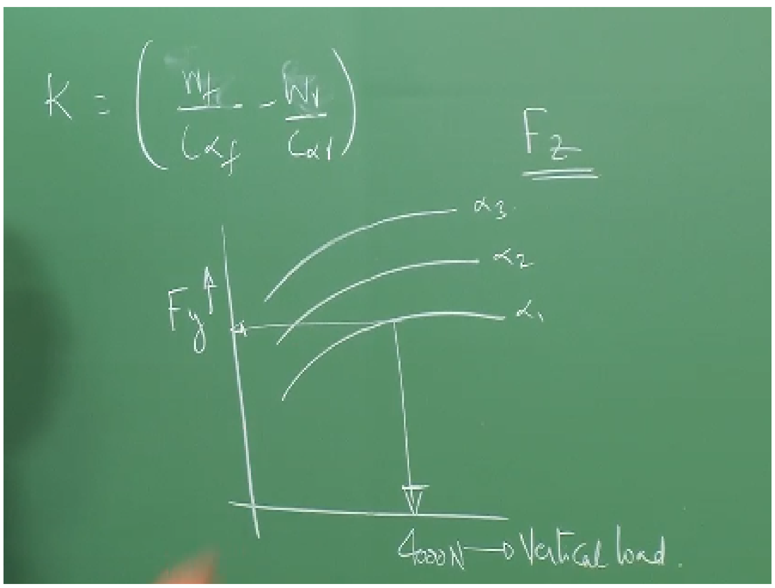
Vehicle Dynamics
Prof. R. Krishnakumar
Department of Engineering Design
Indian Institute of Technology - Madras

Lecture - 23
Mimuro Plot for Lateral Transient Response (Part 2)

Now in the last few classes we have been looking at the handling characteristics of a vehicle. Now in that process we develop of course the understeer gradient and we also brought in a very simple technique called the Mimuro Rhombus in order to understand the vehicle behavior. These are simple techniques but lot more things happen in the actual vehicle in other words.

(Refer Slide Time: 00:49)



The understeer gradients remember one of the definition for understate gradient is given Wf/C alpha f-Wr/C alpha r remember that I wrote it as K2 or something like that, okay, this was multiplied by ay, I remember right this is what we had defined we define 2 ways in which you can do it or the word to be defined in 2 terms the understeer gradient the other the other understeer gradient is mxvc alpha r –ac alpha f/l square x c alpha c alpha f you know that is other term.

So these are very simple definitions which came out by looking at the tyres interaction with the vehicle and the road, but then the tyre interaction should be brought out much more in a much more detailed fashion and that is what we are going to see in this class and we are going to take whatever we are going to learn today to the next topic of subjective and objective evaluation of a vehicle.

Okay, what do we mean by expanding this topic it is very clear that this is affected by the weights that act on the front and the rear. Okay there are other things that happened in the car which is going to affect this Wf and Wr as well as the assumptions that we have made regarding the linear behavior of the tyre remember, what are the assumptions that we had made in our bicycle model?

In order to obtain the definition for this understeer gradient one is that we neglected the roll and number 2 is that we assumed or linear behavior of the tyre right and we neglected all other aspects of suspension and steering handling is all about how the vehicle behaves when i give an input to the steering by now you would have realized that now the input to the steering is of course in under your control and that is what is going to also give you a feedback about the vehicle.

So we have to understand in a much broader perspective these things, okay let us now look at first from a physics point of view what happens when you do not have the tyre which is linear in other words if I remove this assumption of linearity of a tyre let us see what happens okay before we go into the equations, let us see them, I am going to follow Gillespie in this lecture but then we will take over and then look at other things from next class.

And please pay attention to this because this forms the basis for evaluating a car in the words it will be exciting to know how you are going to evaluate a car if you are going to buy a car and you want to drive a car what all you should do in order to understand the behavior of the car is going to be the next lecture and for that this will form the basis right, okay now let us look at the actual diagram for the c alpha f

In other words, how actually the force developed in terms of the slip angle note that we had already said that F is the normal load is going to play a roll in the development of Fy in other words the stiffness of what we call a stiffness depends upon the normal load that is acting okay now why are we harping on this basically because when you say for example when you want to take a turn all of us know that the vehicle is going to roll.

Okay especially initially when it is going straight both the wheels left and the right wheels within small tolerance would say that both of them have the same reactions okay so when there is a roll of the vehicle then these reactions what we call as Wf and Wr are going to change and the left wheel is going to have our different reaction and the right wheel is going to have a different reaction depending upon one is going to higher.

And the lower depending upon they turn to the right of left and so on, right so what is important is to understand what happens due to load transfer for this is one type of load transfer the other type of load transfer happens when you take a turn and start accelerating or decelerating. Okay again all of you know that there is a load transfer from the front in that area so the load transferred to the front and rear.

Whether it is due to braking or traction is again going to have an effect on these things and is going to have an effect on understeer and oversteer gradients so load transfer is going to have an effect and so does the first thing so we would understand how c alpha f varies. Let us say that let me plot it like this let us say that this is the vertical load I am going to plot the graph of remember that we plotted alpha versus Fy.

Fy and I am going to plot ir for this graph slightly differently I am going to say that Fy and I am going to plot it for different alphas let me call that as alpha 1 alpha 2 and alpha 3, okay so for different alphas right now of course alpha 2 is greater than alpha 1 and alpha 3 is greater than alpha 2 and so on right now let us say that I have say 4000 Newton that is acting in a tyre, okay and that produces an Fy.

That is the Fy which produces and that is good enough for me to say is what good enough for me to equilibrate the centripetal accelerations okay right now when there is a roll, why there is a roll, we will see it in a minute when there is a roll in another words when the left and the right are inner and the outer however you want to call it okay get when the loads get redistributed look at what happens okay for the same alpha 1 for the same alpha one which is the slip angle.

Okay when the load comes down say for example when it comes down to 2000 okay the load or the stress sorry the force that is produced comes down okay and for the other tyre which is going to increase to 6000 this is going to increase okay now this gives rise actually the total Fy which is going to be less than less than what is produced before in other words because of the non-linearity of the curve note this carefully because of the non-linearity of the curve.

When there is a change in load the alpha that is produced the alpha that is produced is not good enough to equilibrate the centripetal acceleration it is going to fall short, okay so that has to be a larger alpha that is going to be larger alpha okay please note this is left and right, this is tyre left and right in the front and rear there is left and right here there is a left and right and remember that in our model.

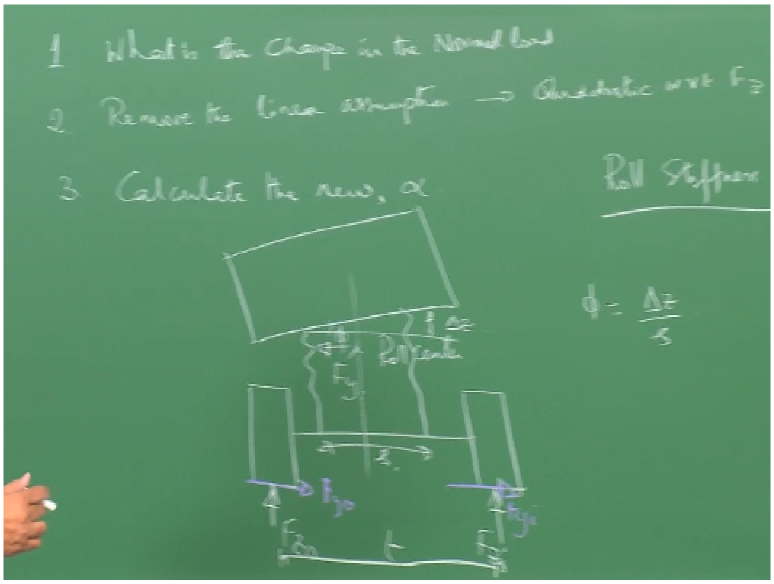
We have c alpha f and c alpha r to depict the stiffness of the front and the rear together in total. So what will happen now because of this it is not good enough the Fy is not good enough to equilibrate centripetal accelerations. So this alpha has to shift up the alpha that is produced has to shift up the right so that if this where the equilibrium so let us say that it is 3800 or something like that is this has to be the equilibrium force.

Okay now I am falling short maybe if we get 3400 okay that difference of 400 has to be generated okay so I have to work at a higher alpha and that is such that okay I will just exaggerate that here so this is the one which is initially was like this okay and I had fallen short so I have to now you increase my alpha such that I come back to that 3800 okay. So in other words in other words the roll redistribution of the forces result in a change in the operating alpha.

Right now look at that closely when there is a change in alpha, alpha f and alpha r that is going to produce a change in the understeer gradient okay after all that is alpha f-alpha remember our delta is equal to alpha f-minus alpha r+L/R and so on so alpha f and alpha r are going to have an effect on the understeer gradient so the very first thing is that due to this roll there is a change in the operation of alphas.

And hence have an additional term to my understeer gradient right so what are all the things that I have to do know first of all I have to find out what is this change what is this change right between the left and the right or outer and inner, let us call it as outer left and right is not the correct one outer and inner right what is the change once I work out this change.

(Refer Slide Time: 12:42)



Then I have to introduce the first thing is the derivation which I will have to put now is what is what is the change in the normal that the first thing I have to find out what is a change in the normal load okay. So the next thing is that remove the linear the linear assumption for the tyre and introduce and introduce a non-linear relationship let us call this as quadratic relationship, let us say quadratic with respect to fz.

Please note when I say linear the linear relationship in the sense that we had only 1 c alpha that is what I mean by a linear relationship and since that the only 1 c alpha yeah it was independent of fz, okay it is independent of fz, but actually it is not independent of fz, so we are in other words Fy is a function of say a x fz so that is the only thing that we said you know now we are going to say that it is a quadratic with respect of fz we have introduce that in a minute.

So note what we mean by that linearity, Linearity is with respect to Fz, okay so once I do this then calculate the new alpha that is all we are going to do, right. Let us look at a very simple diagram we will make this we make a few comments to make it a bit complex. Let us say that I have a solid axle and just for illustration we will see how we it can be looked at for independent suspension.

Okay let us say that is the sprung mass of the vehicle okay and that is the what we call as the roll center we will see what is it in a minute if you know that already roll center let us say that now Fy and Fz forces are acting on tyre. So where will be Fy forces that will be acting Fy will be towards the right or the left, right so this we will call as Fy outer and Fy inner and of course that would be a normal load that would be acting which is which we would call as Fz outer and Fz inner.

So this is yet this is turning like this so actually like this to the right and then roll to the left, that is why I call it as outer and this is inner, I do not want to call it as left and right, okay because it can be the other one, right now the concept of roll center I am sure you would have heard about this roll centers form a roll axis i know that you had heard about this term roll axis and about which the actually the sprung mass rolls.

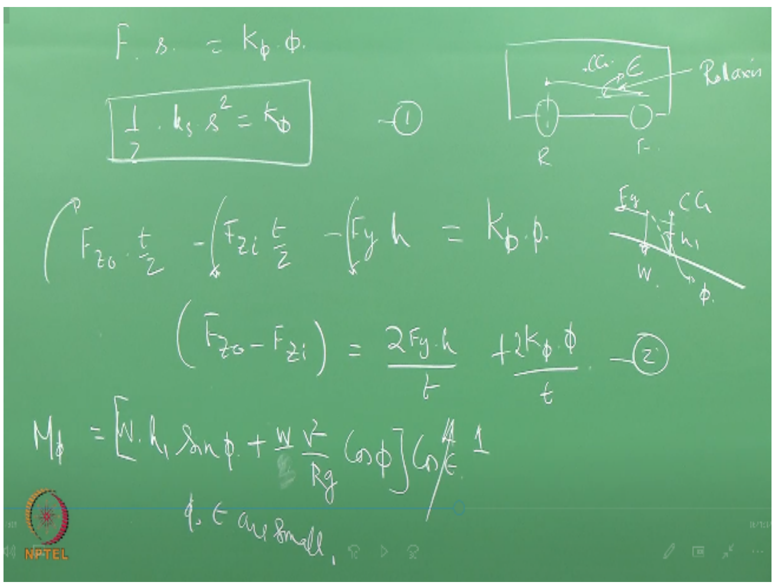
Another way of looking at roll center is the center about which you can split in very simple mundane terms you can split the sprung mass from the un sprung mass So for example if you want to draw a free body diagram of this for the specially for these forces okay then I can say that that is where I would take the reaction for the Fy, in other words the reaction for Fy okay for this would be about that point.

So that would be the Fy okay of course Fy=Fy0 or o outer+inner,so my job is to find out how does this vary okay in order to do that I have to look at what is called as roll stiffness I have to look at the roll stiffness okay as the name indicates it has everything to do with the roll so it relates the moment that acts to the angle of roll fine right okay now we will talk more about this roll center right now we will say that there is a left row center, front row center, rear roll center.

And the joint and what results in the role axis okay and about which takes place. So let us let us say that this is the s and let us say that this is t, whatever derivation I am going to put here for the rigid axle can be extended to any independent axle independent suspension for example that is what you do for a car independent suspension by assuming that all these stiffness are dumped at the wheel position are dumped at the wheel position.

Right so it is very close it is somewhere here you can assume that it is a track which is s clear okay now let us say that that angle okay to which it rolls is phi and that that difference in the height is delta z and hence phi=delta z/s, clear okay now help me to determine what should be the what is the role stiffness note that delta z=F/ks okay right how do you go about this how do you think you can go about this.

(Refer Slide Time: 21:52)



Simple Fxs which is creating the moment okay it is equal to the roll stiffness K phi x phi the moment that is created by this force is actually equal to the K phi x phi right from which you can write down 1/2xksx s square= K phi okay that is substituting for phi from this expression okay and then calculating the delta z by the difference okay I can write down that K phi will be ½ x ks x s square.

So this is we call as the roll stiffness if I now so this is the relationship between the spring stiffness and the roll stiffness. what I essentially did was to find out what all are the forces 2 forces that causes a moment and that moment is what is given in the left hand side in the moment I mean the distance the right hand side is the one which equilibrates it through K phi where K phi is the roll stiffness okay right it is force per degree or radiant multiplied by the degree.

Okay now as I told you the first step is to find out what is the change in the normal load now okay in order to do that let me use this diagram let me take moment okay let me take moment of all the forces that are about the center now the spring stick the normal load and as I said Fy okay when they cut it this is Fy is where the reaction for this is taken, right now how do I write this Fz 0xt/2.

Okay about this and that is the clockwise direction right then –Fz inner x t/2 this is in the anti-clockwise direction right then –Fy right x h right h is the height and that is equal to that is the total moment that is created that is in the anti-clockwise direction that is equal to K phi x phi the moment that is created what causes the roll that is the total moment that is created is what causes the roll.

Okay this is just for one suspension we had said what is the connection between the 2 and so we now have K phi x phi, re arranging these terms F z 0 outer and –Fz inner bring that Fy in the other I mean to the other side so it becomes 2Fy x h/t is a track+K phi x phi x 2/t. Let me call that I have to put the equation numbers properly and let us say that I will call this as an equation 1 and that as the equation 2.

Yes, At this point because I want to find out the difference between Fz outer and Fz inner so that is there that is what It is remember that remember that Fz outer +Fz inner=WF so 2 equations okay so my job is to find out what is Fz outer and Fz inner right that is why I am taking the moment about this so that makes my job easier because my expressions are neat right if you want take it about any other point as long as you get Fz outer-inner it is fine.

No center of gravity locations are different so the roll center is different from the roll center is different from center of gravity okay do you understand it is instant in a center about which does this rolls okay that is not that is not the CG location in fact in fact the roll center I am going to do that now okay the roll axis is at a distance from the center of gravity and that is going to cause a moment okay a moment about the roll axis and so on and which we will go to derive now.

So in other words, if you look at the side of the vehicle right so the roll axis for example this is the center of gravity location then there is a front roll center or rear let us call that as rear and that is front okay there is a front row center and the line joining these 2 is what is called as the roll axis okay so it is an imaginary axis which runs about which this whole body rolls okay so that is the roll axis.

So a moment which is created because of the distance say for example in the center of the locations away from it okay so the center of gravity at which here is the point so let me draw the same you know let me cut that and show it or let me assume that to be a 3 dimensional view i mean this is a just an imaginary line that I have joined I mean I have drawn there to see that that is the center of gravity location that is the center of gravity location okay and this is the roll axis.

Let us say that is the height h right and the role happens about this okay the roll happens about that so the center of gravity location now rolls or moves okay by an angle phi okay and for the whole body you know that that is where weight W acts and we know that that is how say Fy acts right so the moment about the roll axis then is given by the moment created by that W as well as Fy clear any other question? Right

Write down the moment now let us see they do not the moment let us call this as h1 okay the moment is now given by the 2 things h1 x that is phi, so W x h1 let me call that as W x h1 x sin phi, okay that is a sin phi+Fy is v square/r v squared/rg as our practice we will call that as you know we will write it like that so that becomes ay okay x cos phi and that the roll axis is inclined at an angle xi or epsilon rather small angle.

So that also you know the cause of this is this is actually the factor which as an m phi so that multiplied by a look at the geometry you would notice that there are 2 angles so that would be cos epsilon Now I am not going to make it so complex I am going to assume that phi and epsilon are small okay so that becomes 1 this becomes 1 and that becomes phi yes yeah 2 roll centers front role center and the rear role center front roll center.

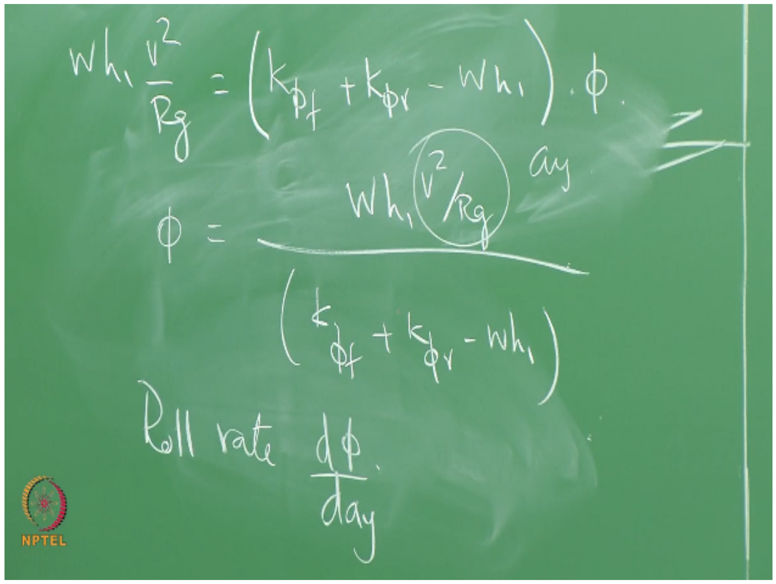
And the rear roll center are characteristics of suspense and there is a method of calculating geometry method of calculating the roll centers that would have studied in the vehicle system. Yeah but now we are saying that it lies right here right at the top because of the symmetry positions okay we assume that it is symmetric okay so roll axis would lie right at the bottom okay right.

These are not this wont be un symmetric so we are looking at there is no reason why this has to be un symmetric right so simplify this so there is equal to what is that take out W h1 so W h1 sin phi becomes phi so v squared/rg+phi that is the moment caused by this phi and this is also= what is that= also= K phi front equal to what is K phi front okay+K phi rear okay that is the roll stiffness multiplied by phi.

Yes, look at that carefully and tell me obviously here I left out h1 okay obviously there will be h1 x cos right so h1 x cos phi right let me simplify this and look at what is phi remember why are we doing this coming back to this graph I want to find out what is the change in alpha. So let me call that or let me give that equation number correctly let me call that as 4 that equation number 2.

Let us call that as equation number 3 they call it as equation number 4 because when I resubstitute it back that would be fine okay now let me rearrange those terms here in this so that I would get phi so my whole idea is to get phi right so Wh1 let me rearrange that.

(Refer Slide Time: 35:59)

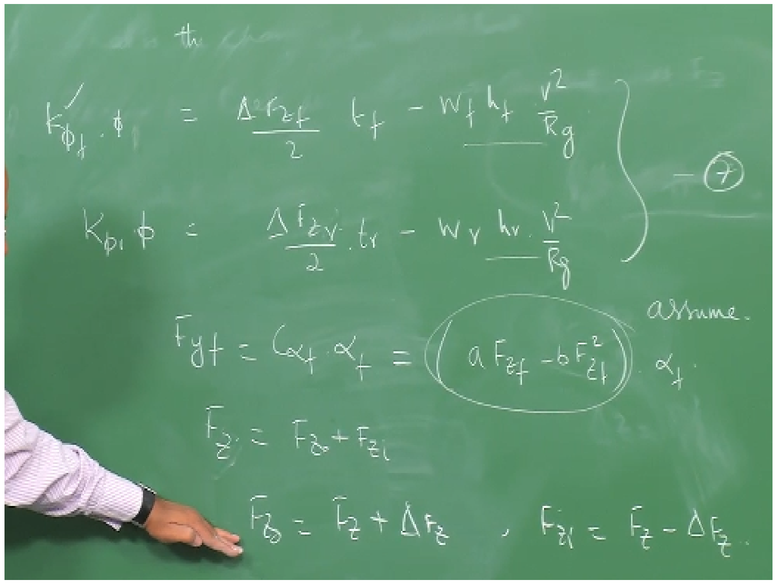


I will say Wh1xvsquare/rg=K phi front+K phi rear-Wh1xphi= so that phi=Wh1v square/rg/K phi f+K phi r –Wh1 all right okay now the automobile engineers define what is called as roll rate the define what is called as a roll rate which is d phi/day note that carefully it is not phi. Though I call this as rolls I mean roll rate it is d phi/day, what is that simple that is ay and so differentiating the first term.

So this would be Wh1/K phi f+K phi r –Wh1 that makes life easier when you want to define and it is about 5 6 degrees usually per g in other words that tells you if you know the roll rate and if you are taking a turn and you say that I am subjecting my vehicle to 0.3 g then just multiply by roll rate you would know how much is the roll right clear now let us find out let us go back to equation number 2.

I have written that correctly, now let us go back to equation number 2. Let me write down that equation to the front and the rear that is generally we have written up to this I have written down generally then the whole vehicle so let me go back to equation number 2 and write it for the front and then write it for the rear.

(Refer Slide Time: 38:52)



Okay I am going to lay down the final that expression and that is quite simple it is not K phi f x phi of or K phi f x phi=delta Fz is at front/2xtf –Wfhf x v squared/rg and K phi r x phi=delta Fz/2 xtf-Wr hr x vsquared/rg okay what is that I have done I have just okay defined now delta what is now delta Fz so this term that is the last term okay that is the last term, what is this term that is the Fy x h/t that is this one okay so Fy have substituted in terms of Wfxv squared rg.

So delta Fz/2 is what I call as Fz0-o-Fzi right okay this is implicit assumption here in this step the rest of it is only this is an implicit assumption, the assumption is that the whole body the whole body is stiff and that there is only one roll, so we said that this is applicable for front and rear actually the whole body of 1 phi okay so that is what is this expression now once I know once I know what is k phi f which is actually the property of the suspension.

And once I know these factors I can easily find out what this delta zf sorry delta zr okay and delta zf and delta zr it is straight forward from these 2 expressions and let me call them as 4 and this expression as say 5 and that as 6 and that as 7, right okay that is what I wanted what does delta f is that front the difference between the outer and the inner and delta f is at rear which is the difference between the outer and the inner and the rear.

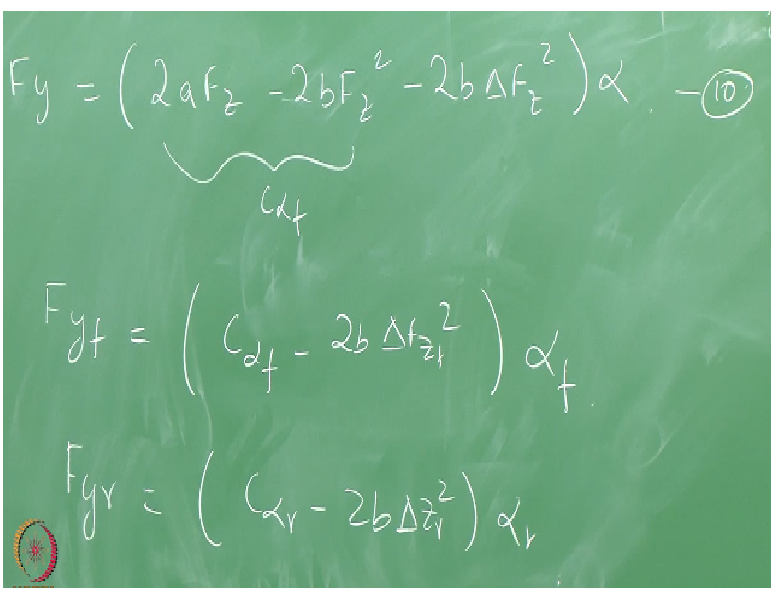
Okay once I know that I am now going to use that in order to determine how my alpha f and alpha r varies, any questions right clear okay now remember that we had this expression in the front as c alpha s x alpha f and at the rear as c alpha r x alpha r okay let us now assume that c alpha f has a quadratic expression okay which i would write as aFzf –bFzf squared okay x so that this would become x alpha f.

Now note that if it is for a single tyre c alpha f is written for single tyre for a bicycle model I to multiply this by 2 so I will get 2a Fz squared-2b sorry Fz-2b xFzf squared right Fy say Fy front or rear is the sum of Fz outer+Fz inner What is it I assume this is what I assume okay is it correct okay it is an approximation actually we do not use in practice this expression okay actually we can find old all these factors say for example by assuming the tyre to be a Pacejka model.

Right and then we can get this correct redistribution okay by this now note that Fz outer=normal Fz which is Wf so I would call that as normal Fz+delta Fz, delta Fz ia that/2 okay and Fz outer sorry inner=Fz-delta Fz, okay now substituting that sorry Fz outer, no this is Fz, now substituting this expression which I would call as this is call as 8 and that we will call as 9 this whole expression substituting 9x8 okay 9x8 and then expanding this expression expand that expression.

I will write the final form I can write down Fyf okay you know much broader expression taking into account the non-linearity with respect to Fz do that you know it is a very straightforward exercise.

(Refer Slide Time: 46:02)



So now substitute it and then simplify it so you will get 2aFz-2bFz square-2b delta Fz squared x alpha remember what is delta Fz okay so it is delta Fz 0-delta Fz I/2 okay so that is what we had there ½ of it so Fy is now given by this expression okay so in other words if there is no load transfer this would have been the c alpha f I told you that because of the load transfer that 2xc alpha f sorry the 2x this that is affected and you have an additional term.

So let me call that as 10 right, so I would write it as Fy, this is Fy genera; Fy okay outer and inner can be in the front as well as in the rear right so Fyf can be written as since I said that is the c alpha f, c alpha f-2b delta Fz front squared x alpha f, Fyr=c alpha r-2b delta Fz rear squared x alpha r and what is this and this is equal to Wfx vsquared /rg and that is the Fyf and that is=Wr x v squared/rg.

Remember what is my delta go back and look at that what is delta? L/R+alpha f-alpha r from which my whole definitions everything came out and remember alpha f was written by Wf/c alpha f and so on right. So that this what will give me my also my definition for K, I am going to substitute let me call that 11 and that is 12 substitute 11 for alpha from 11 to 12 okay and then we will get an expression for a new expression for delta.

What is that essentially we did this recapitulate we will expand that in the next class essentially what did we do we realized that the load transfer is going to change alphas okay and the load transfer is due to roll and that the roll is controlled by what is called us roll stiffness and the roll stiffness comes from the suspension stiffness right okay these are the things so now using that and using simple equilibrium okay due to forces that are acting in the body.

We find out what is phi right and then from this we assuming assuming that that tyre has a nonlinear relationship with respect to Fz okay we find out okay the changes in all alpha f because of introduction of both this nonlinearity as well as the roll okay that is what we did here so that gives the race to an expression to alpha f which is Fy divided by this, Fy divided by that expression okay and alpha r=Fyr/that expression.

Okay so I am going to substitute that and then write down delta=L/R+Fyf/c alpha f-2bxdelta f z squared+Fyr/c alpha r-2b delta Fz r squared oh sorry minus exactly so in this model in other words we are not consider considering it as 4 tyres okay this is a simplified model in practice in actuality okay this would not be the case what you are saying is absolutely right so alpha f front outer would be different from alpha f front inner and so on.

Now we are getting a gross effect here the advantage is that we can look at very easily until you tell what would be the effect of roll the disadvantages that it has it is a limitation of having only one tyre compressed a bulk quantity and one tyre of the rear right so this is a very simplified model the bicycle model but helps us to understand okay what happens when there is a load transfer.

So in actual if you want to really design a car then you take up to a commercial say software that are available like Carsem or Adams or one of them okay in order to understand actually how does this happen now he whole idea here is to illustrate okay in simple terms what happens again we are going to use this in order to look at how to evaluate a car right load transfer has an effect on understeer or oversteer one statement.

How in a simplified form this equation is going to tell us that is it clear okay or we will see the rest of it in the next class.