Vehicle Dynamics

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Lecture - 24
Parameters affecting Vehicle handling characteristics

We were looking at the effects of various parameters on under steer. Remember that vehicle handling which is what we call us lateral dynamics which we are looking at essentially are 2 things for a driver. One is how does a steering behave, how much steering he has to give, okay what is the torque on the steering and so on and where does that vehicle go or how does it head, okay. These are the only 2 things that the driver is concerned with, right.

So how much angle he gives in the steering angle and so on. Of course this can be measured and that is what we called as objective characterization of the vehicle handling. So, let us complete a few of the other things that has an effect on what is called under steer and quickly run through what is the effect of steering on handling.

So that by next class or after that you will be ready with all the answers that I may ask you as to how you will test a car if you gone buy a car. What are the tests that should be done, okay, if you are, now you are an expert in vehicle dynamics what would be the tests? In other words, what would be the logical subjective questions that you would ask if you were to buy a car? Okay.

So, keep all these things in mind so that when I ask those questions 2 days from now you will be able to answer the question as to what all you will do with that car, right. We were looking at, we were half way through in that derivation where we were looking at the roll of the vehicle, the roll, okay not role of the vehicle. Now what we are essentially doing now is to look at all those things that is going to affect my delta.

In other words, the steering angle or other word is the under steer characteristics. As I said, because driver is concerned with delta, right. So, I am going to put down all those things which going to have an effect on delta.

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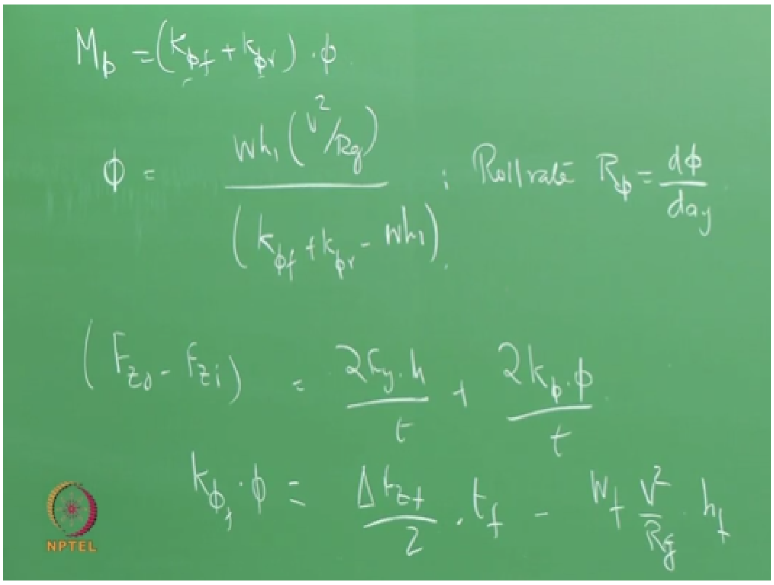

The first effect we were looking at is what is called as a load transfer. Remember that we did not have that as a part of the degree of freedom in our model. So, we are looking at load transfer. This also gives a very good inside as to what happens. I am just going to recapsulate this. This is a small, you know one 2 difference in between what we had done and what say for example Gillespie as book has. So, let us be very clear on that, okay.

That is the front roles stiffness rear role stiffness you know what they are half into Ks*s squared and so on, right. We said that if this were to be an independent suspension then s is taken to be the t, the track. phi is the roll, note that body being stiff. It rolls through only one angle there is no twist of the, there is no torsional stiffness. Please note that you will do this in the next course on automotive structures.

The torsional stiffness over a period of time has increased tremendously and this has helped handling as well. So, torsional stiffness of a body is very important and increased magnitudes of difference 10 force even 50 force over a period of time. So, we found out what is the roll, okay. Essentially what is that we are doing we are finding out what is delta Fz. So, here is what I just wanted to tell you this we wrote down, okay and this we called as delta Fz F/2.

In other words, we call this to be delta Fz and this is delta Fz/2. I think there is a small confusion because in Gillespie this is what is called this is divided by 2 is what is called as delta Fz, okay. It does not matter but as long as you understand you know what we mean these equations are very straight forward there is nothing much, right. Any questions on this? Okay, that is the load transfer between the outer and the inner, right.

So, I am rearranging that equation I am bringing that to the left hand side, okay and we know that Fy is nothing but Wf (()) (04:57) front is Wf*V squared/Rg and that is what I am substituting it here and I am instead of h I am putting hf. So, I am specializing this equation for the front as well as for the rear. So, this is the general expression which we derive and that expression we are specializing for the front and the rear, right

This is what we did in the last class and we continued of course by assuming distribution for or an equation for c alpha f and then we further went down to understand what is delta, right.

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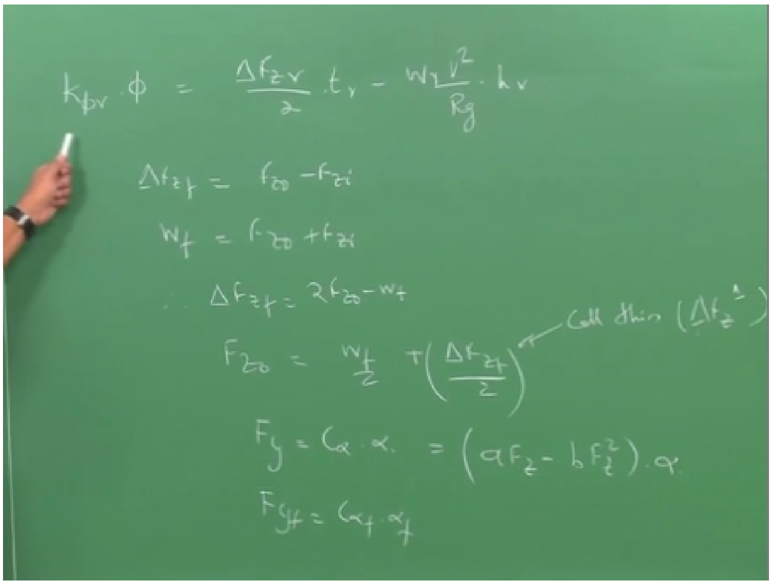

So, how do it we write that expression? Fy = c alpha*alpha c alpha we wrote down, okay. Please note again I am telling you that non-linearity is with respect to z, okay. This anyway we assumed a linear model, okay and so we had put here this but if you want to do this whole thing with say (()) (06:06) it does not matter. Then this will not come up and you can substitute that very more accurately.

So, how did we write this? This we wrote this as afz–b*fz square and that is how we wrote c alpha multiplied by alpha, right. Please again note that since we collapsed the left and the right tyre into one tyre the c alpha which we will be using is 2 times the c alpha of individual tyres, okay. So, it is 2 times or when you are giving the problem be very careful on that, okay.

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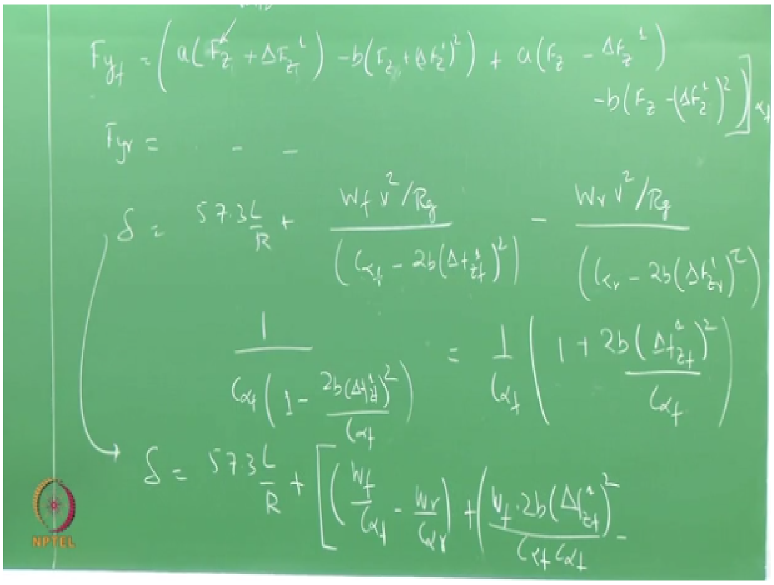

Now what I am going to do is to substitute this, okay. Whatever is done here, so if I have to look at the front tyre then what is that I should do a into I should say what is the front outer and front inner and so let us let me call me this delta fzf/2 as say delta of z with superscript one, okay just to show that this is divided by 2 or else as a 2 which enters and bothers us and just I will write that like that, okay.

So I have to, what I have to do is to substitute fz outer, okay that is for the front+fz outer is what your subsidiary that I will substitute with us. WF is actually fz+delta fz 1–b*fz, fz is actually what is that wf/2. It is the normal force acting on each of the tyres.–b*fz squared fz+delta fz 1 whole squared, okay that is for the outer, so let me call that us outer tyres and then I have to add the inner tyre, okay that is a*fzi.

Okay the inner tyre which would become now what is that look at that so fzi becomes – delta fz which is the same as delta fz outer the same thing. So, I would say delta fz*delta fz outer 1 – b*fz – delta fz outer 1 whole square*alpha, okay. If you do not want to put this as an outer okay you can just remove it and say that it is fz. It does not matter. If you do not want to put that that is also fine because fz outer is nothing but wf/2.

Now we will simplify this and ultimately get fyf front and fyr same thing we will do with fyr as well, right. So, what are we trying to do we have an equation we are substituting for the outer and the inner for fz summing them up that is all, right. So, I am going to write down the final equation check do that yourself. This is nothing this only algebraic equation. Now let me take that one of that 1 – 2b*delta Fzf 1 whole squared divided by c alpha f.

And that can be written as 1/c alpha f*1+2b*delta Fzf 1 whole square/Cf. Let me explain this. This one? So we have Fy this is the general expression, okay. If I had to put front I have to put Fyf = C alpha f*alpha f. And if I have to put rear Fyr = C alpha r into. So what is the C alpha F expression? C alpha F expression is for a tyre which is a*Fz – bFz square, okay. This is for a tyre, right.

So now when I want to look at the front Fyf that is the front tyre or the front of the car front 2 tyres lateral force, okay which is Fyf. Then what should I do? I have to substitute, I will get the sum of 2 tyres contribution, okay. So this first one is the outer tyres contribution and the second one is the inner tyres contribution, okay, multiplied by alpha. Note that this is a bicycle model and so we are compressing both tyres or replacing to 2 tyres with respect to 1 tyre.

The same expression I am going to have for the rear, okay. So I will write down that is why I said I am not going to write down that it is you are running out of time. So you can do the same thing here, okay. Here you will replace it by C alpha r and so on, clear? Yes, what is 57.3, I am replacing radiance by angles, okay. Because usually these are expressed in terms of these alphas are expressed in terms of angles, okay.

So I am converting L/R the radiance into angles that is all, right, okay. So then what I am doing is this is the expression I get. What is that expression? Go back and look at my expression for delta. What was the delta expression? L/R+alpha f – alpha R and that was what was written as Wf/C alpha f – Wr/C alpha r. Remember that we wrote it in that fashion, okay.

And that is now what I am trying to do is since I find out this and then I am now getting that alpha F and substituting that I am simplifying this, you simplify this. You will get that you will see that that is the expression you would get. The C alpha f now is the total C alpha f for one time, okay, clear? So now what I am going to do look at this expression, this expression is different, what is that difference in this expression?

This expression had in the previous case it had Wf/C alpha f – Wr/C alpha R multiplied by V squared/Rg which is ay outside, okay. Now I am going to do a small jugglery. Now that expression is changed. I have denominated. So I am going to now simplify the denominator so that I will be able to separate out things and understand it, right. So it is 1/C alpha f I am talking out 1/1 – x, okay.

Which can be expressed as 1+x+ f squared/2 factor and so on. So, in other words, 1+x+ higher order terms. Removing the higher order terms this term is written as 1+x, okay. Now, so for this term I am going to replace it, for this term I am going to replace it and I am going to write down now a delta term, clear? That is all I am doing. There is nothing very difficult. So, in other words that delta now becomes 57.3 L/R+Wf/c alpha f – Wr/c alpha r+I am going to have additional terms.

What are the additional terms? Wf, note that these are the additional terms. Clear? any questions? So, what is that I should write+let me put a bracket here Wf*2 b*delta Fzf whole squared divided by c alpha f, this is outside c alpha f multiplied by that c alpha f – I will have a term like this for the rear, right. So, that rear would be – Wr*2 b*delta fzr 1 whole squared divided by c alpha r*c alpha r the whole thing multiplied by ay.

Clear? nothing great, simple jugglery, okay. Check whether you have written it correctly I have written it correctly all these things. If you have doubt let us look at Gillespie. Now what is happened? The term ended here but now I have an additional term, okay. Now interpret that additional term that is the most important thing. Now interpret the additional terms, how you will those things you will interpreted.

In terms of designs, simple things, we are not going to unfortunately this course we will not have time to convert this handling characteristics into design, okay. We will try to do that in the next course. But let us interpret that. How do you interpret this? This interpret, interpretation is based on 2 things. What are they? They are of course based on delta z that is the change here, okay and that is affected by what? That is affected by that expression.

So, that is affected by this, okay. Let us take the front you know that in this of course, okay. Let us take the front, okay. What happens if the front stiffness, suspension stiffness increases interpret that. What happens when the front suspension stiffness increases? This increases which means that this increases, okay then so this increases and hence the vehicle becomes under steer, right. So, that is the first thing. The vehicle becomes under steer.

So, if you put a torsion bar or stabilizing bar where the stiffness, front stiffness increases then the vehicle will become more under steer, right. After all we are interpreting under steer in terms of delta the steering angle that has to be given. One of the factors that is important is what? The next thing that is important here. One is of course the stiffness, the other thing is what is important. So, the roll center height, right.

Now what happens when the roll center see this is the front of the vehicle? Let us say that that is the vehicle, okay. The roll axis, okay inclination is one of the issues in the design. The roll axis can be inclined like this, the roll can be inclined like this and so on, right. Now if the roll axis is inclined with nose down as they call it, what happens? If nose down the roll would produce more delta fz in the rear, right because the height that is what causes the roll.

So which means that this guy is going to be higher and so the vehicle is going to be more over steer, okay. So, this is how we interpret it, okay. So, the first thing is that the roll causes a change in the under steer over steer characteristics, okay. Remember that when you take a turn you give a steering and then the vehicle realizes that it is taking a turn and then the roll happens and the roll happens with some time.

So, there is a transient region, okay when the body was straight and as it take a turn the body actually rolls. So, then afterwards it may settle down into steady state when you are talking a steady state cornering. So, there is a transient region under which the roll which was 0 will take on a value.

So in other words the over steer or under steer characteristics of the vehicle or let me say the under steer gradient of the vehicle would now change from one value to another value as you roll or as you go into a maneuver. People sometimes call this as a transient under steer or a transient over steer because in the transient region between running straight and taking a turn, okay, the roll develops.

It is not that immediate when you as soon you give the steering it is not that the roll is immediately going to turn. It is going to take time. So, it is going to develop as you travel. So, there is a transient over steer and a transient under steer, okay. Depending upon when you come out similarly the roll will come back okay the effects will be opposite, right. So, these are transient reaches**,** that is number one, okay.

We are going to talk more about it. So, this is one of the things which you would look at when you look at a vehicle, right. There are many such effects. We will quickly go through all of them, okay. Now, is this clear? We will move out? So, this is due to the roll. Roll also produces 2 effects, okay. The 2 more effects that are important. One of the effects is what is called as the roll steer.

Remember the vehicles have 2 things one is toe and the other camber apart from what we call as caster, okay and so on. Let us now look at these 2 things, okay and understand what happens. After all the suspension is a mechanism. You used the theories of mechanisms in order analyze for example a (()) (24:30) it is a mechanism. So, when there is roll, okay there is a change in camber which you would separately deal with as well as a change in the angle of the wheel, okay.

In other words there is a change in if you want to call it change in toe, okay right. Now, how much it changes? Very very important okay and what is this. Let us understand this clearly. Let us say that,

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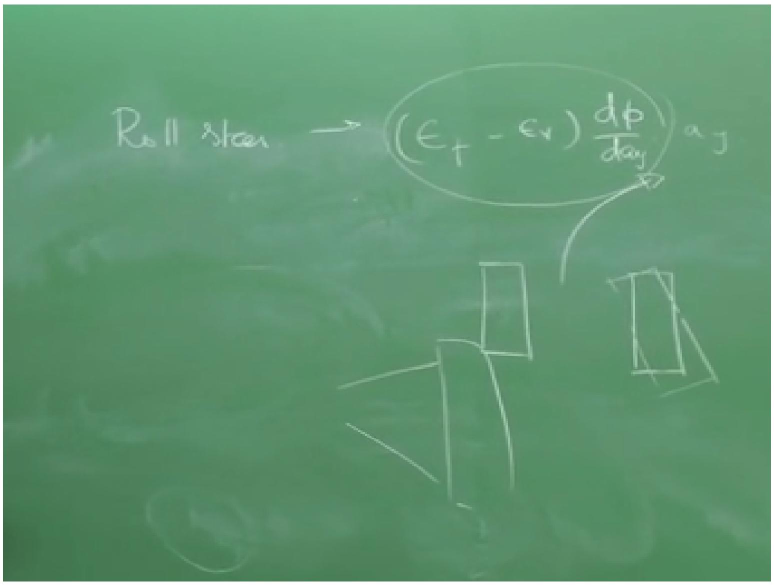

Yes, you will see that in the next course. See, unfortunately that as I said I am going to do that suspension design in the next course. Unfortunately we do not have time, okay. Roll center can be determined by geometric part or if you just use one of the software's you will get it in no time. Yeah, good question, okay. That is a better question for this course, okay.

Yeah, one of the concerns of people who design in a vehicle dynamics are how does many of these factors change dynamically. For example the roll axis does it change? Designers ensure that this does not change, okay. Roll axis does not change roll centers do not. They want ensure. So, they would look at this dynamically, okay either in a software like ADAMS are one of them, okay or they will measure it.

So, many of them they will measure it. You have hit the correct point. For example roll steer, okay. What happens during roll, okay. You have the wheels with the certain what is called toe. You all of you know that it is a toe in or toe out, right. Now because of this suspension kinematics, okay when you take a turn and when there is a roll the linkages change. Say let us say that it is assumed it to be a 4 bar linkage, okay.

That is connecting the wheel and then because of the roll these linkages move. After all they are attach to the body and so they start moving. So, when they move the vehicles or the wheels attitude also changes, right. So when I, now take a turn to the right, okay and look at the front, okay. It is going to have an effect both in the front and the rear and we have take a turn to the right and let us look at the front wheels.

And if the front wheels change of attitude because of the roll happens to be like this, okay. This is what we call as roll steer, okay. When it changes then what would happen? Without you giving an input actually the vehicle would try to go in the other direction. Whereas, you want to go it to the in this direction. So what would you do? You have to give more delta in order to compensate for this roll.

So, this is the thing that as I said dynamically toe will change. So, people are always worried as to how much toe changes because of roll, okay. You will do that in the next semester for vehicle dynamics lab, okay. Keep all this things in mind and when you do that lab you look at you are going to design a suspension and look at actually how toe changes. Let us say that let me put that epsilon is the coefficient for the toe change, okay or under steer coefficient because of steering.

Interestingly look at the rear now, okay. The rear if you understand how that rear turning is, okay suppose you have a rear steered vehicle, okay you will, so that is the rear that is the front, okay. You would actually turn it the other way and this you would turn it this way. So if I now take a turn to the right and if my rear suspensions are such that the wheel turns to the left what is that I would get.

Good for me as far as delta is concerned. In other words I would get an over steer effect, okay. Clear, no? Yeah, so now look at this very carefully. When I take a turn to the right the roll is to the left, okay. The roll is to the left vice versa, okay. So, if the roll is to the left and to the left, now right now I am, this is the one, okay. The roll is to the left and if this wheel happens to now go along with me to the left, okay that would cause an over steer, okay.

So I would a positive epsilon, clear? Of course, that is the fundamental know when you take a turn to the left the centripetal force is acting that is, okay. Right. So we will define like what Gillespie has defined. We would define epsilon F okay to be degree that is we will say epsilon F – epsilon R*d phi by day where this is, so we would say that the delta that is affected is something like this multiplied by ay.

So we define the under steer or the roll steer coefficient as I would call it okay, to be degree per degree multiplied by this is the roll, multiplied by roll per ay. Remember why is that I am doing all these because I have outside ay. So what comes inside this bracket? This is what comes inside the bracket, right? So this is what is the under steer whatever is coming inside is what is the under steer gradient okay.

Which we called as K2 if you remember in one of our earlier classes and that is what would go into that expression, okay, clear? Any questions? So this is called as a roll steer. So, people actually plot how toe varies as you roll, okay. That is the part of your exercise in the next course. So, do not worry about it right now. But you would look at that closely. Ideally speaking I do not want that the toe to vary and give me surprises, okay.

So, this is the next effect. In fact recent studies I have shown that toe effect or in other words roll steer effect is more important than the next effect which I would say is the effect on camber. What, yes, that is the coefficient, okay that is what you will determine by doing a theoretical kinematic analysis, okay or a KnC test rig, okay. There are KnC test rigs go on look at that in the YouTube or in Google for KnC kinematic and compliance.

So, this is called as kinematic and compliance test rigs, okay. What you to do is put the vehicle on this test rig and then measure how much toe changes, camber changes and all that. This you do it in the static case and there are technics by which we will show later that you can measure them in the dynamic case as well. So, in other words there are censors which would measure these changes.

The other, if toe changes camber has to follow. Camber also changes, okay. So, what is the effect of camber change there is an effect why? Because if you remember we introduced the term called camber thrust. Remember that we introduced in effect call camber thrust, right. Remember that this was due to the deformation of tyre, okay. Because when you roll a tyre with camber that introduced camber thrust.

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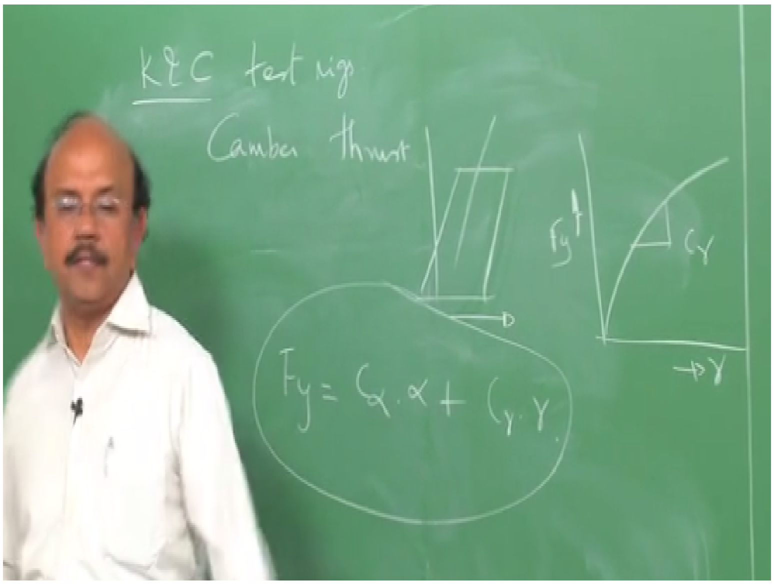

In other words, if this where to be the camber of just exaggerating it, cambers are very close to 0 in most of the modern cars, okay close to about 0.5, 2.75 in many of the trucks. So, that would result in a force f. Fortunately, radial tyres camber thrust is very small, so approximate calculations show that about 10 to 15 degrees of camber variation would produce a force equal to that produced by one degree of slip angle, okay produced by one degree of slip angle, right.

So the camber thrust produce us a force and that force is now going to be a force which I have to look at closely, why? Because after all the most important equation here is this equation, so from where we got all those things. So, now the force that is generated is due to 2 things one is c alpha*alpha + let us say that this is a graph for the force Fy versus gamma which I would say is a camber angle then let us say that the gamma, okay.

So, c gamma*gamma, right. So, this again varies or produces variations in this whole derivations which we had given here and so that would introduce further variations in our under steer and over steer, right, okay. So, the derivation is quite simple I would like to skip that and would refer to the Gillespie for further, this is about 4 line derivation let us because we are running out of time.

So, I am going to only write down the final expression, okay and referred to Gillespie's book for this derivation. Because this a very straight forward expression, very nicely done at in that book.

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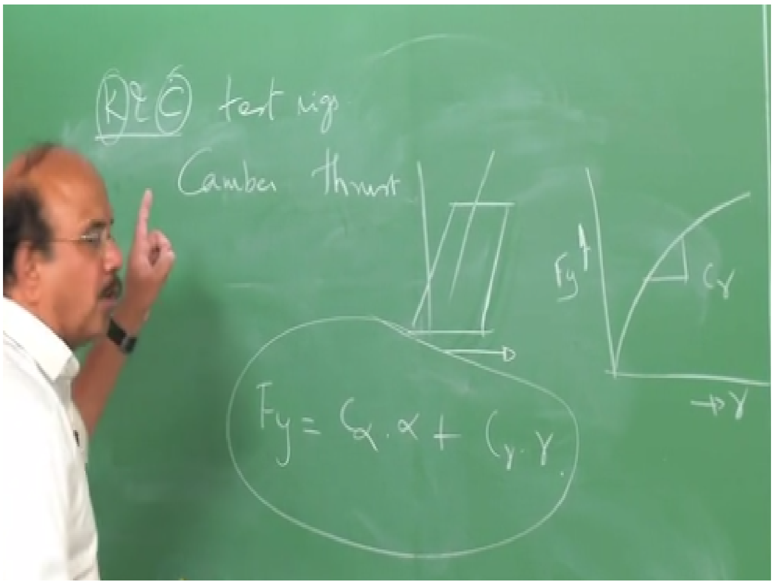

So when I introduce that so, let me introduce the term called so let us say that gamma is the camber angle. So, let me called dou gamma/dou phi = the camber gradient, okay and write down the final expressions straight away. No big deal. I have written this. Write it for front write it for rear, write this down in terms of ay, okay and then rearrange it. So, that would become 57.3*l/r/C alpha f.

Of course this our good old tyre under steer gradient +c gamma f/c alpha f*tau gamma f/dou phi – c gamma r/c alpha r*dou gamma r/dou phi the whole thing multiplied by dou phi/dou ay*ay, okay. So, again here we have not included this roll term no, if I would take into account the roll then I will introduce that as well. Here we are only looking at the effect due to camber alone, okay.

So, when you want to have a complete under steer gradient then you have added the effect of each one of them. So, this is the effect due to camber change. So, we have roll steer and now we have the camber, right. The problem is that many times it is not this gradient is not linear. So, if you now really look at how the camber changes and plot this graph tyre going up and in, so the graph may not be like, I mean may be something like this, okay.

Or the other way about you know it may be like this and so on. So, it is not necessarily a very linear graph nevertheless, okay, this is, there is an expression and this becomes an important factor the camber changes becomes an important factor, okay. Second to us as we see many of the measurements are done second to that roll steer this becomes important. Of course this becomes quite important as well, okay.

One of the other things, so we are looking at how each one of them that are members of the suspension are changing the under steer, okay. Many of them are transient that is why introduce the term called transient under steer or transient over steer, okay. They may or may not. See the whole problem is if you drive in a very calm you passing at 40 kilometers per hour we are not talking about any of these effects.

But if your speeds increase that is where trouble starts then these transient effects now become important unless you are very alert the amount of steering you give now depends upon this transient effect that is why we are looking at these things closely. As I said kinematics and compliance we had looked at kinematic changes. There are other effects due to Compliance. What is this Compliance of the suspension system?

The compliance comes from the rubber bushings, okay or things which are going to deform introduced for various purposes to isolate vibrations and so on in the suspension system, okay. That is why it is called kinematics always it goes together kinematics and compliance.

Now, kinematics and compliance place the roll and next this is what is called as the compliance effect, okay. What is the effect of this Compliance?

What do you think will be the effect of compliance? I said lateral compliance, so obviously this.

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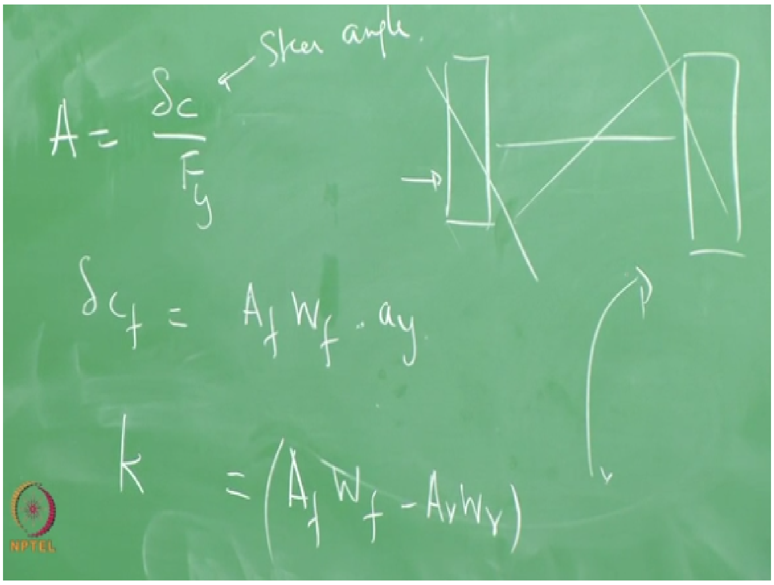

Can you just imagine suppose I have this is the, so let us say that I am just putting this does not matter whether it is an independent suspension. Let us say that, you can consider that as an independent suspension, okay. Let us say that this is the say your, let us say that this is rear, okay. Now this is going to take a turn like that which means that there is going to be a force that is acting in this direction, right.

Now if the force so happens acts and if there are deformations, okay due to which the tyre now is deflected. Say, for example this whole thing is deflected like this, okay. Again it is going to have an effect on obviously on the under steer gradient. Whatever has an effect on the wheel how it is going to move, okay it is going to have an effect on the under steer gradient, right. This is what is called as a lateral compliance steer.

Now will it cause an under steer or over steer? That depends because this would act both in the front as well as in the rear, okay. So, if it acts suppose this is the rear and if this is what you are going to do fine, this would cause an over steer. On the other hand if this happens to be behind, okay and you are taking a turn and the wheels go towards the left when you taking the turn to the right then that would cause under steer.

Sorry first one is over steer and the second one is under steer. First one because you have you want the rear to go like this for making your turn easy. So, when it turns to this it is I think I slipped over steer, this is an over steer. If this happens to be the front and the (()) (45:48) happens to be at the rear and if you are going now take a turn like that and your wheels are now towards the left.

Then you have to give more steering in order to steer that back and so you would cause under steer. Remember delta increases when the steer increases. This is due to lateral compliance steer, okay. This is, so there are 2 things. One is front lateral compliance steer due to front and the other is rear lateral compliance steer. So, front lateral compliance steer and rear lateral compliance steer, right.

So, again this is introduced as a coefficient, okay we will follow Gillespie on this. And so, we will introduce this coefficient as A and let me call that A to be delta c/Fy, okay. So that delta cf, delta cf of course is the steer angle which you get out of this. So, delta cf = Af in the front Wf*Ay. So that the lateral compliance steer of the front and the rear together on the under steer gradient can be written as Af*Wf – Ar*Wr.

The sign of which is obvious because of all the explanations which I had done so far, clear? So, the compliances become in fact some of our work which we had done if I have time we will present it to show that I will also (()) (47:53) paper in the next class show that this is a very important parameter. Front lateral compliance steer seems to be a very important parameters in K.

So, I am adding one more is added, okay. So, under steer gradient we started with only term. We add our second term here, we added third term due to roll steer, we added 4th term due to camber. We are going to add now a fifth term due to lateral compliance steer. So that is why KnC measurement as they are called becomes extremely important, clear? Now, story does not end there. That I want to hold you for a long and I am stop you from having a lunch.

What do you think which is the other effect? Answer that question you are free to go and have lunch. What do you think are the other effects? No, that is, we are talked about pull already know and where it act is what we call as lateral compliance. But there is see this load transfers seems to be the first major thing. So, what is the other thing that is going to cause low transfer?

Correct, so traction or breaking is going to cause now low transfer, okay. So, this low transfer again redistribute alpha f and alpha r, okay. And is going to have an effect on the under steer gradient. It is so important that there are tests to do this. Imagine that if obviously there is, is going to be a change we will see that in the next class what the change is. Imagine that you are taking a turn and you are applying breaks.

So, there is a redistribution of forces because of which this redistribution or there is a change in alphas and when there are changes in alpha obviously you will feel a different steering input is required, okay. So, that is the next effect. And lastly steering itself has its own problems or problems in the sense that they also contribute, okay. Compliances of the steering also has an effect. We will talk about these 2 this load transfer and the steering effect, okay in the next class.