here right correct great. Now let us look at this. Thanks okay now you are clear with the derivation as well. Now let us quickly pick up this and this before we write what is steady state gain?

What is steady state gain? When do we say what is steady state gain? This is nothing, but the transfer function. So when do you call this as steady state gain? When s=0 that is all okay. So that means that r/delta f this term will go off.

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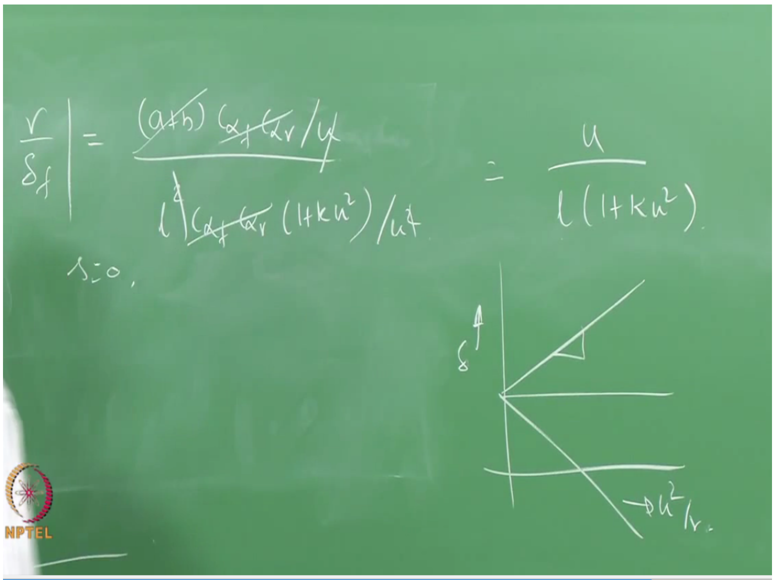

Can you tell me what is the steady state gain? a+b c alpha f c alpha r/u/right the last term here l squared c alpha f c alpha r 1+k u squared/u squared, you can simplify that later that is equal to 0 and similarly you can write down ay steady state gain for a also I mean for the lateral acceleration as well, but before we go further let us look at these 2 expressions and what do you pick up from these 2 expressions?

It is very interesting thing that is happening here. Look at the numerator it is first order polynomial that is in other words you have only s. Here this is second order okay. What about this? Numerator has a second order and denominator has a second order okay. Now all of you know that if I want to convert this into a frequency response function what do I do? S=j omega fantastic.

So I will substitute s=j omega in order to convert it into a frequency response function right. So what do you pick up from that? Say for example when the frequencies are high when the frequency is high okay. So frequency response function, the frequencies are high as the frequency goes higher and higher what happens to this response and what happens to this response?

This would go to 0 yes so the frequency response at high frequencies okay. This would go to 0 which means that there would not be any yaw velocity okay for delta f. What about this? This would become a constant because the first 2 terms would go off okay that would become a constant okay. So at very high frequencies this goes to 0 and this goes to a constant. So if you want to do any of the you know sensors to be placed in order that you want to control okay.

Or you want to look at the automotive control okay then what is the sensor that you would use or what would you pick up? Would you pick up yaw or would you pick up lateral acceleration? You would pick up lateral acceleration because lateral acceleration gain is still you know it is a constant. You would pick up that right okay. So write down for both this as well.

So let me simplify this that is equal to a+b is l so u/l*1+k u squared. Remember that k is the understeer gradient sometimes is called as understeer coefficient okay people use this as understeer gradient or understeer coefficient. Why do we call this as understeer gradient? What is this gradient about? Why is that we call this as understeer gradient? Remember that graph, which we talked about terms of delta versus u squared/r.

You remember, go back and check that graph okay remember that I had delta versus+u squared/r. So in other words delta=k*u squared/r+l/r you know that was the expression that we got right. Remember that we had drawn it u squared/r versus delta and the graph was like this remember like that the graph was like this right. So what is this gradient? In other words, what is the slope of this? That is the k right.

So because of the fact that it is the gradient in the graph between delta versus the lateral acceleration, we call this as this understeer gradient. People sometimes call this as understeer coefficients it does not matter, but remember that that is the word that is used right okay. Let us write down the steady state gain for a1.

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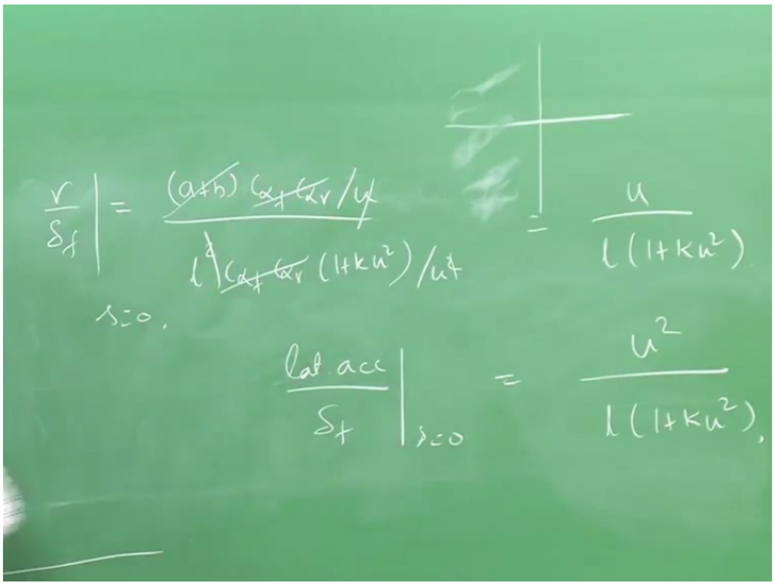

Of course, it can be right, left and right, left half plane or right half plane. That is going to be very interesting when we look at instead of delta f when we look at delta r okay. So in other words, when we are backing you are going to find a very interesting twist to the 0s. What is it? After a minute or after a small derivation we will come back. So to summarize, this is very important for us to determine the poles okay response of which the system is determined by these poles, the position of the poles and that 0s okay.

And what happens if this 0 falls on the right hand side of the plane? Can it fall? What happens? We will see that now okay. Any questions so far? Okay now we are going to look at response of the system. We will generalize it later. This is for the bicycle model. We will generalize it later for a much larger system by expanding what we did so far. I am going to write down these equations, which you already know.

That is the one problem with this course is that you need to keep on writing down the equations, which we derived before and I hope we are in track. If you have any doubts stop me okay we will review this aspect. What I am going to do is to pick up what all I did so far in the state space okay. So just to summarize just to go back again, so if I want the frequency response function for any of these things, no issues at all.

I can get this from these expressions okay just substituting s=j omega okay. So these 2 responses are important as we said okay. What is if I have to take a double lane change? Then the expression which I would be looking for the response I would be looking for is this. If I have to do say for example a cornering, then the gain is this okay. So that is the expression, which I would be looking for right okay.

Now ultimately our aim, what am I going to do? Let me summarize that and maybe we will start the derivation in next class because I am going to go through last derivation. Ultimately, my aim is to understand handling okay and see whether I can put down a quantitative measure for handling okay. Yes, this is fine, you understand the physics, you know you understand that yaw becomes important.

You understand that lateral acceleration becomes important and so on, but ultimately I want to understand handling in a very quantitative fashion what does it mean? It means that if I have 2 or more cars okay, if I come and give you say 3 cars and tell you tell me which of these cars has good handling characteristics. Suppose I am asking a question like that. Tell me which has good handling characteristics okay?

Now you can drive the car of course and then tell me your feel okay, looks like this car is good you know in other words you can give what is called as a subjective evaluation.

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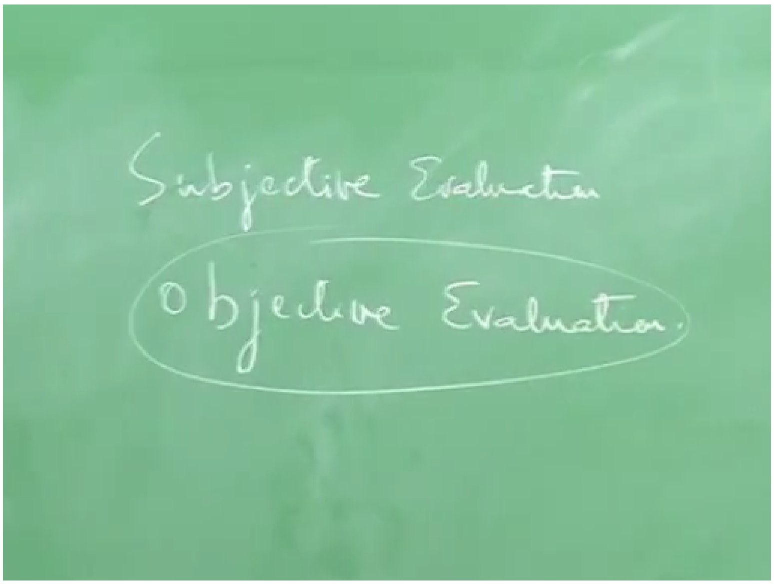

You can go to and give me a subjective evaluation. This is just by driving the car. You drive the car, it takes a turn, you take a double lane change, there are ISO standards for this Slalom test, you do a pulse test, you do all those things and say okay I feel this car A is good than is better than car B and so on. There is other thing which is called as an objective evaluation of the vehicle okay.

Now what is objective evaluation? I have these things yes of course these equations okay. What does it really mean okay when I have 2 or 3 cars? How am I going to look at the response through these equations and tell okay quantitatively car A is better than car B better than car C and so on okay? Why are we interested in an objective evaluation? Because objective evaluation very clearly tells you what is the characteristics of the car, which makes it good or bad.

Subjective evaluation by the driver would only tell you whether this car is good or that car is good in a language, which are not necessarily very technical. Of course, every car company relies on subjective evaluation, but it is very important to now bridge this gap and marry these 2 subjective and objective evaluations. So we are now looking at handling from a very quantitative perspective with all the equations, which we have derived okay.

And put down the rules and say that look at these 4, 5, 6 quantities I have a number of them, I have a yaw, I have a yaw rate, I have some yaw rate gain, I have lateral acceleration, I have beta v/u, the vehicle slip angle and so on so many things, that now I know what is important? How am I going to now combine them and tell that vehicle A is better than vehicle B? So in other words, how am I going to convert these things into characteristics of the vehicle?

That is what we will see and we will pick up one of the models and explain that model okay and understand that model okay and how we can use that for the vehicle evaluation. We will do that in the next class starting with my derivation clear okay. Please revise the state space because I am going to use the state space again clear okay.