**Vehicle Dynamics**
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**Lecture - 29**
**Vertical Dynamics - An Introduction**

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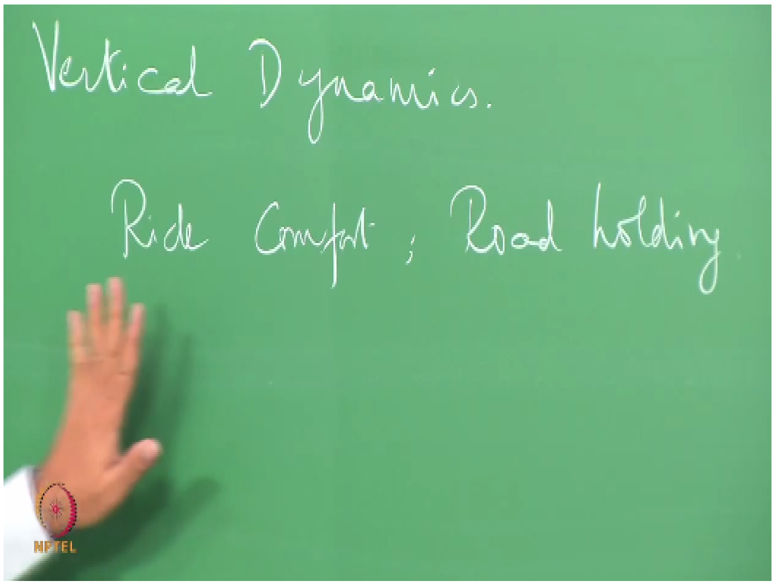


We are looking at or we started looking at Vertical Dynamics and that is what we are going to for the next 5 to 6 classes, and we already noted that there are 2 important things in Vertical Dynamics, 2 important things that we have to look at in Vertical Dynamics; one is what we call ass ride comfort, okay and the other is what is called as road holding are the 2 important things that we are going to see in this part of the lectures.

Now, of course, road holding for a common car may not be an issue, maybe more for formula racing cars, this becomes very important because of certain differences between the usual car that you drive and that of a car which is driven for say Formula One racing. So, of these 2 what is more important for us is what is called as this ride comfort. As we had done earlier in this course where we had looked at the bicycle model and looked at handling from a very simple model in other to concepts, and then of course we did some modifications to this models.

We looked at the assumptions that we had made and then slowly remove that assumptions and then brought out a complete picture. Like that we are going to look at this ride comfort from a very similar perspective and we are going to first put down a very simple model and then draw some conclusions. The point is this that there are a lot of assumptions in this simple model is that it is not valid.

One of the first things you notice is that when you have a very complex model that you may put up in one of the commercially available software like Adam's and so on. They need to first have a model in order that we put it inside the software. So, a first analysis is always good to do before we go to bigger models and that is why these models are important.

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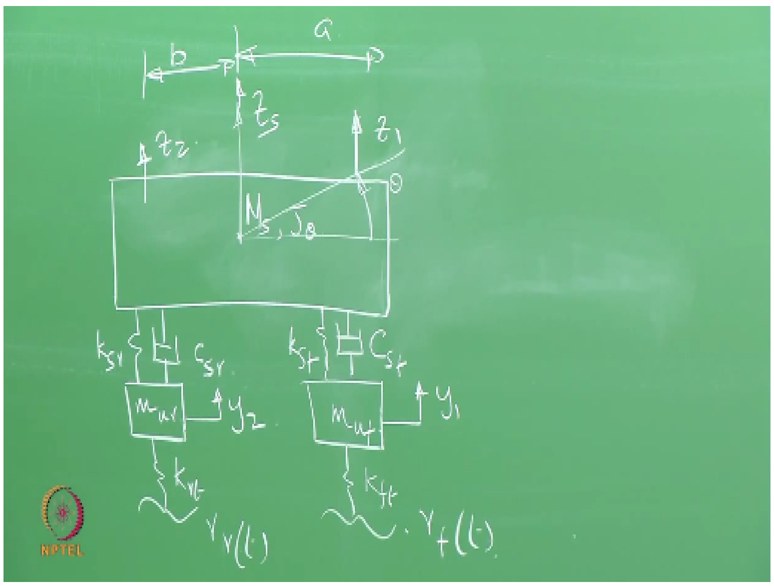


Let us go straight away to this model. There are a number of names to these models, we will know as we go along. What are the names to these models. We will look at what is called as half car and a quarter car model. This is a standard technique, you would have notice that in your control system course where we will going to replace the system with spring mass damper and maybe even in your vibration course.

So, that is our simple model. You would notice that tyres have been replaced with springs which I would call as the spring stiffness K front tyres and K rear tyres. Notice that we have not put a damper to the tyre, though it looks very attractive, but usually the damping effects are not very high and is neglected, okay and this is what is called as our unsprung mass, okay. So, we will call m, unsprung mass in the front and the unsprung mass in the rear.

These are the stiffnesses and the damping characteristics of the suspension system, so we will call that as Ksf, Csf, Ksr and Csr. That is the sprung mass and we call that as M sprung mass and that we also include a J theta. Notice that we are going to have theta which can be considered as one of the degrees of freedom. So, we are also putting J theta in our analysis. So, that you know theta can be considered in the analysis.

We will follow the same thing as we had done before that is this is distance A and that is the distance B from the front and the rear. The input to this is obviously the road input, okay. So, we will call that as a road in the front and road at the rear. What we have done is like we did compressing bicycle model and so on, what we have done is to compress that and put it like this, okay. So, in other words we are designing half the car.

People call this sometimes as half car model because the other half is compressed and put into one thing. So, what is possible to have here is the pitch mode. We are not considering role, in other words, what we consider in this model or the assumptions that we make in the model is that the right input, the right side of the tyre, input is the same as that of the left side of the tyre, both the inputs are the same or in other words the road is such that right and left side are the same here as well as here.

So, of course this is a question of time, that is the input into the model and we assume that there are displacement. Let us call this as y1 and y2, okay and let us just put the displacements here as Az1 and Az2 which can be expressed of course in terms of what I call as z or zs. In India, we have a tendency to call as zs. So, you know why we used to calling this as z because z is never used as z.

You know, if you got zebra z, so if you notice that z is always used as z, so that is the reason why usually people do not call this is as z but they call it as z. We are used to calling this as z. You know when I say z, there are problems for some of you to follow what I mean, so in anyway. So, that is the Az1 and Az2, okay.

So, now what am I going to do, very simple, nothing great. I am going to write down all the governing equilibrium equations, okay which are going to be a second order differential equation, that is all I am going to do, okay. In fact, you can write it down. There is no issue.

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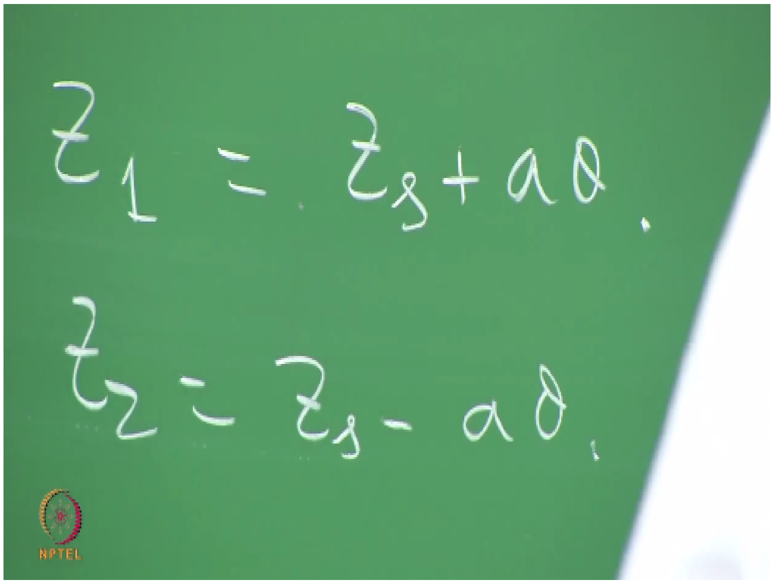


What we are going to do, of course I can write this in terms of z, s, and theta or in other words, it is possible to relate both by means of this relationship, okay or I can write it in terms of zs and theta or in terms of z1 and z2, okay, fine. Now, let me write down the governing equations and you will understand what are the degrees of the freedom, okay. So, I said that if I want I can change it, that is all I want to state, okay.

Fine, now let us consider each of these masses separately and write down the governing equations to it, okay and then of course these are the road input. I just want to give you a small background to this road input. There is a tendency for people to just measure the road profile, just measure the road profile. Road profile measurement projects which I think a couple of students have done before you, maybe 3-4 years ago.

So, road profile measurements are done using laser sensors and so on, and there is a tendency to just input that straightaway into this model, is that correct. **"Professor - student conversation starts"** Is that right. No. Why it is not correct. It can change. No, no that is correct but I am saying a particular road where you are doing measurements, okay. Measured this, I can do that using. Speed also (()) (10:16). Okay, that is very good.

We are going to consider the speed. Tyre may damp (()) (10:24). Correct, tyre acts as a filter, okay. **"Professor - student conversation ends."** Now because tyre that is what is called envelopment characteristics of the tyre. So, the tyre as it goes is not going to be like this but it is going to be like this and in other words it is not the true representation of the road. What the car sees is not the true representation of the road.

So, when I put this Rf and Rr, they are not road inputs which is usually called in many textbooks. It is not that just road profile but it is the profile which the car sees after this high frequency filter called tyre which filter's out not due to its stiffness characteristics, we are not talking about that, that I have already put it here but more due to the envelopment characteristics, so that what goes inside is what is felt.

If you go and sit in this tyre here in this region, what is it that you feel is what we call by Rf and Rr. So, there are models to understand and put that as an input, okay. This is a very important problem been tackled well in tyre models, okay. Envelopment characteristics as is called, if go and look at swift model, then you will see that there is an envelopment characteristic as a part of the model but that is very important.

So, whatever model you put in, it is very important that envelopment, you understand what is envelopment, envelopes okay that is what goes as an input, that becomes an input.

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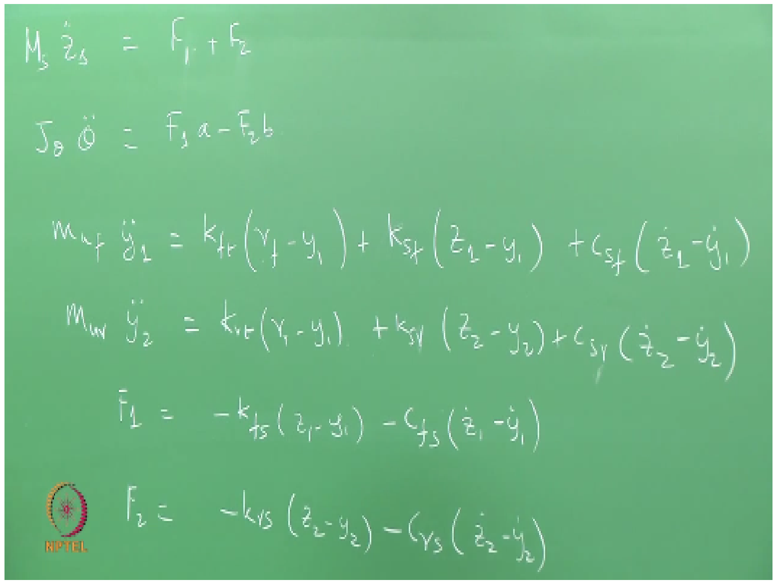

Let us now straightaway write down these equations. Now, so Ms into I would use that is equal to Ff+Fr which I have to be consistent that is way I have to refer to that notes because I do not want to change this. Let me call that as F1 and F2, okay. I have a tendency to change this, you know. I want to be careful, if you do not understand anything. So, let us call that as F1 and F2. So, F1 and F2 are the inputs from the front and the rear respectively.

So, what is the second equation, J theta (()) (12:49) moment equation J theta*theta double dot=moments F1*a-F2*b. So, the next equations 2 equations I am going to write for the unsprung mass, the front and the rear, okay. So, M unsprung mass, the front y1 double dot=so write down the force balance equations, okay Muf, right. So, Muf*y1 double dot, write down the force terms K front tyre multiplied by now that is the spring.

So, all of them are Rf, Rf-y1, all of them are function of time, so I am just removing this T+Ks in the front. So, there are 2 things here. So, y1 and the other side is Z1, okay. So, Z1 is written in terms of course, I already written that, right. So, similarly I can write down Mur*y2 double dot=Krt*Rr-y1+Ksf*z2-y2. I am not going to write down or derive every step because this is algebra. I told you already that you have to workout some of the steps that you have.

**"Professor - student conversation starts"** Yeah. (()) (15:45). Oh, sorry, right. (()) (15:59). What is happening to me, okay. So, good, I appreciably say that I just put it so that I want to know whether all of you are awake, okay. So, all of your are listening and you are understanding, that is very good, okay. **“Professor - student conversation ends.”**

So, what is F1 now, write down F1-Kfs*z1-y1-Cfs*z1 dot-y1 dot, okay and F2=-Krs*z2-y2-Crs, okay. Now, as I said I am not going to do the rearrangement of every step. Obviously, I am going to write this down. **"Professor - student conversation starts”** By the way, what are the degrees of freedom, how many degrees of freedom are there. 4. What are they, y1, y2, z1, z2, zs theta, okay. So, this is basically 4 degrees of freedom model. Let me prompt what I am going to do.

The first thing I am going to do is to cut this and make it into what is called as a quarter car model, okay. **“Professor - student conversation ends.”** I am going to compress that and put that as one quarter car model, okay. In fact, more interestingly what I am going to do is to cut this and consider the masses to be distributed to the front and the rear in the same ratio as we have done before and then consider this as 2 quarter car models, one to the front and one to the rear.

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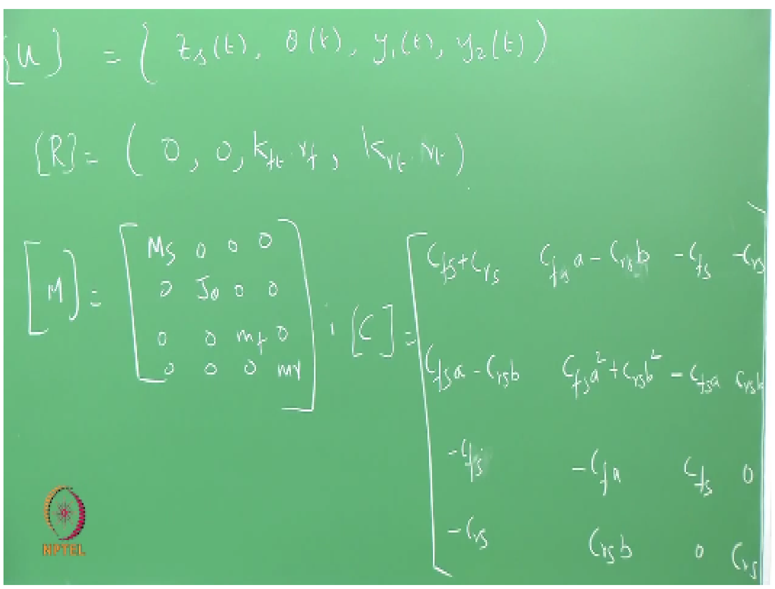

So, what I am essentially going to do is to rearrange this, okay. I am going to write down the results, of course U is equal to and R which is the load. Of course, M is quite straightforward, okay. So, that is quite straightforward you can see. What I am doing is very simple 4 equations, I am just re-writing it in a matrix form, okay. You know why I am doing it. It is quite straightforward. So, the next is the C matrix.

Note, that I am going to replace z1, z2 and all that. So, rearrangement gives me something like this Cfs+Crs Cfs a-Crs b-Cfs-Crs, okay. One of the things which I had pushed in without elaborating is that the spring stiffness is a constant. In other words, when I wrote down this equation, I push the spring as a linear spring which is definitely not the case with most of the suspension system. It is a hardening spring, okay.

So, the suspension spring is not going to be a linear spring but as I told you we want get what we call as first cut results as a reason why we have made these assumptions and that the dumping is characterized by a simple number and the spring stiffness is characterized by simple number, right and K matrix we will write down the K matrix. Here the results are very interesting.

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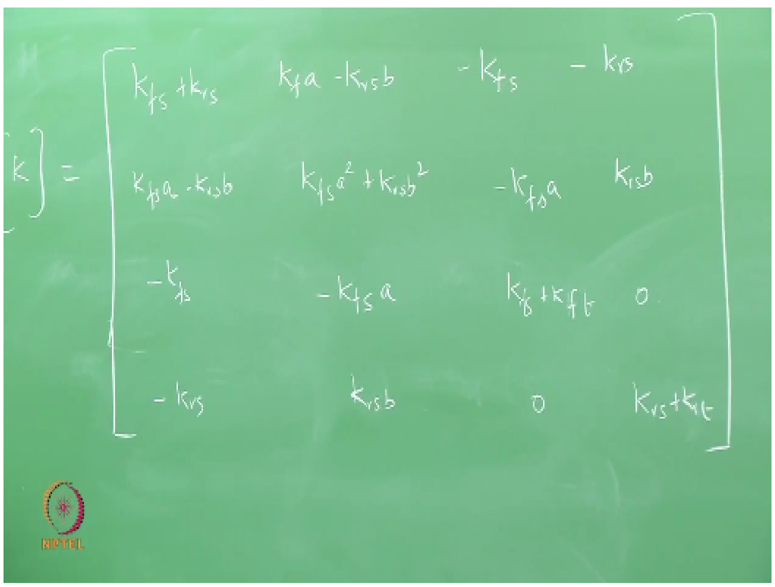

Unfortunately, we cannot jump to the results. So, you would see notice that it is very similar to C matrix because there is no C there for the tyre that is missing, the stiffness of the front tyre are the rear tyre, okay. We can solve this, not a big issue, okay. You have Matlab, okay. You know I give you the functions, you can solve it, but I am not going to do that, okay. As I said we want some results, so this is a complete half car model.

In fact just for the fun of it, write down equations for a full car model, okay and it is possible to solve some of the difficult and solve that as well, but nonetheless you write it down just to get a feel of how we are going to deal with it. In other words, if you go to a full car model, then you have a pitch, then you have a road, right the other direction. So, for the body, the Z, then pitch and the road, right, you can write it down.

Now, what I am going to do is to split this to get some very interesting results. I am going to split this into 2 quarter car models. Later, I am going to consider a similar version for half car models in order to understand more about this model. So, the first thing is that this what most of the companies do as a first cut. Why do they do it, immediately if you to look at suspension tuning, if I want to look at optimised suspension, then it becomes very simple to understand this, right.

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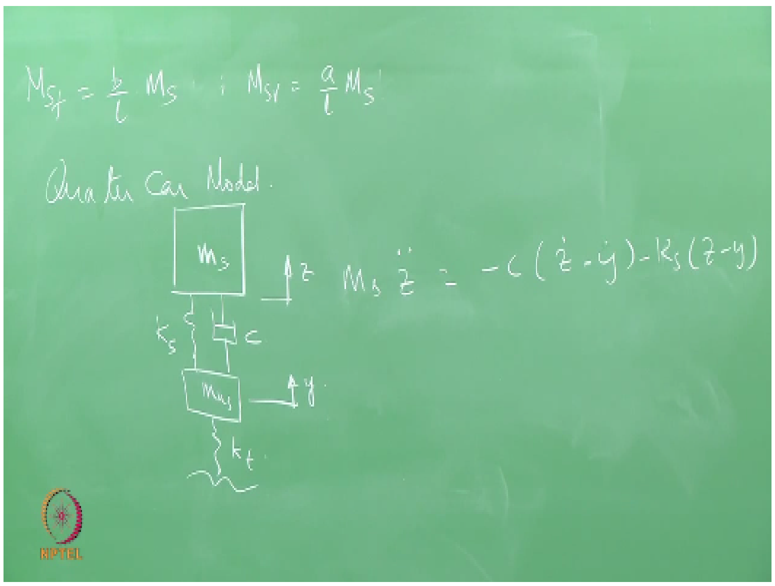


So, we use this for optimisation of the suspension system. So, the first thing I am going to do is to look at what is called as the quarter car model. As I told you before, I am going to re-distribute the mass, right and put that front mass Msf=, how do I put that, b/l*sprung mass and is equal to a/l*sprung mass. So, now the equations are the same, front and the rear. So, I am going to consider only one of them and put that Ms as ms and write down the equations.

So, how does it look like. As I said I am going to cut it here. So, the front and the rear will have the same equations, the only thing is you have to replace corresponding the Msk front, k rear and so on, right. So, we will write down one come thing, that is our sprung mass and then that is our suspension, that is our unsprung mass, that is my tyre, the tyre is on the road. So, that is my what is called as quarter car model.

So, let me write down that. As I told you this one we would call that as z and that as y-C*z dot-y dot. I want you to check this, is that correct. Of course, that is Ks, that is C and that is Kt. **"Professor - student conversation starts"** Is it correct or not, I know because I mean in one of the earlier courses people are confused between. See, it is very easy when I put a force. So, when I have 2 things which are moving, okay.

So, what happens when z is more than y, in other words this is positive, okay. What would happen. Let us just look at the spring. Z is more than y, so what will happen. Pull down. Pull down, okay. So, the force will be in the negative direction, but my m, s and z double dot is in the positive direction, so I have put a negative there, right. I know your question, solved. Any other question, okay. **"Professor - student conversation ends."**

If you want write it like this. If you want write it as Ms=force from the spring + force from the damper and then write down, right.

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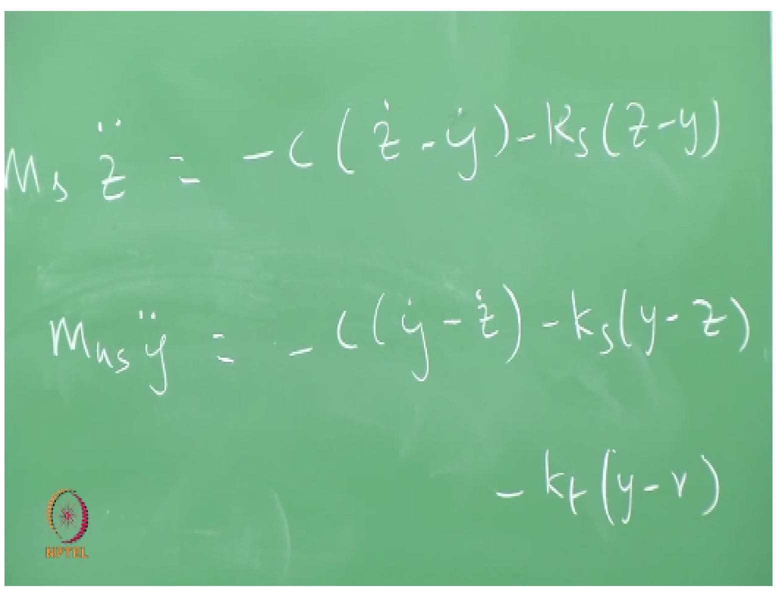


So, note that we are looking at 2 degrees of freedom, z and y. Of course, r is the road input. Note that all of them are functions of time, okay. Now, I have a strategy to look at it, why am I looking at this. One, I said that I want to optimise the suspension. So, in other words, you can look at this simply from point of view natural frequency, determinate the natural frequency, okay and we will see if we can optimise suspensions and so on, right.

In fact, I just want to state even before we go there. So, you will get in simple terms of what you have studied, you would see that there are 2 natural frequencies, one is called as the wheel hop frequency and the other is the body frequency, right. But more important thing also is that I am going to write down the behaviour or the ratio of y in terms of or y to r and z to r, okay, I am going to write that down.

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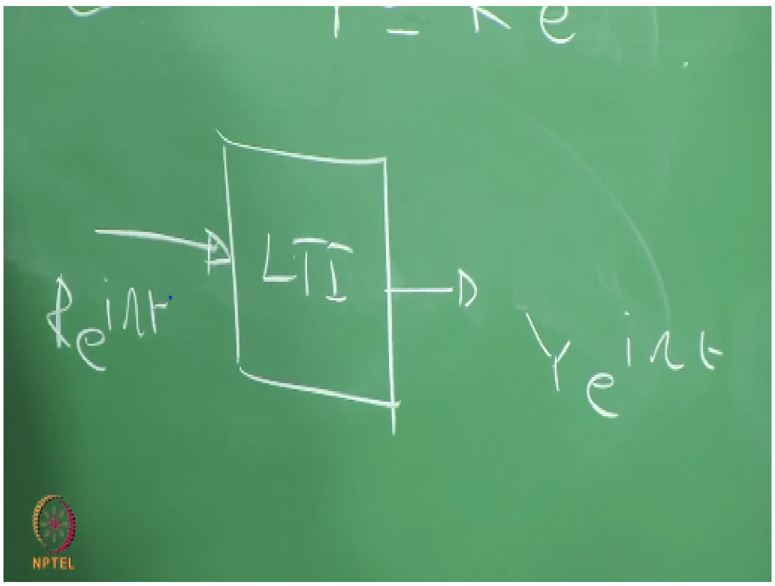


Now, I am going to assume that this r is a complex exponential function which I would write that as Re power I say omega T, okay as a complex exponential function. You would have studied this already in controls, okay. So, what is this system. This is what is called as the linear time linear time invariant system. Nothing is going to change, okay and that if your input is delayed, output is also correspondingly delayed and so on.

The beauty of this system is that if I now give a complex exponential function like Rr power I omega T, the output is also a complex exponential with the same frequency but a face lag or with a face, right. So, let us call that as, say for example this thing, let us call that ye power I omega T+phi which I would combine them together and write that as ye power I omega T. So, the y is actually a complex function takes into account the face as well, right.

Now, why I am doing this or what is the use of this or how do I use this property called linear time invariant function that the complex exponential becomes, how do we use it or why do we put it like this. Basically because whatever be the input in the time domain, you can split it into a complex exponentials or sum them up as complex exponentials. You had seen this for example in 4ier series where you had a periodic function.

The periodic function was expressed in terms of sin and cos which can be expressed also in terms of complex exponential, that is for a periodic function. You can also express, say for example, the road input as in terms of complex exponential, in terms of a number of frequencies using another technique which is very will used called 4ier transforms, okay and the way we are going to calculate that is what we are going to call that as Fast 4ier Transform, okay.

So, a 4ier transform converts whatever be the input into a number of complex exponentials and hence if I now get a solution for one complex exponential, I am using this time with (()) (36:16) principle, it is possible for me to get the output for a number of such complex exponentials. So, that is why I am interested in that complex exponential where y is also a complex function and so, I write it like this, right.

What that background, let me write down. I am going to, as I said, quickly run through because we are running out of time, we have a lot of things to cover. So, input and output are of the same form, substitute it. What do I do, put that in the governing equation, I am going to get omega square here and so on.

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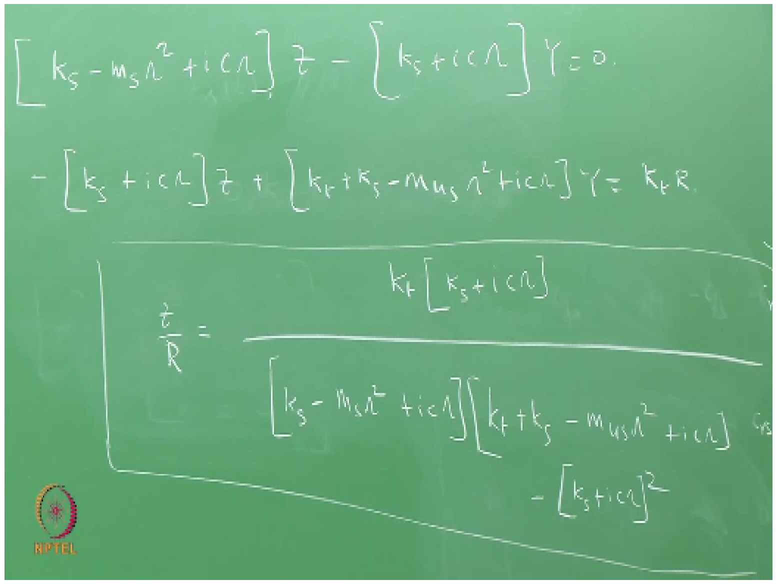


You do that, I am going to express this final form, you know, the 2 equations do that. Of course, r is real from which I am solving this and writing down z/r and y/right, okay. So, that is z/r.

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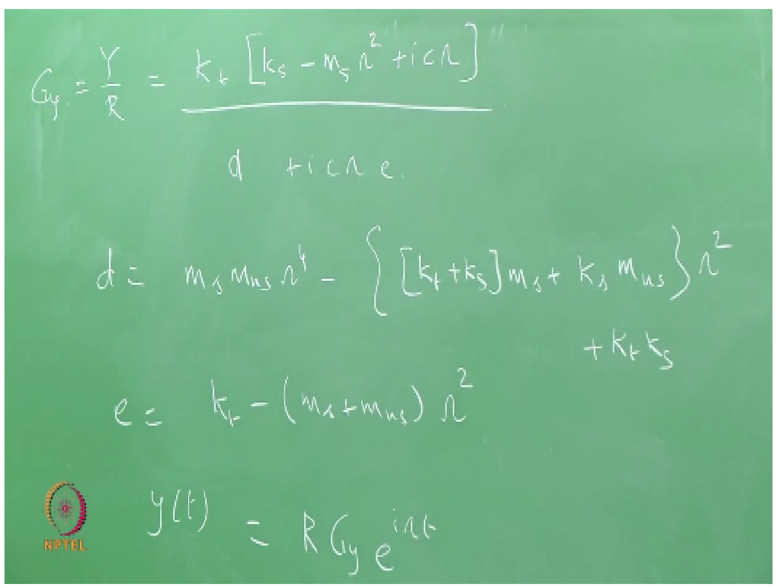


The same thing as denominator, let me write that down as something like this d+IC omega e where d is equal to expanding that. Though this looks formidable, there is nothing that is difficult in this. Because of the number of terms, it looks very complex, okay. So, as is usually the case, I am going to simplify it later, okay. So, of course, if you want a solution look at that everything is known, it is straightforward, there is nothing very difficult.

Notice also that both y and z are complex, okay. We will do the interpretation in the next class, but let us first understand what it is. So, in other words, the very first thing is that given the road input as a complex exponential, I can find out what would be the displacement at the unsprung mass position. What is that. So, let me call that as Gz. Let us call as Gy. You know why I am calling that a Gz and Gy, okay.

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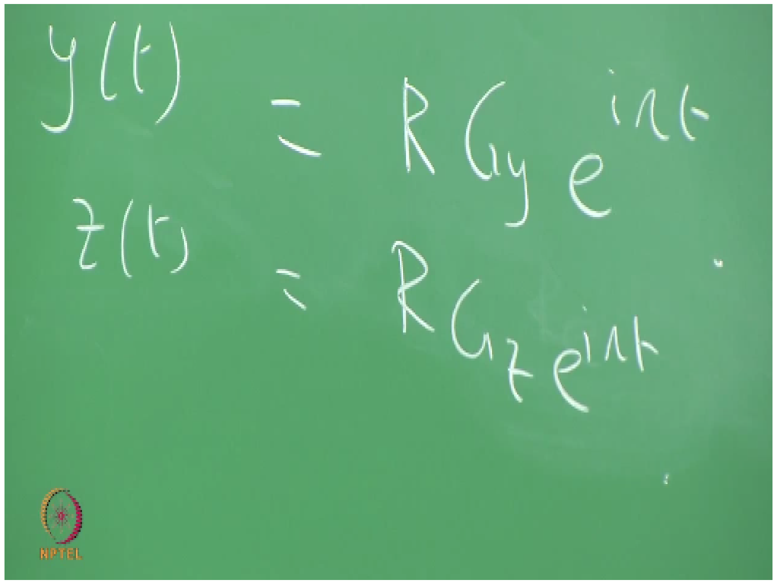


So, y is given as r*Gy*e of I omega T. In same fashion, you can write down z, okay. Suppose I give you this and I want you to do a test. How would you do, practically I want you to measure this, before we go further. This is too much of math, too much of equations, okay. Tell me what are you trying to do, what is that ultimately, I am interested in, okay, right. That is the first question.

I am giving you a car and I want you to measure, first thing is I do not even know you talked about road envelopment characteristics. I do not even know how I get this road envelopment characteristics and so on, okay. Give me a car that we first do some experiments to understand what is happening, first question.

**"Professor - student conversation starts"** So, what are those things that you have as an equipment as a sensor in order to do an experiment. (()) (43:21). You have an accelerometer, right, okay, very good. All of you have used the accelerometers. So, the first thing you would do is yes I have an accelerometer, a data acquisition system, right. **"Professor - student**

**conversation ends."**

So, what do I do. I put an accelerometer here and I put an accelerometer here and then start recording the acceleration levels. So, now acceleration levels are obviously can be determined this by this, y dot=R Gy e power I omega T and I square that is in other words from here it is minus of this quality minus of that quantity. So, once you measure this. Suppose I want to know how much of road input has gone in, I measured it.

In other words, I know what this quantity is, right. Let us say that I do Fft, I do all those things. Let us take one frequency. I know what is gone in here, so I can easily find out what is because I know I measured it. I know the other quantities, I can find out what this my actual r. So, this is for a particular that is a constant and so I can use that for any other vehicle, I can use r*e power I omega T as an input for the road, right.

So, the first thing is that from here I am interested to determine the acceleration and I am going to look at how my tyre is filtering and also more importantly how my suspension is filtering all that input, right. Why I am interested in acceleration, because you would later see that acceleration is the deciding factor for right comfort. We would see that the tolerance level for a human being would be expressed in terms of accelerations.

So, that you can do it by measurement as well as you can do it by a quarter car model. We are going give a small twist to it. Is it exactly acceleration, is it something else. We will come to that, we will give a small twist to it. But nevertheless, from here what we are interested in is the acceleration, right.

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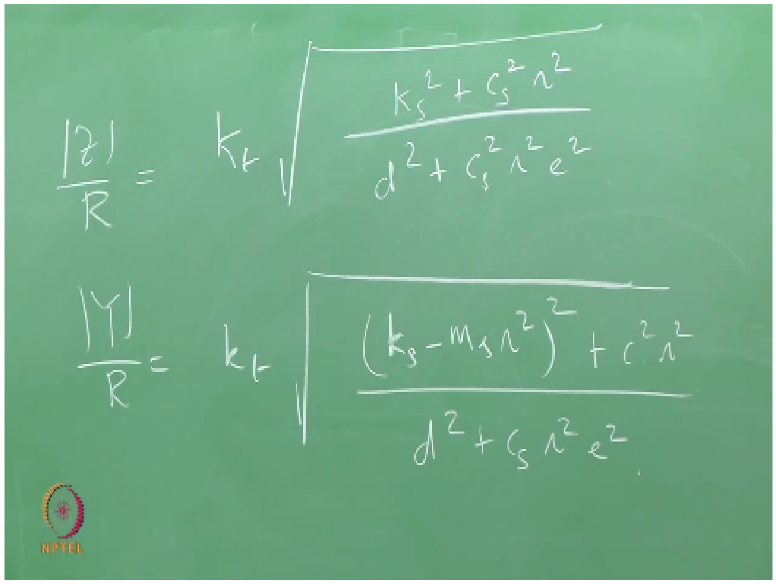

If you want the magnitudes of z and r, we will finish this class with that magnitudes of z and r. Of course, you know how to get the magnitude, get the root of real+imaginary. So, that is the 2 vertical magnitudes. The magnitude of acceleration for these 2 are given by omega square

multiplied by this omega square multiplied by that would give you the 2 magnitude of acceleration levels.

**"Professor - student conversation starts"** Clear, any questions here. How is it like (()) (47:26) will be different for different speed and you also. Wait, wait, wait. You are absolutely right. You know, the problem in this is that I have to derive it. I have to show you the equation and then and then come to interpretation. I know slightly impatient of why are you doing this, right. That is why I immediately jumped and talked about acceleration.

You are absolutely right that this is what we are talking about road profile is a spatial in time domain, okay. **"Professor - student conversation ends"**. As a matter of distance, spatial domain , right. It is a matter of distance, actual profile. When I put this as an input, I am converting the spatial domain into time domain. So, what is that linking factor, speed. So, what is that you get you know, with the last statement we finish this class, okay.

So, what is that you get. You see if I go in the same road, very important point brought out, if you go in the same road at different speeds, I am not going to get obviously the same output, very simple, you see that everyday, right. At particular speeds, you would see that suddenly your car starts jumping and after sometime it will calm down, okay. So, when is it going to jump, that is an important question, okay. That is what we are going to see, right.

So, in other words, for the same road speed give you different inputs with respect to time, absolutely correct. How is it, we see that in the next class. Does that answer your question. So the speeds are important. I have not yet tied this up. Speeds are important, how, put that down. You know the velocity in terms of meters per second, you know spatial distribution in terms of distances as meters, connect them together and we will see that in the next class.