Vehicle Dynamics
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Lecture – 28
Rollover Prevention (contd..) and Vertical Dynamics

In the last class, we were looking at rollover or in other words, we started discussions on rollover. So this will be our last topic on lateral dynamics before we move over to vertical dynamics.

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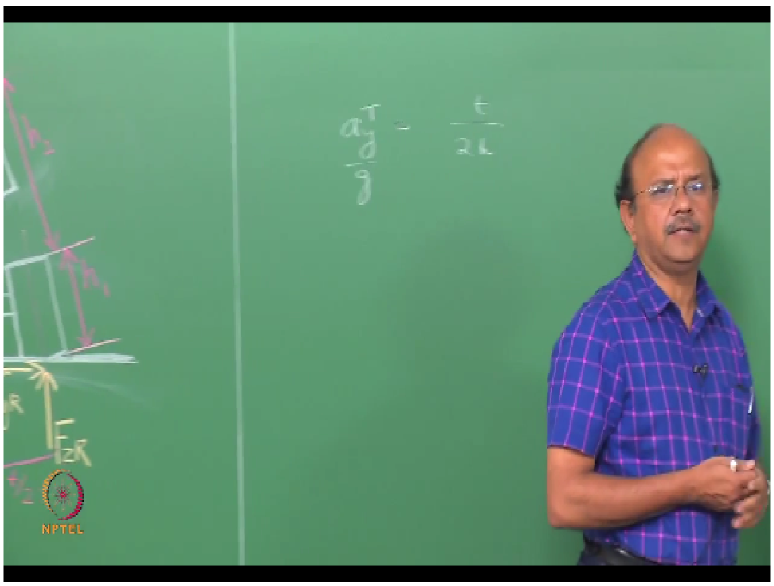

One of the important results we got yesterday, okay, based on the static condition of rollover, is a condition where we said that the critical threshold for ay or even ay/g, this t indicates threshold, =t/2h, okay. So what is t, t is the track, okay and h is the height and very intuitive, the larger the height, you know that, the ay will or the rollover propensity is high and that larger the track, larger this one and smaller will be this ratio.

So larger will be the propensity, larger the track, okay, larger will be this ratio and lesser will be the propensity for vehicle to rollover, okay. So this we arrived at with a simple, very simple equilibrium, okay, of moment which we had taken about one of the tires and remember also that when we talked about rollover, we were just talking about the, what is the condition that the tire just leaves the ground.

In other words, we said that inside tire had or will not have any reaction force, it will just off the ground, okay. We are not interested whether it will rollover further or whether it will really rollover or come back and so on, okay, but from a design point of view, we look at a condition where the wheel just is off the ground, okay. Actually we are not very justified in only analysing it from a static perspective.

We have to do this from a dynamic perspective as I told you in the last class, basically the dynamic perspective is also to get controllers which would control this rollover, right. So in other words, you can write these equations, the dynamic equations we are talking, okay, we can do that and maybe you can write it even in state space form and then you can design a controller for this. Unfortunate this course we will not have time to go into the dynamic case.

Because we have to move over to vertical dynamics. We hardly have about 7-8 classes remaining. So we need to go and look at vertical dynamics.

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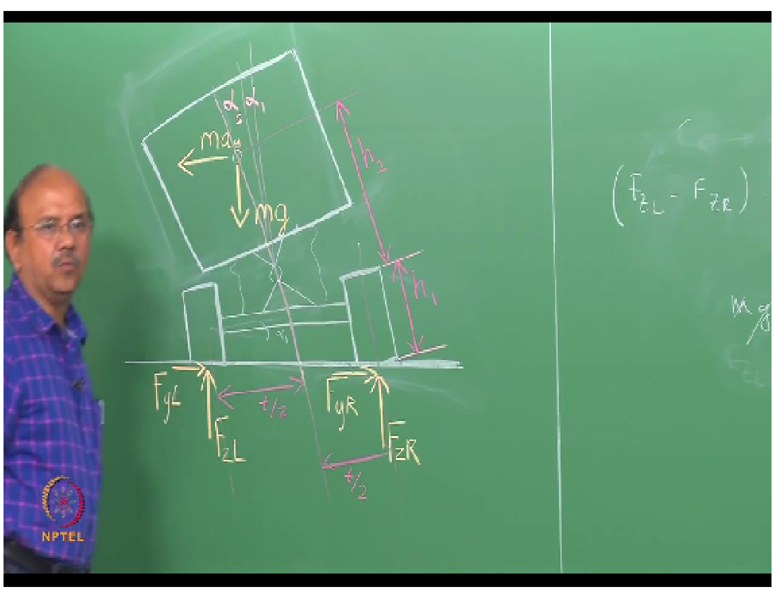

So we will only concentrate on the static case. We also said that there are various factors which would change this t and h and in fact, I would suggest, I would say that the first thing that any designer would look at is that ratio, okay, especially if you are looking at trucks as I told yesterday. So the first thing that you have to look at is what are the parameters or what are the conditions under which both T and h are affected.

What are the most important conditions under which this h, the factor h is affected, is by the rolling of the vehicle, okay. Now let us concentrate on that, that is a very important factor and in fact, you would see how that is going to have an effect, okay or in other words what are the additional terms that is going to come. Let us understand this figure. Now we have 2 rolls, okay. Here the sprung mass, okay, which rolls about the roll axis has a roll angle of alpha 2 which is superposed on the alpha 1 of the axle, okay.

So we have alpha 1, this angle through which this axle rotates as alpha 1 and then we have alpha 2 as the angle through which the sprung mass rolls, okay. Now what we are going to do is, is very simple but the equations are going to be complex because we are going to bring right-hand side to left-hand side, left-hand side to right-hand side and so on, okay. That is the only condition by which, in order to which, it is going to be difficult.

The rest of it is going to be very simple. **"Professor - student conversation starts"** Alpha 1 (()) (05:08). Alpha 1 is the angle, okay. **"Professor - student conversation ends"** Let me explain this alpha 1, I thought I will do that later but let me explain that. Remember that when it rolls, obviously there is going to be a difference in the load that is carried on the say outer and inner or called as left and right here or you can call it as outer and inner, okay.

So there is going to be a difference in load.

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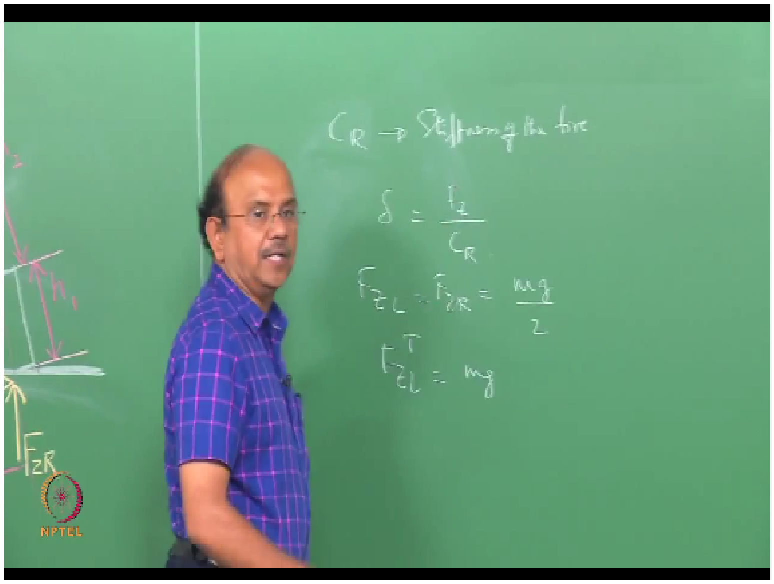

If there is stiffness to the tire, let us call that stiffness to be CR, okay, then the deformation of the tire is given by, this being the stiffness of the tire or I call that as Kt as I said that I am following George Rill. So I am using the same thing. So if this is the stiffness of the tire, the deflection of the tire, okay, is given by the load that is acting/this stiffness.

Now since this vehicle rolls and there is a distribution of loads, okay, the deformation of these tires are not going to be the same or in other words, they are going to be different. And that when there is propensity to rollover, in other words, the whole of this tire lifts off the ground and no reaction is taking place, all that load actually comes to this tire. In other words, if Fz left=FzR=m*g/2, okay the weight, the total weight that is acting mg/2 in the usual case, this steady state running condition.

Then if under the threshold condition, this whole thing is going to be mg, which means that the deformation here is 0 of this tire because it is not going to be deformed due to the load.

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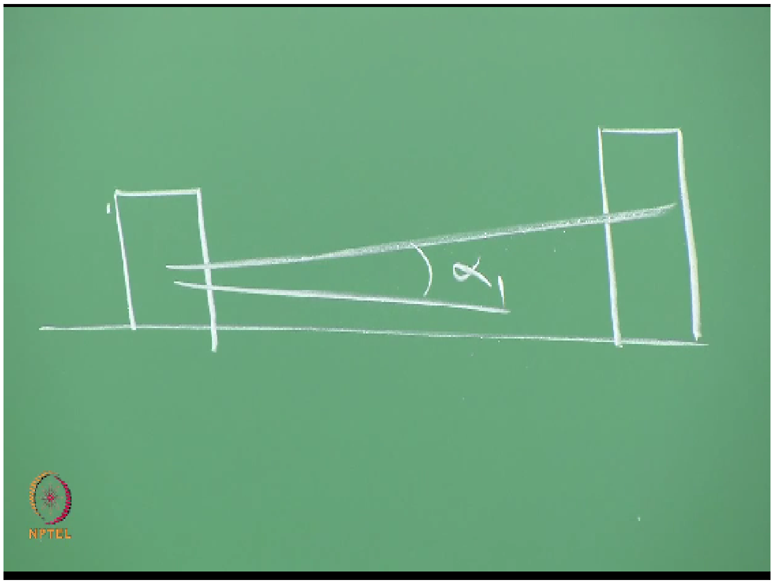

And the deformation here is going to be, Fz going to be mg with the result that if this is the ground, I am just exaggerating that figure. So the tire would be like this, okay, because this tire would be compressed, road being the same. So there will be, so one tire has no deformation, the other tire has deformation. So the whole axle is going to be rotated and that is what we call by alpha 1.

"Professor - student conversation starts" (()) (07:59) No that is why I said that we are looking at static condition, we are not looking at dynamic case, okay. So there is no time factor involved, it is the first thing I said, right, okay. Listen carefully to what we say. So there is no time factor involved. We are looking only at the static case. **"Professor - student conversation ends"** So alpha 1 is that deformation. So in any of these derivations, there are going to be assumptions.

So these assumptions take shape as we go step-by-step into derivation. So to be careful, we will link that with the derivation. So that is the alpha 1 which we are talking about. Of course, alpha 1 and alpha 2 are going to be small and we are going to use that as well, that both of them are small, okay.

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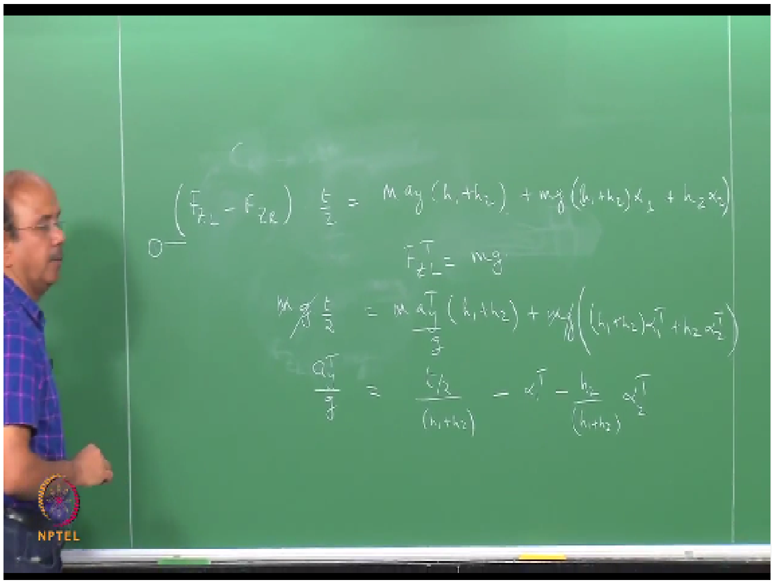

So let us or look at, what is that we had written FzR, okay, *t/2 that is we take the moment of the centre*t/2=write down the moment due to may and as well as due to mg, okay. Note that this is h1 and that is h2 and note that, we are talking about small alpha 1 and alpha 2. So we can write down that to be m*ay*h1+h2, okay, this is in the anticlockwise direction.

Then you have one more term which is again in the anticlockwise direction +mg*h1+h2, you split this up because this alpha is up to this point, alpha 1, so *alpha 1+the other one is h2, h2*alpha 2, right and bringing it to the other side -, since g has been taken over that g is out there, so in other words, okay, -h1+h2, I have divided that, so that means that is gone, -h2*h1+h2*alpha 2 thresholds, yes.

"Professor - student conversation starts" Sir, h2 is height of roll center, right. Height from the roll center to the Cg location. What is the… H2, this is the Cg location, that is the roll center, okay, about which it rolls, that is h2. In other words, I was just going to comment. In other words, h1+h2 is our h in the previous derivation. So the t/2h is actually you have got, okay and that is modified by 2 things, one is alpha 1 and the other is alpha 2, right.

So if I neglect this roll, I get back to the previous derivations. Remove this condition, I mean these things, we will rearrange it and we will write ay threshold/g, okay, = t/2*h1+h2, this is the first term, right, clear. **"Professor - student conversation ends"**.

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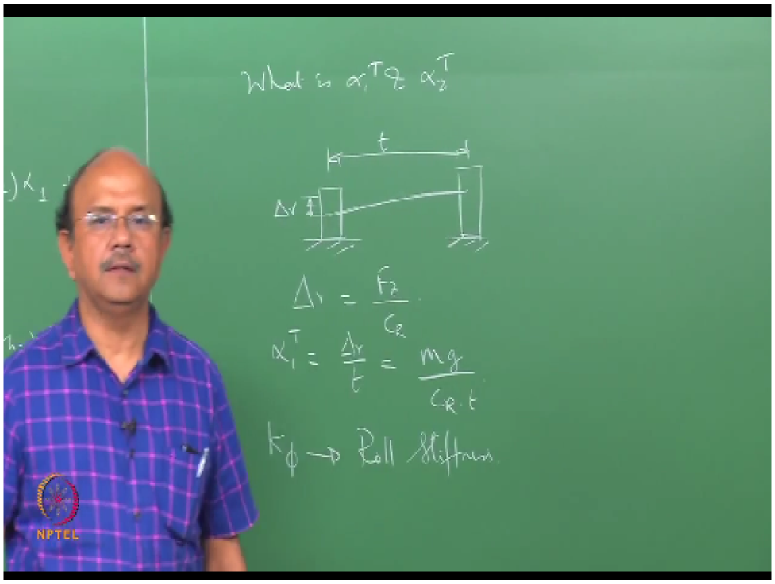

Now my job is now to find out what is alpha 1 threshold and alpha 2 threshold, okay. So we already know what alpha 1 threshold is, so we said that, this is going to be a difference in deformation, okay. Let us call that as delta r because there is a radial deformation, okay and that angle alpha can be written as alpha 1 threshold=, what is this, this is t, =delta r/t and that is equal to, what is Fz, now mg because the whole is taken there, so mg/CR*t, clear, okay.

So what is my next job, my next job is to find out what is alpha 2, okay. Alpha 2 is promoted by what is called as the roll stiffness K phi which is the roll stiffness. We already know how to calculate roll stiffness from the spring, okay, 1/2*Ks*S square, right. In other words, this S=t. So you again t and S are the same. You know S is approximately the same. We should not say that they are the same because the s is actually that length.

When do they become the same, when it is a commercial vehicle with the suspension springs, okay, which are basically leave springs and when they have a rigid axle, then the distance between the springs, okay, becomes S and if it happens to be an independent suspension, then S and p are the same, okay. Note that carefully and that we had already seen. So we will now write this whole thing in terms of roll stiffness, clear, okay.

I think let me, I will put in some number so that I will remember these numbers and my most important equation is this equation.

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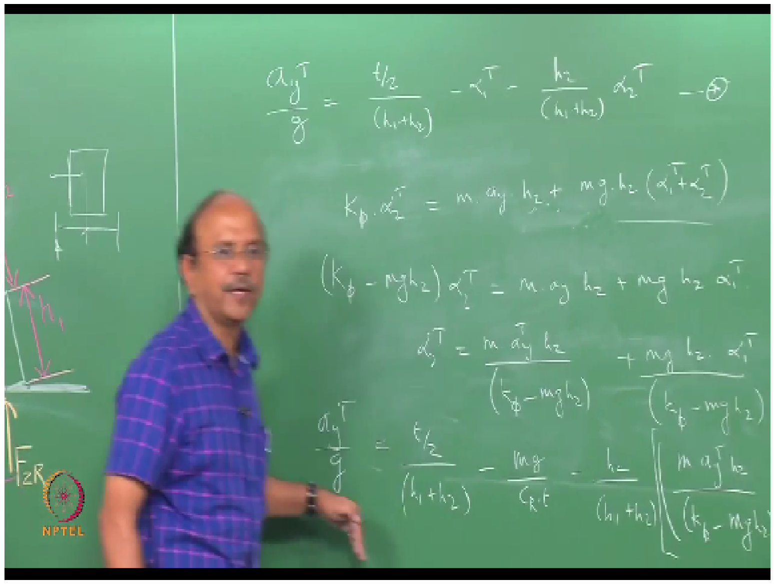

So let me write that equation here and not remove that so that I will, when I want to substitute it, I would use that later. One essential I am going to do and is very very simple. I have already calculator this. I have to calculate this, substitute it, refer, I mean rearrange it and get you one big picture, that is it, right. That is the most importantly equation which we have used it after we get this alpha 2 threshold, okay.

Now how I am going to get alpha 2 threshold, K phi*alpha 2, okay, that is the reaction force, in to alpha 2, is equal to what are the things that go into this, m*ay*h2, again look at the assumption, I am making that they are small angles, cos of that angle is approximately equal to 1, sine of the angle is equal to that angle, all those things I am putting. So be very careful in that we are going to work in degrees or radiance, we have to be very very careful.

Because when we make that approximation, okay, you put that in terms of radiance, +mg*h2*alpha 1+alpha 2, okay. Since we are looking at the threshold, let us call that as T. I have an expression for alpha 1T already. So I will just keep it and then substitute it later, right. Now what are we going to do, we are going to rearrange it, okay. Let us see how we are going to rearrange it.

So we will just going to bring that alpha to the other side, -mgh2*alpha 2T = m*ay*h2 + mgh2*alpha 1T. In other words, alpha 2T=m*ay*h2/K phi-mg*h2+mg*h2*alpha 1/K phi-mgh2.

“Professor - student conversation starts” You have talked about (()) (19:26) what is that equation. Which one, this one.(()) (19:30). What is this equation? (()) (19:36) balance. Balance equation. So this is the action reaction equation, about which point, obviously about this roll, okay, where there is a roll stiffness is, K phi*phi, we wrote this already. Remember that we wrote this equation already in our previous class, go back and look at your notes.

We had written this as K phi*phi, remember that we wrote that as k phi*phi, the moment balance equation, that is exactly what we are writing now, okay. **“Professor - student conversation ends”**. So now what is my next step. I am going to substitute this into my, what we called that as third and I am just going to rearrange it, that is all, right. So that let me write that equation, ay, that is a gruelling task of simplifying this expression, =t/2/h1+h2-, alpha 1.

I had already written there, mg/CR*t, correct me if I am wrong, if I am going to make a mistake because this is going to be lot more, -h2/h1+h2*the first term, put it like that, m*ay threshold*h2/K phi-mgh2+mg*hs/K phi-mgh2*mg, right. **“Professor - student conversation starts”** Let us talk about that point, we have assumed the alpha 1 and alpha 2 are small angles, right. That is what I have been telling.

So may (()) (22:13) due to the roll, it will also have (()) (22:18). See the whole, this is attached to this body only. So this is, the roll is not, that is why we had written alpha 2, the roll is only with respect to this and then this whole body, you know, with respect to alpha 1+alpha 2. Are you saying may*h2…Okay, we will discuss that later, let us finish this first, okay. Let us finish this, because this is going to be lot more things that you are going to do and then we will discuss it, okay. **“Professor - student conversation ends”**.

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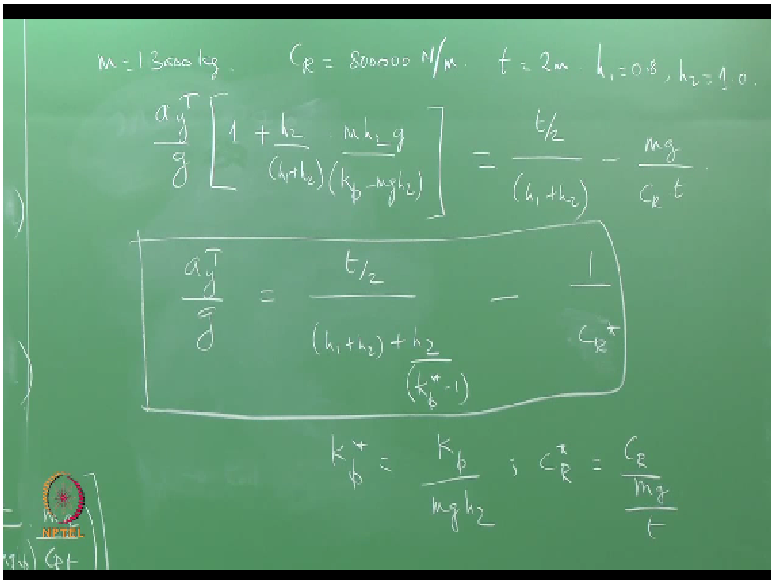

So now let us just rearrange this terms, take a minute to rearrange it, what is that I am going to do, I am going to bring this to the left-hand side and rearrange this term. So let me do that *1+what is that alpha ay, the first term, 1+h2/h1+h2*m*h2/K phi-mg*h2, right. **“Professor - student conversation starts”** (()) (23:59). which is the one. (()) (24:04). Okay. So g is not in the room, so I had to put g there, right, okay and then the rest of it in the right-hand side, okay.

So what are the terms, the first term is there, t/2*h1+h2-mg*CR*t, right, okay. Now take a minute to rearrange this. By the time, let us look at the question which he had asked, okay and let us look at what is the question, yes. Sir if mg is contributing only because of the roll, right, otherwise, mg would pass right through the point where we are (()) (25:11). Mg is contributing? Yes of course. No, no, no, no, no, no, no, no. Wait, wait, wait, wait.

We already saw yesterday in the last class, mg is contributing by how? Even if it does not rotate, okay, that is the restoring moment. You remember last time, we saw that how did we take the moment balance.

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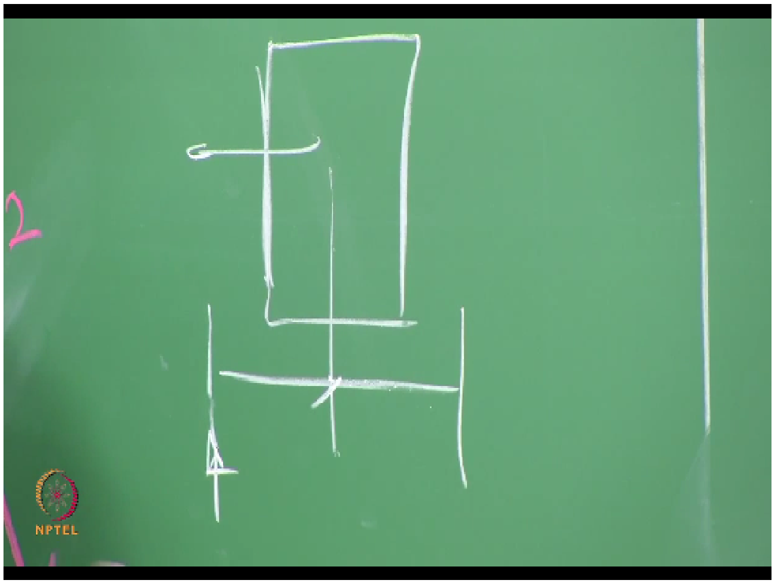

Last time, you took (()) (25:34) No, no whatever it is, it is not a question of taking, it is participating, okay. This and this, okay. So what we are trying to say is that there is one moment which is due to the centrifugal force, in other words, d'Alembert's force which is trying to rotate it, okay, in one direction and then this is going to rotate in other direction, okay. So the equilibrium is because of these 2 forces, right, okay.

It does not matter whether we took it about this or not because that is how it is going to rotate. Now that, 1 minute, now that we have taken it about this, okay, of course mg is, the contribution from mg comes into picture because of roll. If I had taken this one about this point obviously, no, no, that is exactly what I am trying to say. This mg would have acted here and again I would have had mg*t/2, so that will not go anywhere.

No sir. So the question is, whether you take it about this and then use this mg in order to compensate or in order to get back the system to equilibrium or you take it from here, it does not matter, okay. Sir that was not my question. Question is, so right now mg is alpha 1 alpha 2 are small but still (()) (27:04) significant though. Mgh2*alpha 1+alpha 2. Similarly, due to the roll, mayh2, the contribution because of that…May*h2, yes. (()) (27:17).

No may, will not contribute completely. There will be a component which will not contribute to the work, (()) (27:27). Yes, alpha may*h2. Both contribute completely. So what would be the compensation? There will be, I will have to work out but there should be a component which is comparable to mgh2alpha 1T+alpha 2T. Okay. Is that correct? Is that okay. Anyway it is cos component, it will be only for that. Exactly.

So cos of, please note this is exactly that is why listen to every question. Please note that what is, actually you should multiply this with. Cos of alpha 1. Cos of, cos of, so this I exactly said this, this is sine of alpha 1+alpha 2. This would have been cos of alpha 2. So I said that that is equal to 1, sine, both of the angles are small. So when multiplied by cos, see this actually what is that you do, this horizontal distance is what you are going to take.

The horizontal distance is cos of this angle, okay. Cos of that angle=1 we take. So actually this is multiplied by 1, okay. So the angles themselves have no meaning. So it is the sine of that, in fact what I should have written it, written is to write this as cos of alpha 1+alpha 2, then sine of this, then make that is equal to 1, this is equal to, that is why I also made a comment that, can you do that with degree?

No sir but…So you have to do that with the radiance. If sine is small component, it is going to be significant then what we are neglecting there will be (()) (29:14). No, that is the whole assumption fundamental that when alpha is small, when alpha is small, okay. What is the assumption that you usually make? Cos of alpha=1 and sine alpha=alpha, okay. This is the assumption we have made here.

So that is why I said this derivation, see there are 2 things in this derivation. One is I know this when you asked the question, I know this. I did not want to confuse it in the middle because I wanted to go through this and then come back, because I had already made a comment that I am going to make all this alpha as, very good that you made the statement.

So that is why I said there is a difference between concepts in every line and algebra, okay, of taking into the right-hand side, dividing it, okay rearranging it, all those things as high school stuff, that is different but what is important is what we make assumptions here. **“Professor - student conversation ends”**. Now the more important question you should have asked is, how will you know alpha 1 and alpha 2 are small, okay, that is the concept, how will you know alpha 1 and alpha are small. Very important question.

In fact, this is the question first I asked, you know, we had one of the consulting work which we did, this is the problem and I in fact interviewed a driver who had a big accident. The guy escaped unhurt in rollover, okay. So I wanted to have a feel, you know, because you can measure it. Of course, there are thing to measure but this was about a decade ago when we did not yet, you know, understand lot of things about rollover.

So I went and interviewed the driver. He told me that, this is in the desert area in Rajasthan. So he said that he did not even feel the vehicle is going to rollover. He said he was going very slowly, slowly according to him, okay, a turn, he did not, he said that if I had known I am rolling, okay, I would have stopped it. But I did not even know that there was, I was rolling and the vehicle just turned over or rolled over.

So usually it has been measured also that these alphas are small and that is why we make that kind of assumption that cos alpha is equal to 1 and sine of that angle is equal to 1, okay, clear. Get back and rearrange this term. In every problem in vehicle dynamic, we are going to see that in the next derivation also. It is going to be so much of assumptions. Basically that is also because when we workout with hand, the first cut methods.

We cannot use very sophisticated methods. Of course, when you design it when detail design comes, you would do that in a much more sophisticated fashion, okay. Rearrange it and let me write down that, I am going to leave this to you because I know you are also bored, I am also bored to rearrange that, these terms and I will write down the final expression, right. So that is a much more interesting stuff then doing that, t/2/h1+h2+.

You know from where it is coming, okay, h2/K phi start-1-1/K phi star or CR start just a second, let me check that, okay, CR star, work this out where we call this K phi star to be K phi/mg*h2 and CR star=we use that CR, right, CR/mg/t, okay. Rearrange these terms and you get this expression. I am leaving this as an exercise for you to do that, okay. So that is the expression. So when I neglect these terms, okay.

I go back to my t/2/h1+h2, which is the height of the roll center from the ground, okay. So both of them are going to have an effect, okay both the tire stiffness, normalized tire stiffness I will call it and normalized roll stiffness, both of them, okay, has an effect on this role. In fact, in the book, there is a small problem, it is very interesting problem, may be you can try it out, okay.

Heavy truck, it is an heavy truck, twin tire axle, is loaded, it is a 13 tonner, what we call in India, it is a 13 tonner truck. So m=13000 kg, okay. The radial stiffness of the tire, CR is 800,000 Newton per metre, t=the track is equal to 2 meters, h1=0.9 and h2=1.0, okay. So suppose now I calculate from these values, the roll angles as well as these values of ay and so on, this is clear, any questions, very straightforward, there is nothing very difficult about this.

You do that, what I suggest is this is an exercise which I would like you to do. Just plot, I am not going to do that here.

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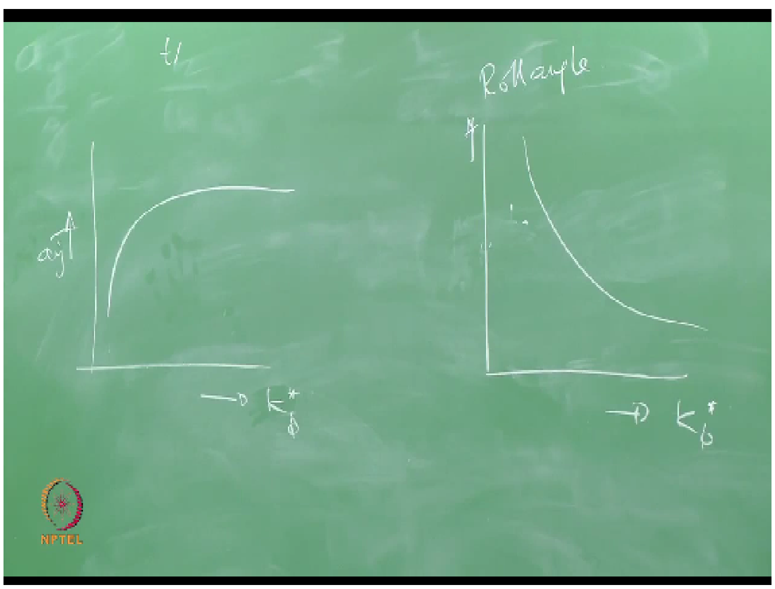

Say for example if you plot this versus K phi versus the ay overturning limit, how would the graph be and if I plot the K phi versus roll angle, how this is going to be, okay, just plot it, okay, that will be an exercise for you. So what happens when the roll stiffness increases, calculate that. So that is why you have, is there an effect. The questions I am going to ask and you are going to answer them, yes.

If I have an antiroll bar put, okay. For example, I increase the roll stiffness, okay, how do I, is there going to be an effect, okay. How do I realise whether that is going to stop my tendency to rollover. A very important question because many people think, for example many people who built this, the car carriers, they think that just keep on increasing the roll stiffness, okay, would stop them from or prevent the rollover of these trucks, okay.

So I had this, again from my experience, I see that lot of people think that just increase the roll stiffness, it will not roll at all. No, in fact if you plot this graph, you would notice that the graph would be, yes, it will be small, there will be an effect. After some time, there will not be any effect of roll stiffness, okay and the roll angle would actually come down like that and after sometime, there will not be any effect because of the contribution of these terms, okay.

Do that, it is a good exercise to do and you will understand about the roll stiffness and so on, okay. So we will now switch gears, we are running out of time. We will move over to, is there any questions. **"Professor - student conversation starts"** What about the front and rear roll time difference in… Good question. What do you think will be an affect? It will be (()) (39:17). See one of the things in handling, it is a good question.

One of the things in handling is that you should also look at the effects of other things, okay. Rollover is one thing, right. What are the other factors, what are the other things that will have an effect on rolls, I mean effect due to variation of roll stiffness. Change in (()) (39:57). Yes, correct, but exactly what, what is the effect due to load transfer.

So in other words, the roll stiffness is going to have an effect on load transfer. So load transfer is going to have an effect on slip angle or in other words is going to have an effect on understeer gradient, okay. Understeer gradient, the slip angle, the delta is the result of variation of this understeer gradient which would have an effect on alpha, F and so on. So in other words, okay, we cannot delink every aspect though for in order to study, okay.

I say that handling and it is over, I am going to shift to rollover. **"Professor - student conversation ends"**. Now I am going to say that this is all over, I am going to go to vertical dynamics, does not mean that they can be looked at in compartments, right, fine. So you should think about what happens, that is a very good question. I am not going to answer it now. What happens, when do I put and what happens when I put, maybe I will answer it in the next class.

But I want you to think about it. What happens when you put the front roll stiffness is higher for example, roll bar in the front for front wheel driven car, okay. When I put it, if I change it and put a roll stiff in the rear, then what happens, okay. Think about it, we will answer that in the next class. Let us quickly get into what is called as vertical dynamics. Let us begin with vertical dynamics, okay.

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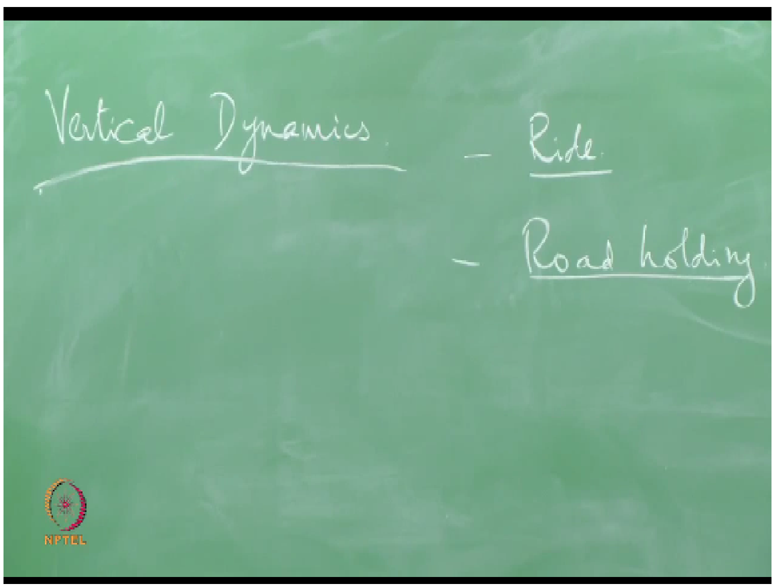

Now vertical dynamics has a very important role to play in what is called as ride, okay. Sometimes even people call this loosely is ride dynamics. In fact, there are 2 things that become important or we study vertical dynamics from 2 perspective. One is to understand ride and the other is to understand what is called as road holding. We want the tire to hug the road all the time, okay.

Obviously, if the tire loses contact with the road, okay, we know that our grip is lost and stability is affected. That way actually vertical dynamics cannot be totally uncoupled from the lateral dynamics but for the purpose of studying this, we got to decouple it, then we are going to study that vertical dynamics, okay separately, right. So we are going to look at both ride and road holding. These are the two things that become important in vertical dynamics, clear.

This is totally frequency driven, right. In other words, there is again a lot of confusion to look at what is meant by vibration, okay and what is noise because when I go on a road, not only there is a vibration that I feel but also the noise which I hear, right. Both of them are there.

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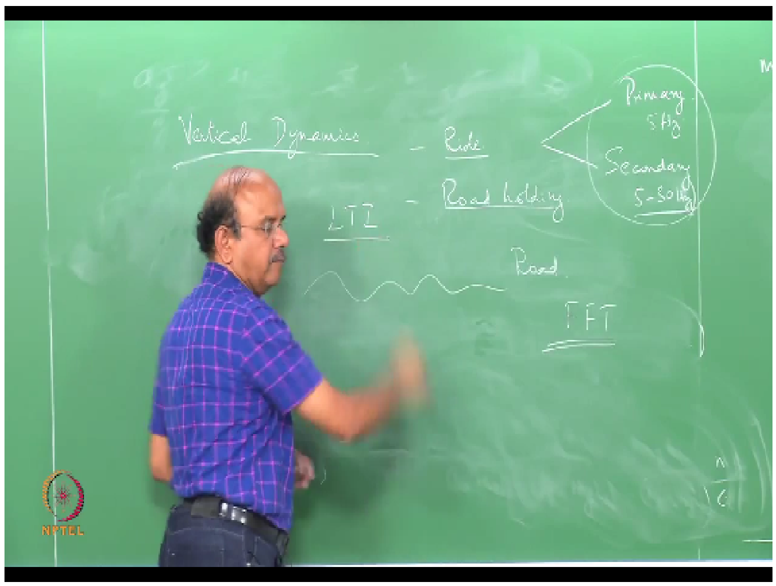

So we usually distinguish, okay, between in ride dynamics, we classify it into what is called as the primary ride and what is called as the secondary ride. Primary ride is from up to 5 Hertz, may be 1-5 Hertz and the secondary ride is from 5-30, so we will put it as 25 but 5 to 30 hertz. So when we talk about vibration of the vehicle, our frequencies are in this region.

So frequency of vibration becomes very important not only because the ease with which we are going to study it, okay, but also ultimately our aim is to understand the effect of these on us humans who travel in the vehicle. So in other words, what is our tolerance level for this vibrations, okay. Is there a relationship between the frequency and our tolerance level, okay. How are our fatigue, we develop fatigue in, physical fatigue when we drive and so on.

So all these things become important, okay. For example, you would have heard about seasickness. This is when you go in a ship, okay, people develop what is called as seasickness. Why is it, because the frequency of excitation, okay, of the ship is about 0.5 Hz which we cannot tolerate unless we get used to it, we cannot tolerate it. But there is a very comfortable frequency, okay, where we do not feel sick at all.

So when I walk, I can keep walking, I feel tired but I will not feel the nausea. So there is again a frequency, okay, 1 Hz where we are comfortable when we walk. So there are frequencies where we feel totally sick, frequencies where we are comfortable, there are frequencies which when we are exposed to say so many hours, we feel comfortable and so on. So in other words, the frequencies are related to our well-being.

So we are going to study that frequency becomes important, okay. So the other thing is this. We are going to do a very linear analysis. This is one of the questions, first question is that what is that we are giving this as an input, okay, how are we going to study model this. Obviously, our input is the road, okay our input is the road. How are we going to, what is random totally, how are we going to characterise the road, okay?

I know all of you know what is called as Fast Fourier transformer, FFT. I know that you know there is a link between the time domain and the frequency domain and that, probably a few of you at least are familiar with Fourier series where you had looked at say periodic signals and connection between the time and the frequency domain.

So we are going to use a few concepts from FFT, okay or in other words, the results what FFT can give and we are also going to look at in this course, this whole system of vehicle as what is called as the linear time invariant systems. So in other words, we are again, look at that, you know, assumptions are very important. Listen to these assumptions carefully because these assumptions may not be valid under certain circumstances.

So one of the first assumptions I am going to make is that the stiffness is what we are considering are going to be linear, okay. In other words, my superposition principle which I am going to apply, okay, by splitting this or by summing up using what is so-called complex exponential functions, simple words sines and cos is valid only if I consider my system to be linear.

So that is the next assumption I am going to make, right, fine. So with that as we go along, look at all our assumptions. With that, let us start or let us see how we are going to attack this problem. We will go into the full derivation in the next class.

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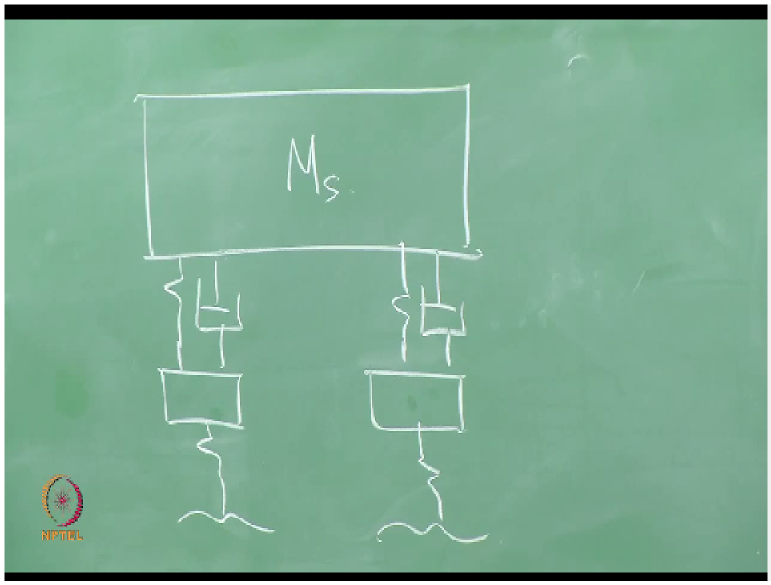

What we are going to do is to consider what is called as the sprung mass, okay, which sit on 2 suspension systems, okay. We will is call that as the sprung mass and these 2 are the unsprung mass and that is the tire, okay and that is the road input. We will consider this system, we will go into the details, okay. First derive with what is called as 4 degree of freedom system. Then simplify to 2 degrees of freedom system and understand these or in other words simplify or split it into 2 systems, okay.

Come to what is called as a quarter car model, okay, and look at what we can get out of this quarter car model. How do I now tune the suspension and that is going to be our major issue here. We will give an introduction about the road, its statistical characteristics but we are able to go beyond that. We will look at these things, the vibration and so on in a more detailed fashion in our next course and maybe a course on NVH towards give a lot more inputs into it.

So we will stop and we will continue with this model in the next class.