Vehicle Dynamics
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Lecture - 03
Vehicle Load Distribution? Acceleration and Braking

Let us continue what we were discussing in the last class. We were looking at the forces that act on a vehicle as a vehicle goes over a ramp or a slope. What are the key things that we discussed in the last class was about the rolling resistance of the tire.

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There are number of questions after the class I would like to explain that again so that you are clear about how we have converted the phenomena of rolling resistance into a force. In fact, this is the key question that is asked after the class. Let me explain that again. We talked about equilibrium with the inflation pressure. We will come to that later. The only point which I wanted to make is that every part of the tire has to be under equilibrium with the load contact pressure that is acting as well as the inflation pressure.

So at the right of the center when there is nothing else is happening, then the contact pressure should be equilibrated by the inflation pressure. We will talk about that when we come to the tire. I just wanted to make a passing comment. We will explain it later.

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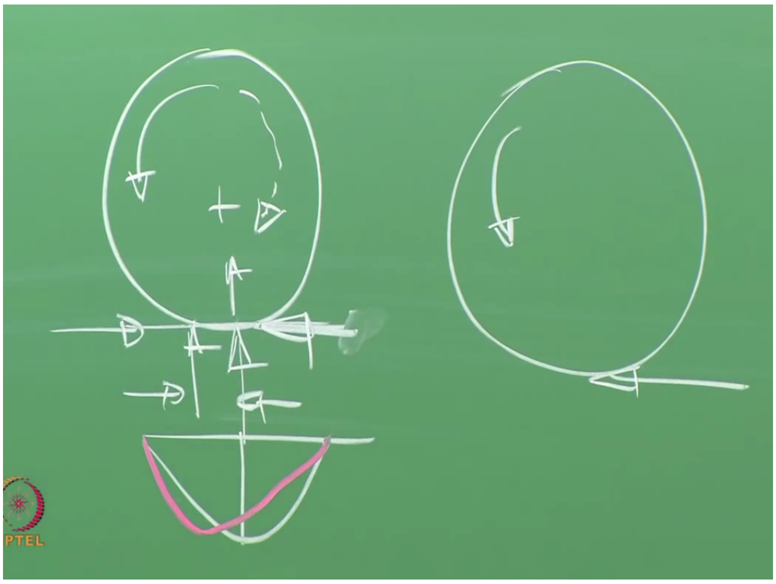

Now let us come back to this topic of rolling resistance. We said that the rolling resistance is due to viscoelasticity right.

And we said that that also results in an unsymmetric pressure distribution right. We said that when the tire is stationary standing then okay it has to be a symmetric pressure distribution where a number of tread blocks are involved and when it is rolling okay when we follow a tread block, the pressure distribution due to a tread block will not be symmetric because the loading and the unloading curves of a viscoelastic materials are different.

And so it has to be unsymmetric and we said loading curves being higher than the unloading curves. We said it has to be like this and that this results in an unsymmetric force. The force that is exerted by the ground on to the wheel okay that force is not going to be symmetric right at the center, but will be away from the center. We will call this later you know we will call this we will give specific names to these things. We will not do it right now.

But let us understand first that there is a shift okay. There is a tread. Now this force is the reality because of viscoelasticity. So if I now want to shift the force to another place right at the center, I will have to do it with the force okay as well as the moment that is acting right. So if this is our direction of rotation and hence the driving torque's direction okay. Then this force would give rise to a torque, which opposes this driving torque.

And so will be in that direction okay. It opposes the driving torque. The question that was asked is how did I now replace this force by this force like this okay? And before that how did I replace the driving torque force okay by a force at the ground? This was the question that is asked. Please I want to take you back to the free body diagrams, very simple things so we will explain this with a very simple example okay.

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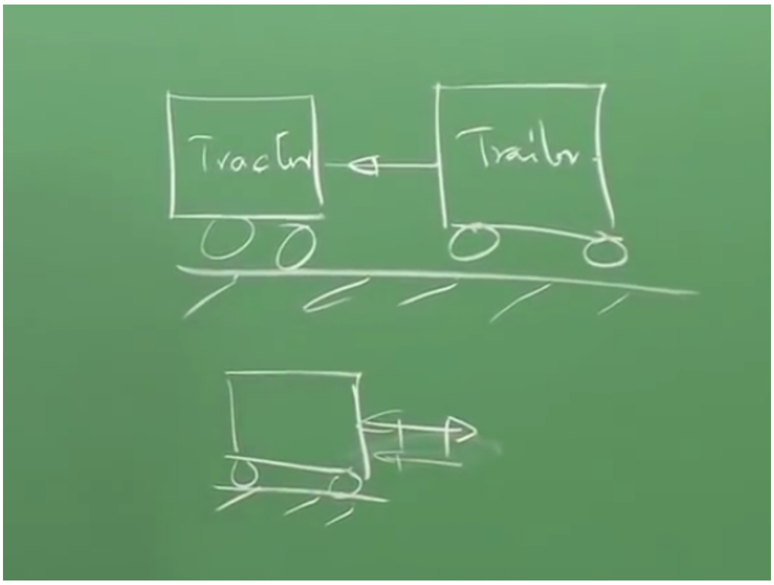

So that you understand small niceties of drawing a free body diagram. Suppose let us take that I have a tractor okay pulling a trailer, a horse drawing a cart, which is a very early example okay. Now there is a link okay and now this tractor is pulling this trailer okay, which means that this tractor applies a force okay in this direction to the trailer. Now if you want to draw the free body diagram of this tractor what will you do?

Apart from other forces we will just neglect the other forces. If I have to draw a free body diagram what will be the direction of the force? It will be in this direction okay. So this will be the direction in which I will draw this free body diagram. So in other words I will look at the action of the trailer onto the tractor okay. So what does the trailer do to the tractor? This is what I will look at and apply this.

I will neither apply 2 forces in this direction that means it cancels that is ridiculous okay and I would not apply a force in this direction then in that case it looks like trailer is pushing it okay there is no resistance. So I will apply it obviously only in this direction. So that is the reaction that you apply.

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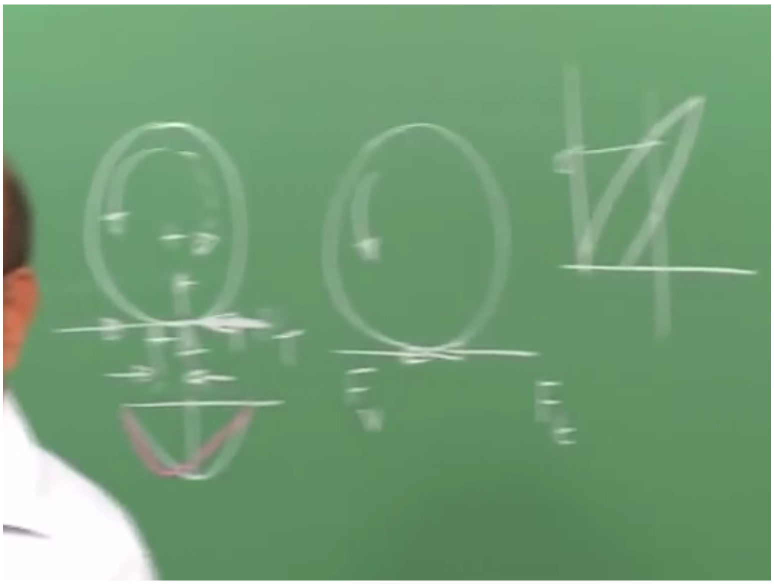

In the same fashion, when the tire rolls okay it actually pushes the ground you can imagine that it pushes the ground you would have seen this when there is water over the sand, which pushes the ground okay and so the force that it exerts on the ground will be in this direction okay like the force that is exerted here is in this direction. So when I want to put the free body diagram now with the force that has to be acting on the tire okay, which I want to represent now.

In other words, the effect of road like we have effect of trailer on tractor okay effect of road onto the vehicle I have to put a force in the opposite direction. So hence this force now becomes this clear. It forces the force becomes like that. So this is now the traction force and that is due to the friction okay. We will see how friction holds this right. That is the first one.

Now since the traction force is in this direction, I have substituted it with the force like this okay because of the effect of the ground onto the wheel I have substituted it like that. I do the same thing for rolling resistance torque, which is produced by this force okay. So when I shifted it here I had a force in this direction and I also had that moment and now that is very similar to the traction force.

So I replace it with another force that is acting like this okay so it is equivalent know same thing same torque that is the F rolling resistance and that is the F traction clear. So that is why we have 2 of them okay the front traction force and the rolling resistance force which opposes the motion clear. **"Professor - student conversation starts"** Any questions? Yes, so what is friction? What is friction coefficient? We will come to that in a minute.

How does the friction act? We will come to that in a minute. Let us understand the forces that are acting okay so now that is the rolling resistance force right okay. Why did not there be a shift? Yes, that is what we explained last class that we will have a shift because of the viscoelastic effect and that the loading you know when tread block goes through loading and unloading then the path followed are different okay.

One is loading and another unloading so for the same displacements okay or the same strain the forces that are given sigma are different okay and hence there cannot be the same for the same whether it is loading or unloading will decide whether the stresses are higher or lower because of its forces are higher or lower and that is the reason why it cannot be the same so that is why there is a shift okay **"Professor - student conversation ends"**

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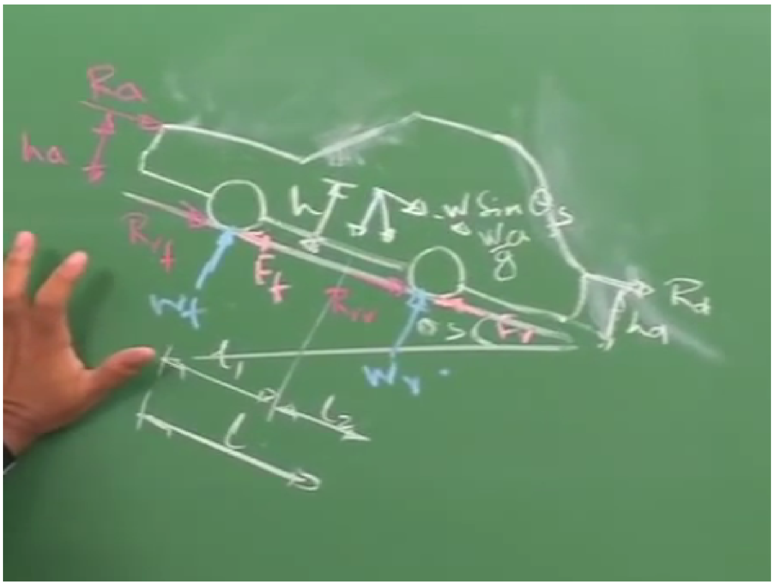

Now having come to this we will go back to our equilibrium equations and write down the equilibrium equations for these things. **"Professor - student conversation starts"** Sir what decides the direction when you draw that rolling resistance component? Yes, yes so what decides what is this one so what is this force, what is the direction of this force? What is the direction of this force okay? That is the question.

Obviously when I am giving a driving torque, the force is in this direction. If I am giving a braking torque okay, it will be in the opposite direction right obviously that is the thing. On other hand, rolling resistance depends upon the direction of rolling.

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I am rolling it like this. The rolling resistance does not care whether you are actually accelerating, decelerating or doing neither of the 2 or either of the 2 and travelling at the same velocity, it does not matter to it as long as there is compression and relaxation okay. **“Professor - student conversation ends”** We are going into the viscoelastic effect and that will give the rolling resistance right.

In either case, you will have that force here. So interestingly what really happens which we pointed out in the last class that the rolling resistance is something like braking and so helps braking when the vehicle brakes and it decelerates the accelerating vehicle on other words it is let say shall we say a bad force is acting, either way since it opposes the motion okay we are to compensate for this energy lost from the engine and so it is a gas-guzzler.

It consumes energy, so it is a gas-guzzler. So it is a very, very important force now especially if you look at passenger car in commercial vehicles especially in passenger car today everyone wants very, very low rolling resistance okay and when rolling resistance have to be low you also get into one more problem because I said that it acts in the same direction of the breaking force.

So the breaking force takes a beating, total force takes a beating okay. So you have to be careful. In other words, you will see later that the tire itself the tire design itself is like a spider diagram. You cannot improve everything. When we improve something that has to be a sacrifice on something else right. So we will continue now. We will write down the equations here.

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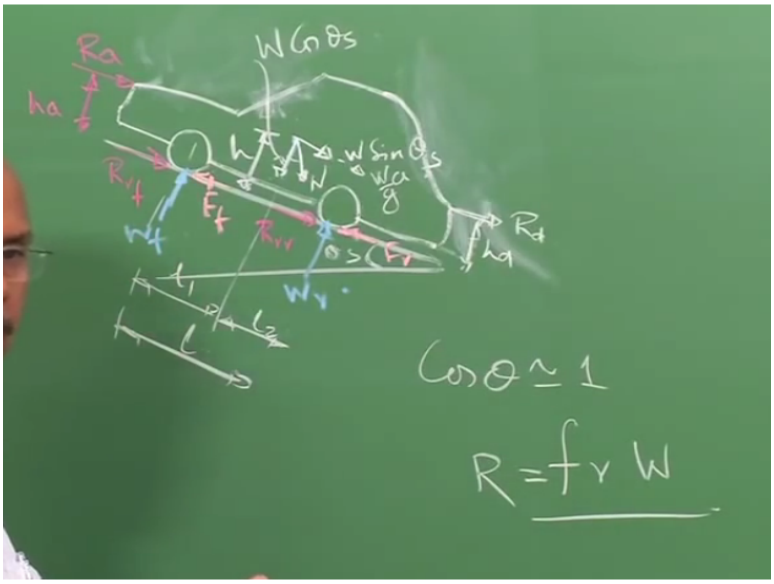

Note that I am up the hill so I have W, this is W that is acting and that is the theta s which is the slope okay so I resolve that so that becomes this force here it becomes W cos theta s and that becomes W sin theta s and so on. What are the assumptions that we have made you know obviously is that cos theta is approximately = 1 in order to write this. Strictly speaking okay, I have to now resolve it and put it.

But I am going to write that in other words I said that I will replace the rolling resistance force by means of a rolling coefficient or rolling resistance coefficient Fr*W. Actually, the W has to be in this direction and so I have to actually resolve it and put it as W cos theta s okay, but I am assuming there that theta is very small so cos theta=1 so I am just replacing this Rrf as Fr*Wf and so on okay.

Now again the rolling resistance coefficient we are assuming that is the same is the front and rear okay. These are small assumptions now we will write down the equilibrium equations.
"Professor - student conversation starts" Yes, this is normal force of course exactly.
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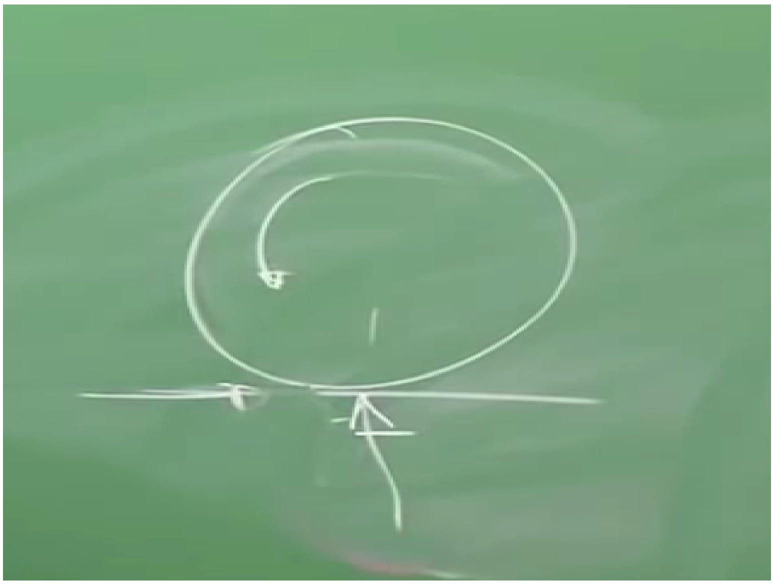

So that is what I just pointed out that we had a force, which is away when I want to now shift it to the center then we will have a force, normal force will still be there plus a moment and the moment becomes the horizontal force and the normal force is still there so that is the reaction force has to be there okay that is the reaction force. **"Professor - student conversation ends"**

Let us continue now let us write down 2 things we are going to do, 1 is of course the well-known F=ma okay. We will write down F=ma equation and the other one, which we are going to write down that is the first set of equations.

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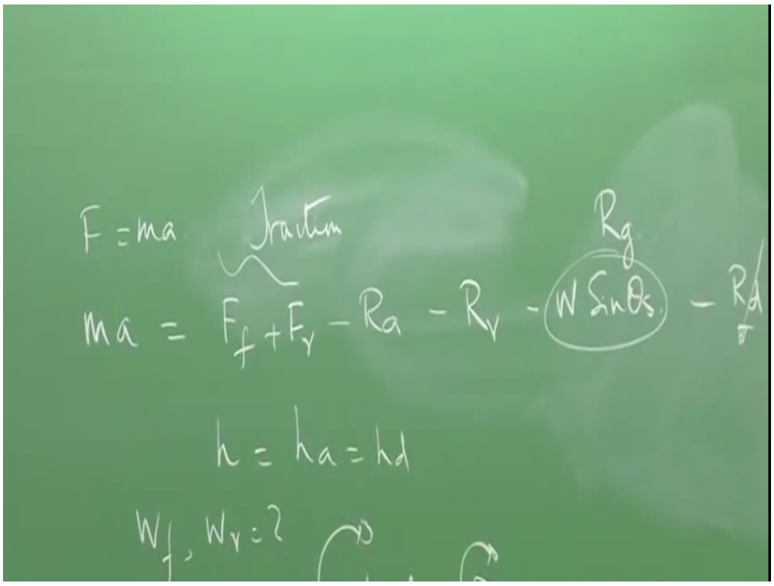

So let me write the other way ma=the forces that are acting, I said that this will be the x direction so we are writing this F=ma okay in the x direction. So what are the forces that are acting? The 2 forces that are going to aid the vehicle to accelerate or the traction forces Ff and Fr. So these are the traction forces and then the forces that are going to oppose the motion are the aerodynamic forces.

Aerodynamic forces as we are going to see now later is that it is proportional to v squared velocity squared. So it is a parabolic distribution okay. So at higher speeds that going to be important. So there is a drag coefficient Cd and you are going to look at the projected area a and you are going to look at v squared okay. So half (()) (16:32) Cd a v squared. We will see that bit later.

So we are now dumping everything as Ra. Then I have the rolling resistance force front and rear I am combining them and writing them as the rolling resistance force. So what is the other force that is acting against it? It is W sin theta s which we will call as a gravitational force, resistance due to gravitational force, you can call that as Rg if you want right and the other force which opposes is the drawbar pull.

We will neglect that we can add it if you want okay. Later we will neglect it okay. Drawbar pull is due to the force is applied if you have a trailer attached to it and so that is the drawbar pull. The next assumption which we are going to make is that h which is the center of gravity height=ha=hd. It is quite valid for a passenger car. So we are going to make an assumption like that right.

Now my next job is to find out Wf and Wr and Wf and Wr are determined by taking moments above that point A and this point B. **"Professor - student conversation starts."** What is that Rd? That is the drawbar pull. In other words, if you have trailer attached to it okay then that is the force so I am neglecting that. **"Professor - student conversation ends"** So how do you calculate Wf and Wr?

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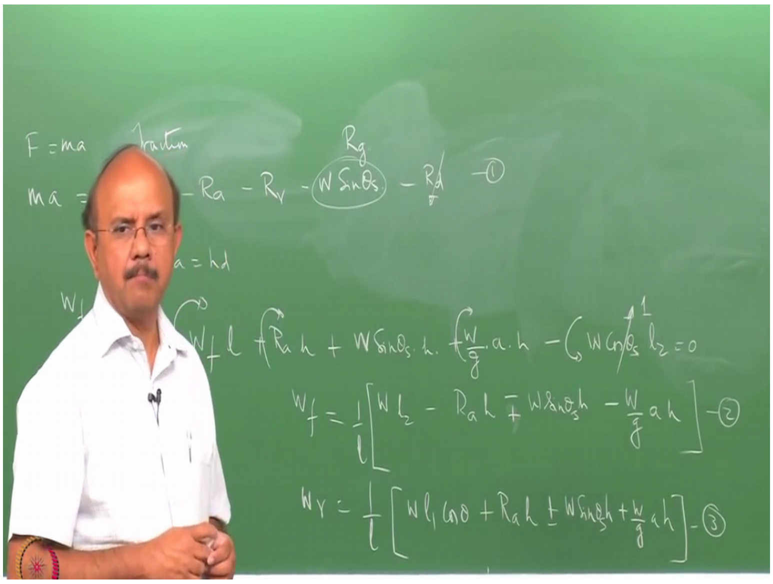

Just take the moment about A okay. Let us get that, I hope we do not we miss out anything so Wf let us look at this Wf*l okay in the clockwise direction okay+Ra*hy+that is also in the clockwise direction look at the direction if I make a mistake so it is up to you to point out okay (()) (19:11) okay. So then what are the other forces? Rg, W sin theta that is also in the clockwise direction*h.

Now I am going to replace this ma by means of that is this force by means of a d'Alembert's force, which is W/g*a right. So when I replace that then that is also in the clockwise direction so W/g*a*h. I also have this force which is W cost theta okay that force and that force acts in the anticlockwise direction. So I got to write that as because it is in the anticlockwise direction write this as W cos theta s right*l2=0.

Rearrange it okay, but only comment, which I am going to make and which will be valid for this class and next class is that you have to be careful with the sign here it would vary whether you are going uphill or downhill okay. So you have to be careful in that so right now we are going uphill okay and so it opposes so it is the same direction as that of Ra okay. If it is downhill, the direction will be different okay will be opposite to Ra.

So good thing to do is to look at Ra and where we stand and what is the direction right. So write down other thing, which we said is that we will make this to be equal to 1 sin theta will retain it okay so that we will get W1/l*W l2-the rest of the quantity why not you write that okay-Ra*h-W sin theta*h-W/g a*h okay. So I am just going to put - or + here to take into account whether it is going uphill or downhill right.

Write down for now Wr. I will give a minute. Write it down for Wr. How will it be? Just take the moment about B now okay. 1/l will have Wl1*cos theta Ra will become now +Ra*h so all the directions will be different+-W sin theta h+W/g*a*h. So let me call this equation as 1, that equation is 2, this is 3. The simple equations nothing in it, all of you can derive it very easily.

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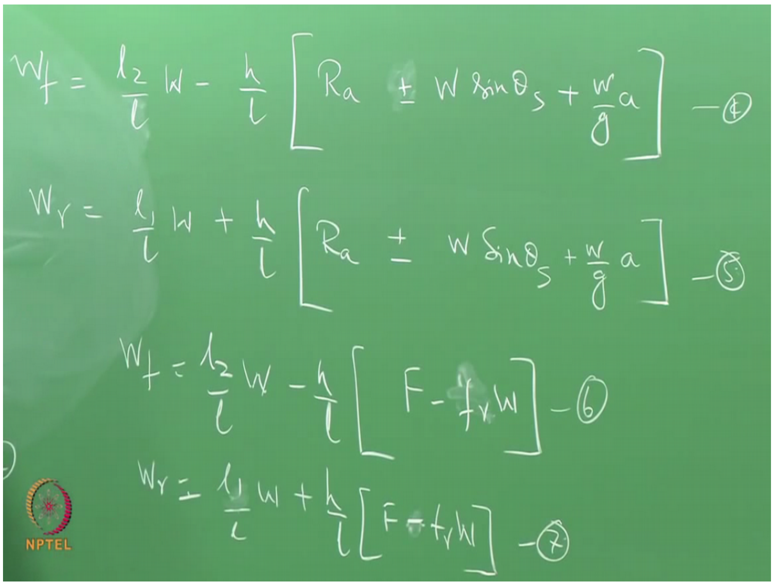

Okay let us rearrange this equation in a fashion that will be easier for us to interpret okay. So I will write down that as Wf=I will take the first term l2/l okay the first term so I will put a -sign there -h/l*let me write down the rest of them Ra+so how should it be just the same thing, but so that the sum has to be we will consistently use just + or – okay so that you will know when uphill, downhill you will accordingly you know you will do that.

So that -++- there will be a confusion. You can very easily find out which is + which is – so I will consistently use +-, according to the situation you will put + and – okay. Sin theta s+W/g*a right okay. Now let me call this as 4 and 5. I will simplify the 4 and 5 equation by using equation 1. Substitute that from first equation so that Wf=l2/l*w-h/l when I want to substitute that so that I have F okay and when I substitute that what will be there?

For Ra, W and you know Rd of course we have made it 0 and Rg so this will be F so what will that be? When I substitute there in that expression, I will have to bring all those rest of it the 3 terms to the right hand side okay. So F-Rr right which I will replace by Fr*W.

Obviously, Wr will be l1/l W+h/l*F-. Obviously, when I add these 2 equations or these 2 equations 6 and 7 or 5 and 6.

I am happy that you very active, lunch time has not dampened your enthusiasm, I am very happy. So that is this l1. So now obvious that when I sum them up Wf+Wl has to be W that is what has happened here. So you see a -sign here and a +sign here, which we experience every day when you do an acceleration that your weight actually goes back or there is a redistribution of weights okay in the vehicle.

And so the front wheel loses some of the reactions and the rear gains this reaction okay. The sum of them=0 okay. Simple I do not think there will be any questions on this. We will go to the next step. Now yes we have written acceleration you see in every advertisement of cars that 0 to 60 in 6 seconds or 0 to 90 in 8 seconds and so on and so forth okay. So what are the limitations of this driving force Rf okay?

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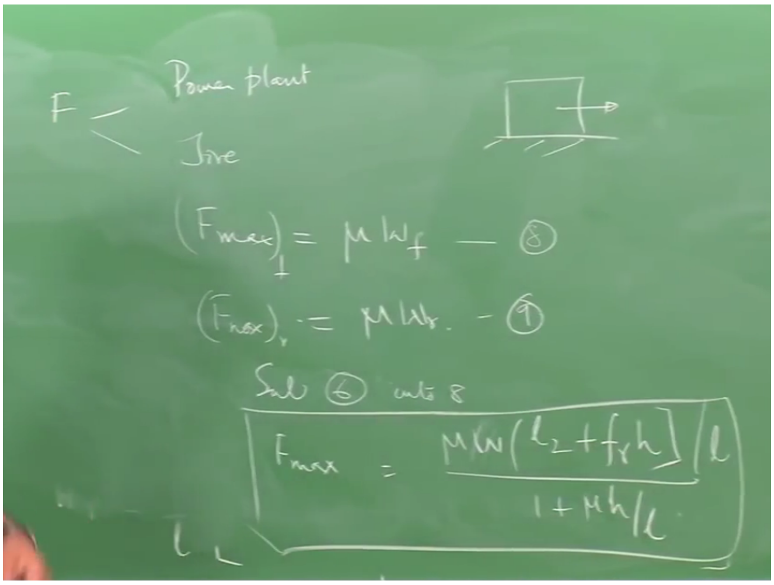

Can I have unlimited F okay that is our first question or is it limited or is that a F max? Of course, the tractive force as it is called is controlled by 2 things, one is the ability of your power plant or the engine to generate the kind of torque okay, which is applicable or which is available at the wheel that is the first thing, maybe your car would not be able to develop that is the first thing, but more important thing is the ability of the tire to take that force okay.

The Ff without what would happen if it is more okay, you would have seen it that for example if it is a wet road or an ice when you have traction okay, the wheel starts rotating, slips okay we have to be very careful in using these terms slip has another meaning as far as the tire is concerned. Right now, we say that we use an English word called slip, but we will then refine it when we study okay tires right.

So the ability of the tire to support that traction force or in other words the tire without as you said slipping. So tire is the next reason for it. So we can write that the F max okay in an analogy to a Coulomb friction. We can write this to be mu*W, analogy to Coulomb friction we can write it as mu*W. Is it Coulomb friction that acts? Is this mu very similar to what I know okay?

Some box sliding okay I had a friction coefficient mu acting okay. These debates are very important and interesting and we will go into those debates when we look at tire mechanics okay. So when I say mu*W, mu is a sort of a dumped quantity okay. So mu is equivalent I would say equivalent Coulomb friction okay. So that law brings us to the difference between a front wheel drive and rear wheel drive.

So obviously when it is a front wheel drive okay or both it can also be 4 wheel drive. Let us now look at only the front wheel and the rear wheel drive okay. So mu=Wf, which is the front wheel drive, maximum force and if it happens to be a rear wheel drive then mu Wr will be the maximum force. So what I now going to do? I am going to do is to now substitute for F it is a front wheel drive or a rear wheel drive okay.

Mu Wf=F max okay I will substitute for F there here okay if it is a front wheel or the rear wheel drive and then rearrange these quantities. So that I will know what is the F max or a/g. Very simple now, the first step is I will just substitute in this equation. Let me call that as 8 and 9 okay, substitute equation 6 there and I will rearrange the terms for F max okay because as an F there rearrange the term F max.

So I will just write down what is F max, you can do that as an substituting okay or I will say using 6 and 7 into 8 and 9 if you want so I can say F max maybe you can quickly do that okay this for a front wheel drive=mu W*l2 is there. So W I have taken it out so l2+Fr*h okay there was a l there divided by W since I had rearranged the terms I will get divided by 1+mu h/l.

So this is the maximum force that is developed okay if it happens to be a front wheel drive. If it happens to be a rear wheel drive, I will do the same thing with the second equation and I will get something like this for the rear wheel drive.

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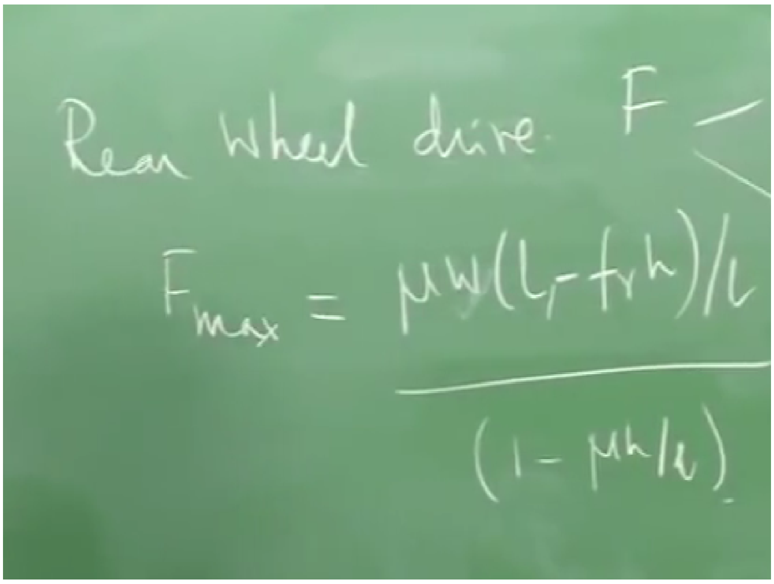

When maximum F max=mu W*l1-Fr*h/l whole thing/1-mu h/l okay verify this I hope this is correct. by Fr*w so the W is what we are taking it out so that is why we get a W here okay if I put RR then it would not come right okay right that is l1. **"Professor - student conversation starts."** Please note we have substitute Rr by Fr*W so the W is what we are taking it out so that is why we get a W here. If I put Rr, then it would not come right okay right that is l1. **"Professor - student conversation ends"**

Now this is the front wheel and the rear wheel drive, the maximum force that is developed okay. So beyond this the wheel will not develop a force okay and we will not be able to reach the kind of deceleration that we want okay. Now what we are going to do is to change track and look at braking okay. So this is the maximum force and braking becomes very important and we will change it and look at braking.

The forces that are acting all other forces are the same for braking except that there is not going to be a traction force, but there is going to be a braking force. So the braking force will be in the opposite direction.

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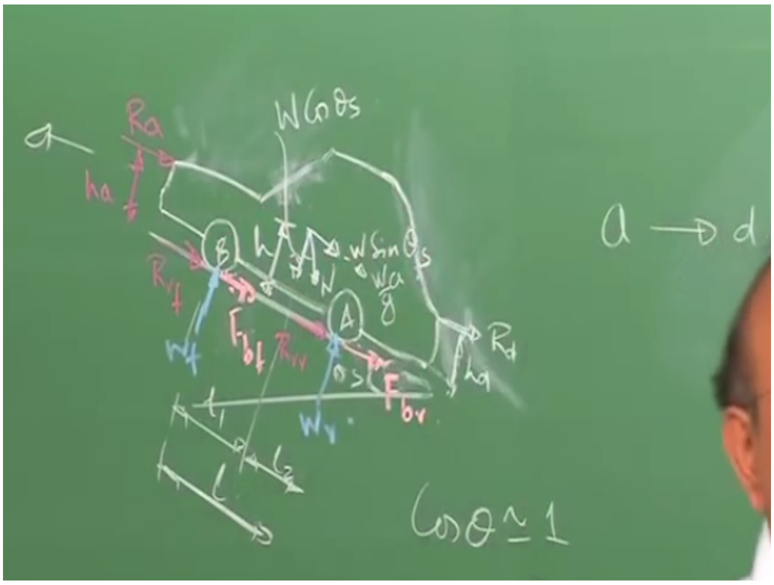

So that will be the braking force Fb front and that will be the braking force Fb rear. Is that the only difference? One more difference is that we are going to decelerate okay. So I will replace though it is not the right thing to do, it is not theoretically very correct I should have a and put it as –a that is what I should be doing, but just for ease of understanding every time you have to look at a then you have to say that actually I am in –a when I am braking, +a when I am accelerating and so on.

I will replace –a by d okay. So I would replace a with d where d is nothing, but deceleration that makes my job much simpler. So when I am decelerating what happens to my d'Alembert's force then it will be in the switch. This will switch right. Now let us write down same thing I am going to follow the same procedure nothing very difficult okay. So let us write down the forces.

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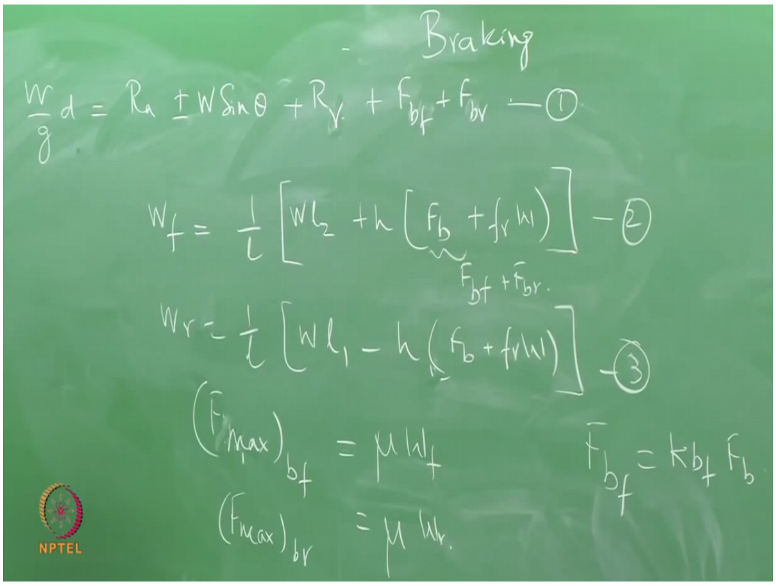

I will do that in a minute, now you can very easily look at this. The only thing I am going to write the first equation as W/g*d I will write okay=rolling resistance force Ra then what is that it is supposing I will put this as +or-W sin theta okay you know the physics okay. So that would be the second opposing force and rolling resistance force is now opposing the motion and of course I have the braking force the front braking force+braking force of the rear.

Yes I am neglecting the drawbar pull and all this because I did not want to add some more on to this. **"Professor - student conversation starts"** Sir at least positive here we are still accelerating. No, that is the accelerating direction I am now decelerating okay I am decelerating so I am braking so I assume that I am decelerating, assume in sense that I am of course decelerating okay.

So the deceleration okay is written like this actually how should I write this I should have written like this W/g*a=-Ra-Rr and so on okay. So what I have done is actually I should have written it as when I write F=ma if I write the directions here I am explicitly writing the directions here okay. Obviously a will become negative okay hence I am replacing that negative a with d.

So in other words, I am replacing all the negative forces by +forces okay just to understand this that is all. No, please not that –a is what I am replacing with d. I said I would not use the symbol a, but I will use it d okay d=-a that is what I said okay. If you want –a=d, the symbol d indicates deceleration obviously it is –a. There is no questions about it. **"Professor - student conversation ends"**

Do the same thing, take the a moment about b in order to find out the Wf and Wr okay. I am not going into those details again. I think I hope I am sure all of you can do that, it is not a problem. So I will only write the final result for Wf okay and this will be 1/l*what will be the case W*l2 -or+it has to be +because the force transfer will be such that, there will be more load to the front when you decelerate.

So this will be +h*Fb, I am putting this as a total braking force +Fr*W right. So once I write this you can write Wr okay. So I am skipping steps here. So this is on braking and so I am calling that as 1. I will call this as 2, I will call this as 3. Strictly speaking, I have to split this. I am going to do that as Fb front+Fb rear.

The maximum braking force, which I would call as Fbf max or F max braking force in the front and the maximum braking force at the rear are similarly controlled by mu okay because it is that quantity called friction, which gives the ability for the braking force to be developed. You know this all the time okay and so that is equal to mu*Wf and maximum braking force at the rear=mu*Wr.

Now this is an interesting factor here in the previous case, we looked at the vehicle and said it is a front wheel drive or a rear wheel drive. Now we cannot do that in braking because braking is applied to both the wheels okay. So if I say that total force braking force is Fb, a fraction of the total braking force is applied to the front and a fraction of the braking force is applied to the rear okay.

Let me call that fraction as front to be Kbf*Fb and the fraction of the rear to be Fbr so Kbr*Fb. Obviously, Kbr has to be 1-Kbf. So the key question, which we are going to answer in the next class is how should the braking force be distributed between the front and the rear okay? That is the question which we are going to answer in the next class okay. We will stop here and we will continue in the next class.