Vehicle Dynamics
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Lecture - 22
Mimuro Plot for Lateral Transient Response (Part 1)

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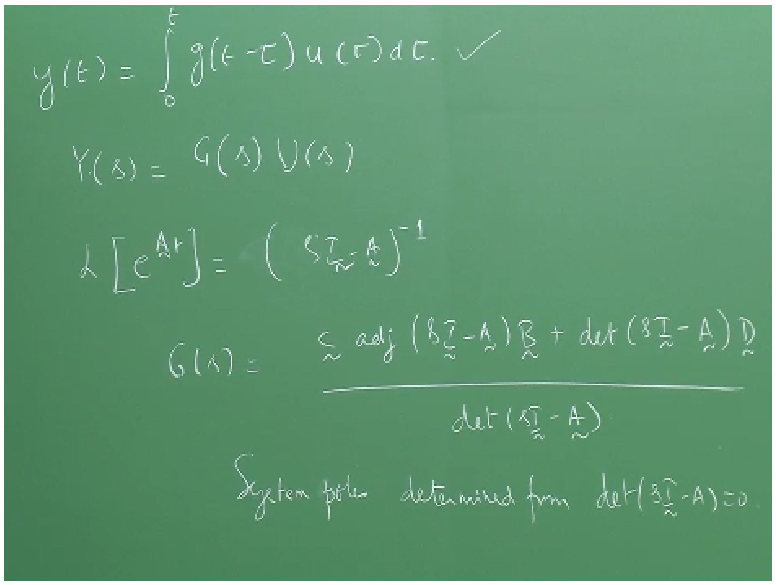

Let us review quickly what you know about the state space technique. So that we will go forward applying this of course we had already discussed a few of the poles we will just put down all the questions just to revise what all you have done in the previous classes on controls okay, so this all of you know this is the response y(t) okay and the impulse response function and this is the convolution term.

The convolution part of the response which when we take the Laplace transform of that equation you get okay the multiplication of this convolution term or the convolution integral and we also saw that the Laplace transform of E power KT I is si-a inverse right and that the G(s) which is the impulse in a transfer function is given by this equation go back and look at it these usually go 0 in all cases so that can go off.

Okay and the standard equation X. =ax+bu y(t)=cx+du okay from that we get this and the poles of the system is determined from this determinant determinant=0 okay and you know the importance of poles and once let us call P1P2 to be the poles of the system okay and if P1 p2 are the poles of the system and GFS is written say for example by simple fraction form and that if you now take the inverse Laplace transform you get GFT.

Okay which is given by this function and the words the poles play a role okay which are loosely called as the eigenvalues of this poles Eigen values, one of the questions which people have especially when they come from a vibration background is that is that of the connection between the natural frequency and these poles is always a confusion as to what we mean by natural frequency.

And damping in other words the poles of the system are expressed in a slightly different fashion okay as omega n zeta for example omega is a natural frequency and zeta is the damping ratio. So the relationship between say for example if the poles of course you know that it has to be a complex conjugate okay or it can be only real in which case you are looking at the time for realtion.66.

And so on you know that is given by this time and more importantly if you want to pick the relationship between the poles the natural frequency and damping then the relationship is given by this form, okay so it is people usually say that natural frequencies is the eigenvalue or poles and so on they are not strictly correct yes one comes from the other but please note that if these are the poles okay the complex conjugate poles then this is what is the natural frequency.

Okay now you understand the natural frequency and the poles you know what are or important okay, further than this about were the effect of poles you all know it.

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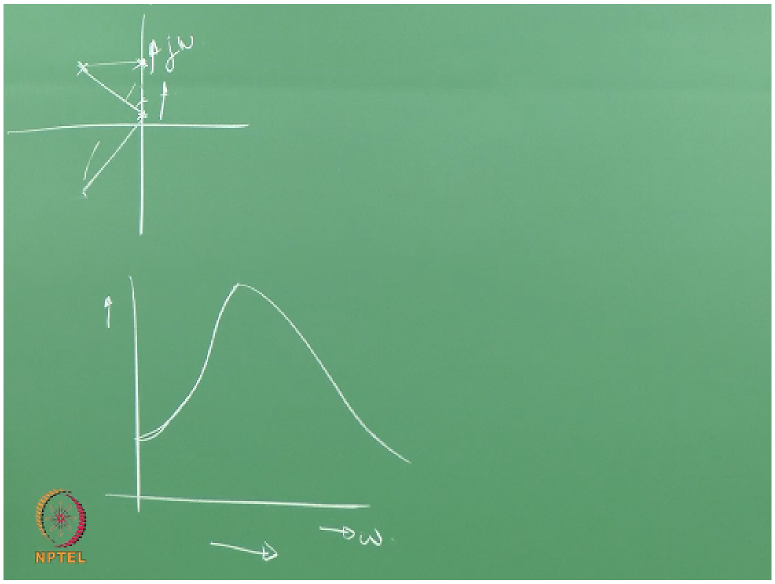

And that you know they are in the S plane okay if this is the left of s plane okay then the poles are situated to the left of S planes, S plane the system is stable basically because you have the E power term here and hence when it is negative okay the E power-term-xt would now start decreasing with time and hence okay the system becomes stable, okay. Electrical engineers have a very interesting analogy very interesting.

We have interpretation not analogy but it is interesting we have interpreted into produce and how they interpreted this so this is the J omega okay, so you would notice that that if I want to look at at the frequency response what is that I do, I replace S by J omega, right in other words when I want to plot the frequency response then I what do I do I plot omega here okay and the response in the Y direction or in the Y axis.

So you would notice that actually as I go along omega I go up that axis okay I in various places okay I go up the axis as they go up the axis if you look at what actually I calculate okay I calculate the distances between the poles between that poles okay between that point and the poles that is what is meant by the denominator actually that is the distance you would notice that as I go along above this.

And when I come to a minimum distance point a minimum distance point okay assuming that these distances are large to the other pole to the other pole the distances are large okay since it is in the denominator okay as I move along or meager my response actually keeps increasing because the distance keeps decreasing okay so when I come to this point because I say go remember as they go along omega actually I am going along the Y axis.

So when the distance is least I get the maximum response okay in the words that is why you have that remember that is the omega that you would get the maximum response so then as I move away response now starts dropping okay so when there is no real term which means that your poles lie in the imaginary axis what would happen that it would go to infinity and so you would have a graph which goes like that and comes down.

So this is another interpretation okay especially used in filter design where it is very simple to understand okay the filters which is low pass filter or high pass filter or bandpass filter for example get those things can be bandpass filter can be understood very easily by these distances okay which are in the denominator right and which 1 by the denominator 1 by the distances small then the response is large.

Okay after all you are looking at the response in the left hand side, okay, so that is a neat interpretation for the frequency response function okay in the last class any questions okay Please revise once the state space formulation if you have any doubts in the last class we were looking at remember the transfer functions right we said that the 2 important transfer functions that we had looked is for lateral acceleration.

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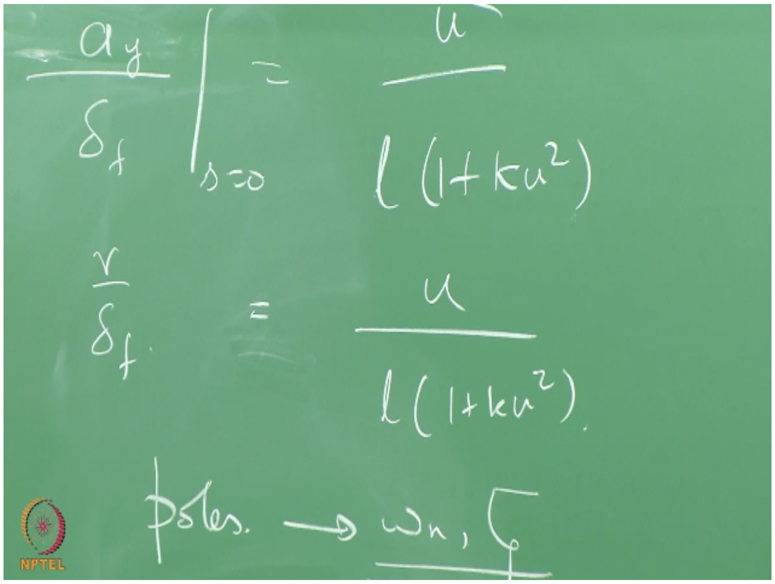

Lateral acceleration we in fact what we did was to find out the lateral acceleration versus delta front okay s well as what is the other one we did r/delta, so I can write this as ay just for simplicity ay/delta and are better remember that we had only the front delta f right now we also if you remember right we also wrote down the steady state gain okay right we wrote down the steady state gain.

The steady state gain at S=0 u square/L(1+ku squared) and that is=u/Lx1+k u squared) okay right this is what we wrote down right now what I am going to is to apply whatever we have done here okay to whatever we have done so far in other words I am interested to find out the poles of the system and then determine the omega n and the damping characteristics of the system this is what I am going to do now.

Okay why are we doing this remember in the last class we said that we need to understand this whole concept of the response from an objective perspective okay from an objective perspective and hence we need to understand how actually the system behaves okay and can be written in terms of omega n and delta, I am going to write down a series of the equations very simple algebra.

Okay if you so I am not going to derive that step by step but that is going to take a lot of time so I am going to write down all these equations because interpretation becomes important so I am going to do that later okay one of the things I promise that we have to look at also what are called as the zeroes right what are zeroes the numerator okay the same what you did with the denominator you do that with the numerator.

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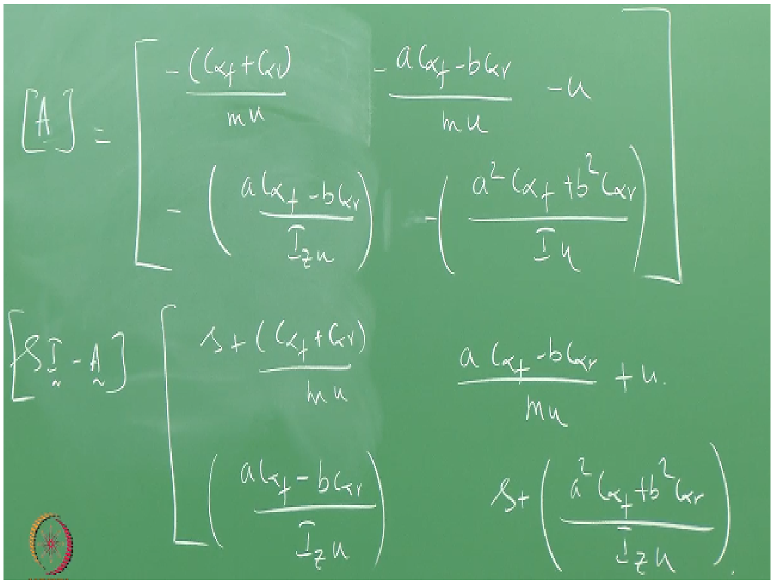

Okay we have not looked at the zeros yet we will have a very interesting thing what happens to the zeroes that we will do it okay after we finish this division we are going to follow for objective of the number of objective metrics as they are called we are going to follow this paper by Mimuro Mimuro SAE 910 sorry 901734 this not the only way of looking at objective evaluation but it looks you know quite simple quite appealing and hence I am taking this paper.

Alright right so the first step let me write down what all you already have so I do not have it in memory so I am going to write down what is A matrix for our problem if there is any doubt go back and refer to this is all there in your notes right so this is this is actually if you look at lot of lecture there a lot of confusion in C alpha and-alpha right I was looking at the thesis the other day there is lot of confusion people have put c alpha f and alpha r to be negative.

Okay the signs are given in such a fashion that C alpha F and C alpha R are given as a negative in which case the whole understeer oversteer gradient definitions change okay so this looks like an accepted practice but I would like you to I would like to warn you that when you read paper or a book be clear okay like how was this definition or given you know what does this definition we have taken –alpha f.

So ultimately that equation becomes you know becomes +C alpha F okay sorry we have taken-alpha F and so C alpha F we have kept it as positive okay so that we get the force centripetal force accordingly but there are people who have written this alpha F to be positive and written C alpha F to be negative okay in which case their K definition would change and so just a bit of bit of warning.

So when you read a book ultimately everything will boil down to the same thing but then the same interpretations but then you would be careful on the science because when you do a problem or you want to understand it you know things would be different you have to be careful on that right so the next step is SI-A, put that there c alpha f+ alpha r/mu ac alpha f – bc alpha R /mu+u ac alpha f- bc alpha r /izu s+a squared c alpha f+ b squared alpha r /izu.

So that makes i-a so the determinant of SI-a=0 is what I have to find out okay that is my next step I am going to do that modern vehicle dynamics follows the language of controls very closely in fact if you look at older books okay the approach would be very different state space approach is usually not followed in older vehicle dynamics books today because of the importance of automotive controls most people have shifted to the language of controls.

And hence we are talking the language of state space okay makes also very this whole subject to be very elegant because you are just applying what you studied in controls to vehicle dynamics.

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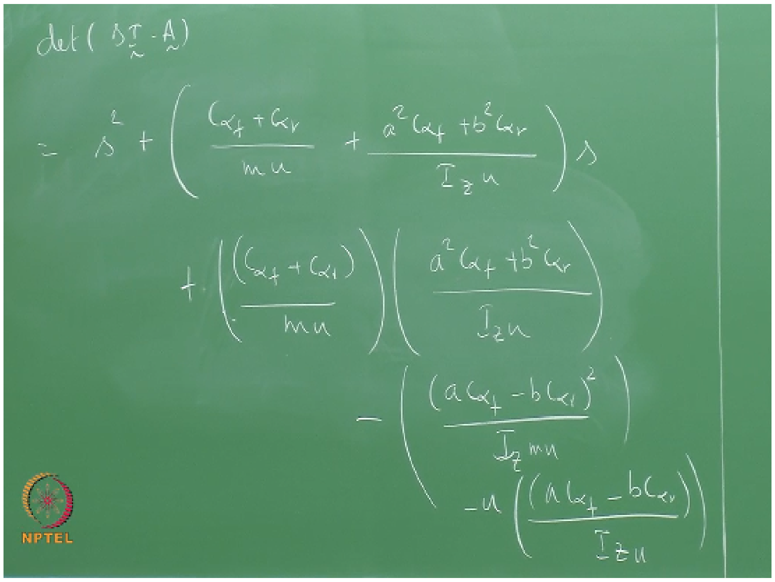

So determinant of SI-A is my next step do that follow me and I hope I am correct s squared+ c alpha f+ c alpha r and forward one step I am just removing it mu+a squared c alpha f+b squared c alpha r/izu S+ c alpha f+ c alpha r/mux a squared c alpha f and b squared c alpha r /izu- this is the last term a c alpha f –bc alpha r/+ izmu+uxac alpha f –bc alpha r /izu, I hope is it correct just check this I think I am correct on that yeah not really difficult to check just check that.

Which one this will become S+c alpha f, c alpha r ,a squared correct then c alpha f alpha r/mu right+a squared alpha+izu yeah yes right+c alpha f c alpha r there is a last term is from there ux+ so- is taken out so that would be – is outside this one now what do you want to say which one this term right yeah clear okay the problem is there are so many terms you have to be careful does nothing that are going to difficult about it yes right.

Most of the times M is usually accompanied by u squared you are right okay that should be the case Now let us concentrate on this last term, let me simplify this last term okay that is what I am going to do simplify it for me simplify this for me.

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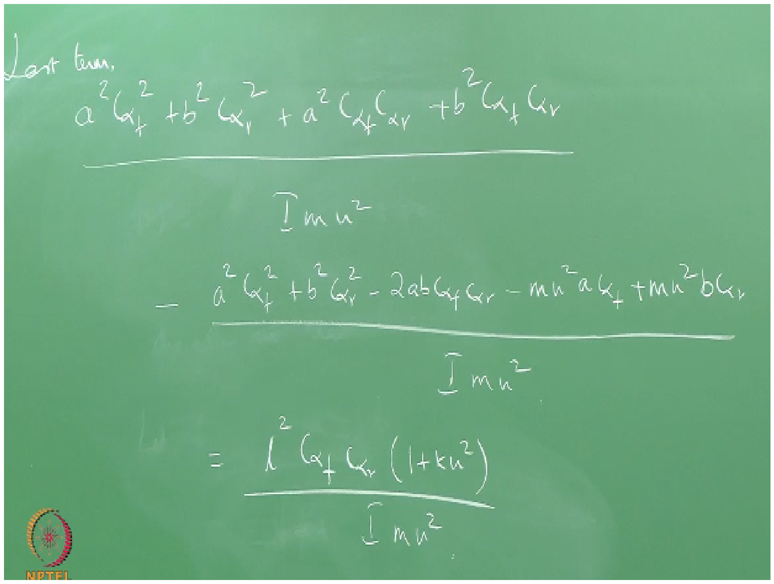

So this would be just follow me very good excellent just follow me and tell me whether the steps are right a squared c alpha f squared+ I know the last step is correct but let us see that all the steps are correct+b squared c alpha f, c alpha r /Imu squared-a squared c alpha f squared+b squared-c alpha r squard-2ab c alpha f, c alpha r –mu squared ac alpha f+mu squared bc alpha r whole thing/imu squared simplify it beauty of it is ultimately is this the third term.

The last term ultimately this happens to be l squared c alpha f c alpha r x1+k u squared/Imu squared right check clear, you have to bear with me till we finish all this.

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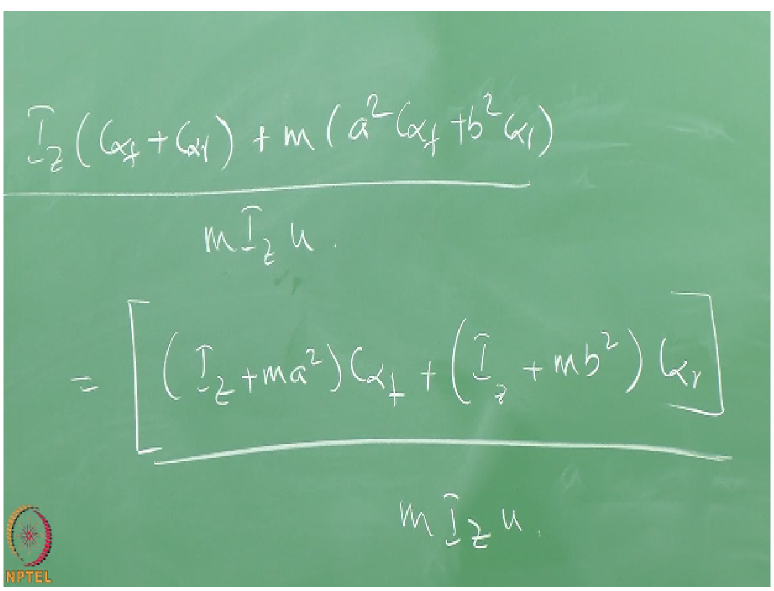

Then we take the second term and I am going to simplify it remember what is my goal my goal is to find out ultimately omega n and zeta I want to find out the poles from poles I am going to find out I am going to develop a much simpler technique in order to find out the natural frequency and the damping ratio okay right.

So the second term is that is the term so let me take mIzu squared as common m Right, sorry mIzu is common okay so multiply the first term by IZ, so that I get IZx c alpha f+c alpha r that is the first term+ the second term is mx a squared c alpha f+b squared c alpha r okay that is the second term let me rearrange them so that I will write that as Tz+ma squared c alpha f+Iz+m b squared xc alpha r /mIzu right nothing difficult just rearranging them.

So now I am going to put these 2 terms simplify terms into my determinant okay and write this down as s squared+this whole term yz+m a squared c alpha f+Izm b squared c alpha /m Izu s+ last term I squared c alpha f c alpha r 1+k u squared /Imu squared=0 okay now from this I am going to find out the poles okay and then write down omega n and the damping ratio zeta. Right I am going to follow a slightly simpler technique.

Okay which all of you know it so I am going to just write this in this terms you know which to all of you know I am just going to write the final result because I want to quickly get into the interpretation part.

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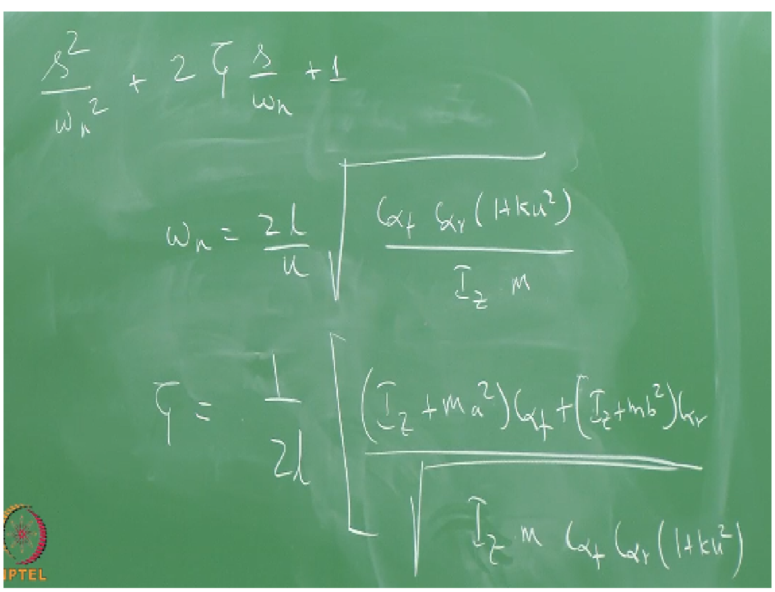

So that I can write this down as s squared/omega n squared+2 zeta s/omega n+1 okay in this form right that last term in this form from which I am going to rewrite that, anyway that is=0 so when I rewrite in this form I will get omega n to be 2/u root of c alpha f c alpha r x1+k u squared/Izxm and zeta to be 1/2lxIz+m a squared c alpha Iz+m p squared c alpha r /root of Izxm c alpha f c alpha r 1+k v squared okay now done we will get into why we did this?

Omega n will be 1/s squared note that 2 zeta omega n shown by omega n squared l squared so l by okay think right now why did we do this or what is the I hope I am just check quite simple just check whether it is okay now here we pick up a physical interpretation what does the physical interpretation of these things there are for quantities of interest one of the quantities I will write the expression later there are 4 quantities of interest.

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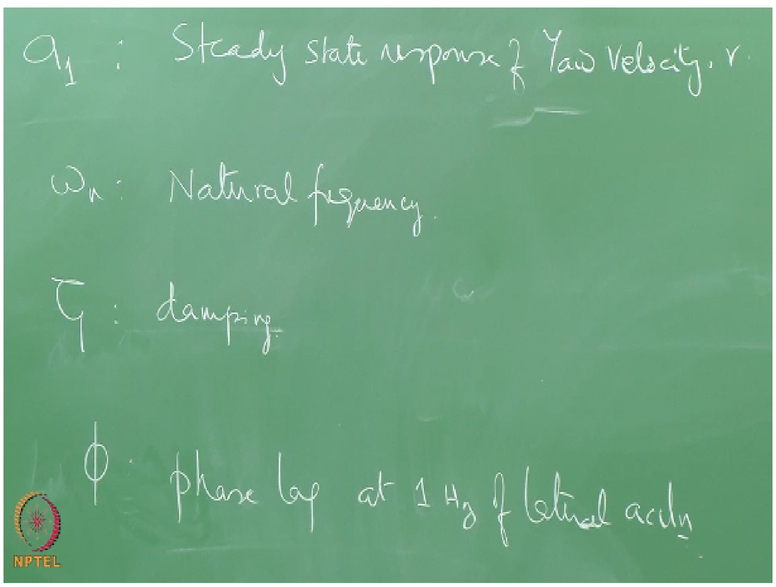

One is what is let me call it as a1 all of this we have already derived okay one is what is called as the steady state response of your velocity r remember what this quantity was v/lx1+ku squared which we just now the second quantity of interest is omega n okay which is actually the natural frequency of the system the third quantity of interest is damping though Mimuro calls this as natural frequency of your natural frequency of damping.

I would resist from calling this because they are just the poles okay of the system I would not like to give them a name let us say that omega n is the natural frequency of system and then let me introduce a 4th quantity which I would call as phi which is the phase leg at 1 hertz of lateral accelerations okay now what is the physical meaning of the first one I know the expressions.

Okay let us not worry about the expressions forgive your system will be able to find it so what is the physical meaning of steady state response of your velocity okay what do you think is the steady state response to your velocity what does it mean that is a ration between your r to delta okay so higher it is what does it mean easily r would be developed or in other words you can interpret that to be the heading easiness.

I want to head how easy I am going to head okay so that was how easily I am going to have the yaw for the car in order to get into the maneuver which I would like to have so this would give you what is called as heading easiness right okay now all of you know that a natural frequency you can express in the terms of radiant per second or you can express it in terms of hertz okay all of you know something about the natural frequency of the system.

What is that you know about the natural frequency of the system you know or understand the responsiveness of the system clear the larger the natural frequency of the system remember that in your earlier classes for a simple spring dashpot system a second order system okay you were able to get what is the rise time, what is the peak time TP, what is the settling time TS okay over shoot all those things remember that you did those things for a second order system.

And that those were functions of 1/omega n okay so omega n in other words gives me the responsiveness of the system okay heading for example now it is the heading because that is what we use the word heading in order to understand the maneuverability so we get what is called this heading responsiveness of the system okay yeah what is what is the all the factors like settling time okay which is 4/omega nx zeta.

And the peak time all those factors or all functions of omega n okay larger the omega n smaller it is smaller is to say for example the peak time so smaller the peak time means I will reach I will reach what I want faster okay so it is ease with which I will go or a response of the system is now better when omega n is increased right damping obviously all of us know it is responsiveness as well as directional damping.

So we would not oscillate larger it is better that it would not get me an oscillating system right and the last thing is the phase lag what is the importance of 1 hertz because most of the systems have you know the frequencies of exaggeration natural frequencies neat 1 hertz, 1 hertz is just given but what is this face like what does that tell you what is what you understand by the word phase lag.

So in other words intuitively I know that if the phase lag is small what I give is immediately heard by the system or responded by the system or in other words it is easier for me to control the system okay so I would call this as controllability right so these are the 4 yes, no this is for lateral acceleration phase lag of lateral so how do I find this out remember we had the transfer function yesterday.

Okay in that transfer function we have to substitute s=j omega and then you would get that 1 hertz and that is what you would get and what Mimuro did this is not really difficult physics all of you know this what Mimuro did was to plot this as a rhombus he plotted this as a rhombus So how did he do that in a very.

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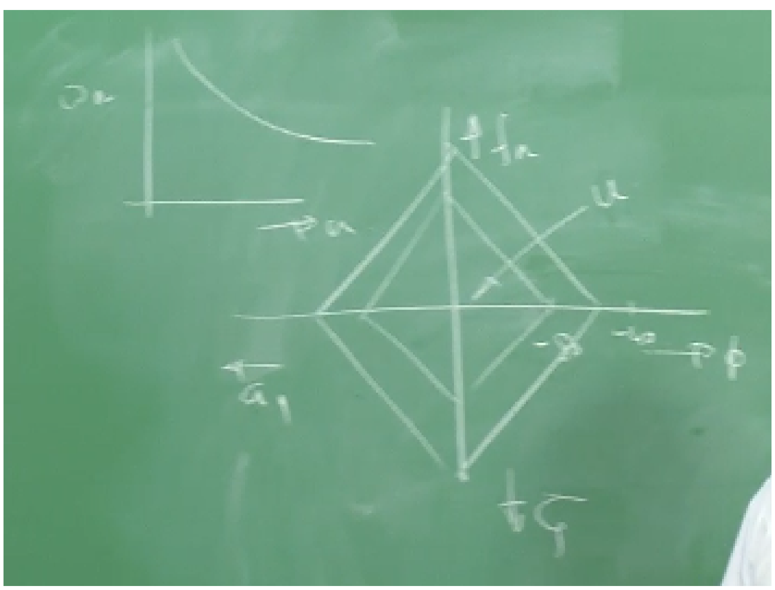

I would say interesting fashion okay he had the natural frequency fn in one of the since there is no 2 axis look at this carefully the positive side you had fn, he had phi increasing like this usually phi can be-80-40 and so on okay that is how he plotted phi, he plotted the damping coefficient Zeta in this axis right ad of course the other one a1 here, why did he plot it like this this very simple that if I now plot that for a car like this.

Okay it simply means that understanding the physics of all this 4 it simply means that larger this rhombus is larger means f1 is larger good for me phi is larger that means that look at how it is going-40-80-40 and so on okay so usually the minus larger it is which means that your yes response is very fast so the lag is more okay larger it is good for me damping and larger it is my first term which is the heading easiness is better.

So he simply said that plot this like this when you plot it like this then the larger this rhombus better is the vehicle handling simple okay it is very easy to now understand or else it you have to go into too much of details but here is the first or the easiest way to understand vehicle handling look at the area of this rhombus and I am going to find out how good is my vehicle okay but look at all the equations.

Which maybe some of them I have written here some of them you do not have here a1 and 5 ok phase lag, I have to write down that equation that is a tricky equation let me write down al also so let me write down a1 here a1 which you already have you divided by lx1+k u squared and phi is r tan al c alpha r /l c alpha r u/2 phi- phi Izu that is a lengthy expression but I want you to check this slightly doubtful whether there is a 2 here or not fn/ fn squared- just check this.

I think it should be right check if it is correct now there is one small way a small one issue what is the issue with all these expressions what is the issue the interesting issue here is that it not only depends upon the car of course it depends upon the tyres, so the tyres are going to play a great role in handling no issues okay in fact we have if you remember we have linearized the tyre performance or the tyre characteristics and just put that initial slope okay.

Not bad not the best way because we need to get this other tyre models okay but we usually assume that we are we are not going to be away from the linear case we will come to how he determined this experimentally in a minute okay but before that let us look at these things carefully one of the peculiarities yes this in this has an effect more importantly velocities seem to have an effect okay the velocities are important.

So all these things are functions of velocity you had written down area and velocities are important in other words a vehicle handling also depends upon the velocity so if you want to test a vehicle so it is not enough if you test it at 80 kilometers per hour it is not enough if you draw this rhombus at 80 kilometers per hour so we also need to plot that for different velocities and what do you expect look at that carefully.

And tell me what would what would happen to a1 what would happen to other factors for a particular k most instances actually with the increase in velocity v or u rather in that case as we increase the velocity rhombus now goes down in other words the handling characteristics depend upon the velocity vehicle which performs very well at 80 kilometer per hour okay we will see a deterioration with velocities.

Yes the typical value, see I have this formula when we do a problem in one of the assignments I have a formula here okay I know the typical values of k, so substitute it with vary it with velocity in fact in other words if I want to now plot omega n works as I need omega n versus u if I now plot it the graph would look something like this clear so if I now plot each of these quantities with velocities for example omega n if I plot it would be like this.

So now I get those I have already the formulas there I get those and then plot it here right very simple nothing very difficult. So now the other one is the understeer and the over steered vehicle, how is the understeered and the oversteered vehicle going to be affected, now what would happen to an understood old as the vehicle becomes more and more understeer how will that go look at that formulas and tell me.

As the big vehicle becomes understeer what would increase okay omega n would increase a1 will decrease so it will move like this and obviously the oversteer vehicles move in the opposite direction right now how did Mimuro determine for cars in other words what is the experiment he did he recommended this group recommended an experiment called the pulse steer, a pulse steer test.

In a pulse steer test in a matter 0.4 seconds a steering is given an input okay of about 60 degrees and back so that is the pulse to give a pulse actually to the system okay so you get a pulse to the system which is late I is like giving an impulse okay so the pulse exceeds number frequencies that is the reason why we went to the pulse of the system here or we can look at why pulse and so on from FFT point of view.

But I am not going to do that this class but go back and read and maybe when you have time in the later part of the course when we talk about ride and when we talk about FFT we will look at the beauty of the pulse okay so he gave a pulse to a system and said that at least about in 0.4 seconds okay this was done this was done so that the lateral acceleration or in the order of about0.25 to 0.3.

So that the assumption that we made regarding the tyres that they are linear is valid clear in fact we have done a lot of work on this and we we found that this with an expert driver this with a rhombus okay which we call as Mimuro rhombus and the subjective evaluation for driver matched very closely so in other words we found this to be excellent objective evaluation tool tyres.

So now you know you see the importance of tyres and we will see the importance of other things has to how they are going to be affected right any questions you want depending upon the value of K this would be and so I know the K values that you have come again what is suggest is that okay I understand that how much to vary we will do a problem okay that is the easiest way to understand.

Okay how for example you are asking a whole big is the K would it have an effect okay if a large or small right we are talking about a typical car what is the K value which is an understeer or coefficient car so what we will do is maybe we will do that in the next class okay I will work out and I will put some values to this so that you will understand how exactly that varies that is your doubt how much it varies.

We will put typical values in the next class and we will work out a problem and give you an idea as to how it varies right in fact the experimental set up or the experiment in the order we will do that may be in the next class this experiment we will do it in your vehicle dynamics lab class okay you will do this in the lab class that is the reason why we lot of experiments that need to be explained this course is too short we cannot do everything here.

So this course is more on theory and the experiment of vehicle dynamics you will do in vehicle dynamics lab right anyway we will pick up we will give you dope on this in the next class okay I am going to give you an exercise okay do this you remember that we defined beta okay v/u remember that find out an expression for the transfer function for beta okay that will be an exercise and we will follow it up in the next class stop here and we will follow it.