Vehicle Dynamics
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Lecture - 31
Noise, Vibration and Harshness – Random Processes

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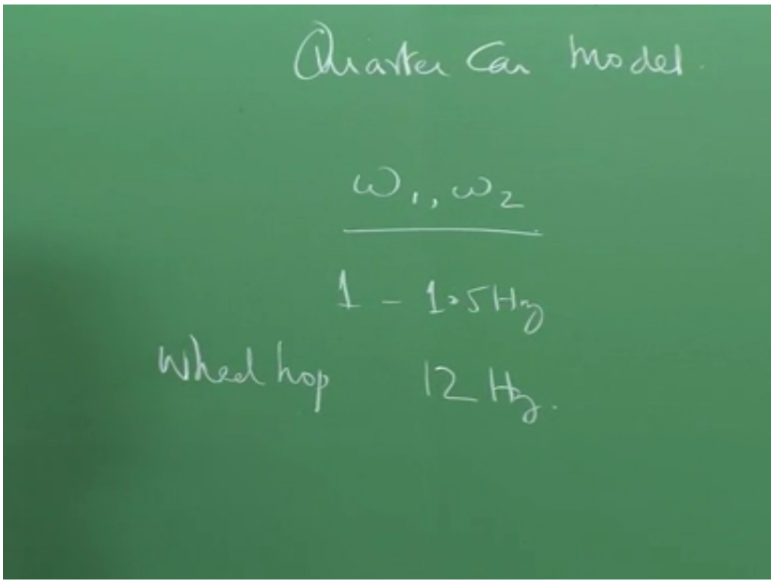

In the last few classes we have looked at the quarter curve model. Remember that we reduced this from a half car model. We said that we would be cutting it into front and rear. It is possible to put them also together and that we distributed the mass of this whole vehicle into 2 parts, front and rear part and we had, what are the 2 dots we have or 2 masses which we said? Unsprung and the sprung mass.

So with unsprung and sprung mass we were able to get certain equations which included the natural frequencies, the 2 natural frequencies. Remember that we can look at this from a very traditional perspective from vibrational controls. We can also get, we can write down the characteristics equation, we can do all those things. In fact we can express this whole thing in states space, may be this time we will do it in one of the later classes.

So once we did this quarter curve model, we also found out 2 natural frequencies, omega 1 and omega 2 and we said that this natural frequencies are spaced such that one of the natural frequencies, the first omega 1, lower one is between 1 to 1.5 hertz, typically they are 1 to 1.5

hertz and that has everything to do with the body of the vehicle. The second natural frequency, omega 2.

We said that this is what is called as the wheel hop frequency, closer to around 12 hertz and may vary between say 11 to 14 and so on. So these are the 2 natural frequencies. One of the difficulties we had, we wanted to now later go and look at and we also developed by the way, we also developed a simple equation for omega 1 and omega 2 taking into account undamped system and when we talked about the damped systems, we had difficulty in arriving at the optimum damping as well.

One of the things we found is that, the damping does not have a very uniform effect throughout and that whatever be the damping, high or low, some of them had better results in one range and others have better results in the other range and so on. So we said that we went back to the 50s and we had one optimum damping. And that completes our one-minute review of what we did.

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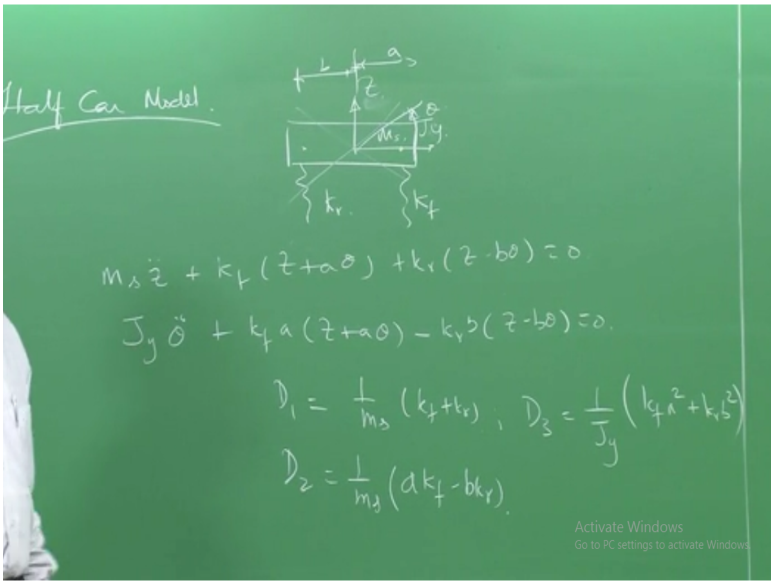

And then we moved over to what is called as the half car model, right? So in half car model, we said that we will take a very simple case, we have say for example, we are going to remove, in fact the unsprung mass. We will consider only the sprung mass and that the sprung mass along with the moment of inertia perpendicular that is J theta form the 2 constants of the car, of the vehicle.

And the variables are the bounce as well as the angle theta. We said that this is going to be supported on 2 springs, the front and the rear. And that we said we will remove also, the unsprung mass, that this is the equivalent springs, when we take into account both the suspension as well as the tyre. Simple model, yes. This is just to illustrate certain things. We can make it more complex, use the well-known MATLAB, software in order to solve it and so on.

Write down the governing equations, either in the state space form, by now you should be familiar as to how to write it in the state space form and then you can solve it, much more complex. But I am not going to do that because I am just going to derive certain important aspects from a simple model which will be useful to you which surprisingly give very good results. It is not that the results are not accurate, the results are quite accurate.

And we can use a simple model to understand certain aspects. If you quickly reviewed that let me write down the equation in the same form as I done in the last class. Remember, that was a and that is b. Jy, sorry, we will put Jy instead of J theta. That is the moment balance equation. So these together the equations. Let us call following one, let us call D1 equals 1/ms kf + kr, D2 is 1/ms aKf-bKr. D3=1/Jy, Kf is Kr.

Okay, and let us substitute this back into this expression. All algebra, so I am just going to write down only the final results.

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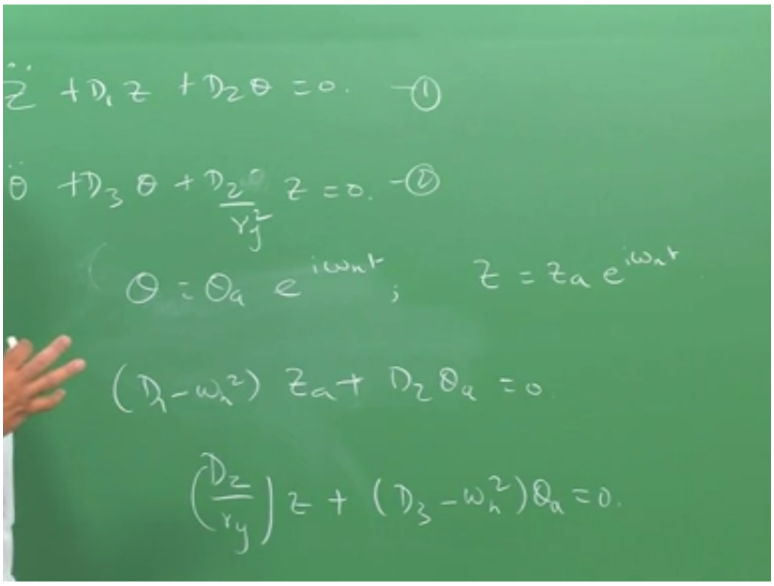

Substituted and I am going to write this down, something like this. Of course what we are doing is free vibration problem because I am trying to find out the i n values in the (()) (07:40). So as I told you in the last class, I am going to look at the (()) (07:52) so that is what going to do now.

What I have essentially done is to calculate or replace Jy by means of m*Ry squared, m R squares is the momentum of inertia term. So radius of that is Ry, so I have written this in this passing. Till now it is straight. There is nothing, just going to substitute it. Look at this 2 equations. Obviously it is coupled because let us call this as 1 and 2 because both the equations has z in the theta term.

So if I have to uncouple this equation, if this were to be uncoupled which I said is not a very desirable quantity, a desirable quality, what is that I should do? if I put D2 to be equal to 0, obviously both these equations get uncoupled, which means that a*kf=b*kr, then this 2 equations will get uncoupled. What is meant by uncoupling? Then in which case this body which is our sprung mass, would now start for the 2 frequencies which you would get and for 2 frequencies.

For one frequency there would only be a pure bounce mode and for another there will be a pure pitch mode, okay? We said that this is not a very desirable thing because we are not going to sit at the center of gravity location and the front and the rear, Center of gravity location would be slightly behind the front seat, but it may vary from car to car. It is lightly behind the front seat or almost maybe one third of the distance between the front and the rear seat.

You can calculate that. In fact, one of the exercises you can see is how to calculate the moment of our outlook. Determine the moment o inertia from a simple experiment. Position of the momentum a and b, good experiment to do. So in which case if I am going to sit on either side, and if it is going to bounce and pitch for any small differences whether it is front or rear, I am not going to be very comfortable, okay? So this is not a very desirable mode.

All of you know how to calculate the natural frequencies. I am not going to do that, because of lack of time, but we can write down the natural frequencies, 2 natural frequencies for this by writing down.

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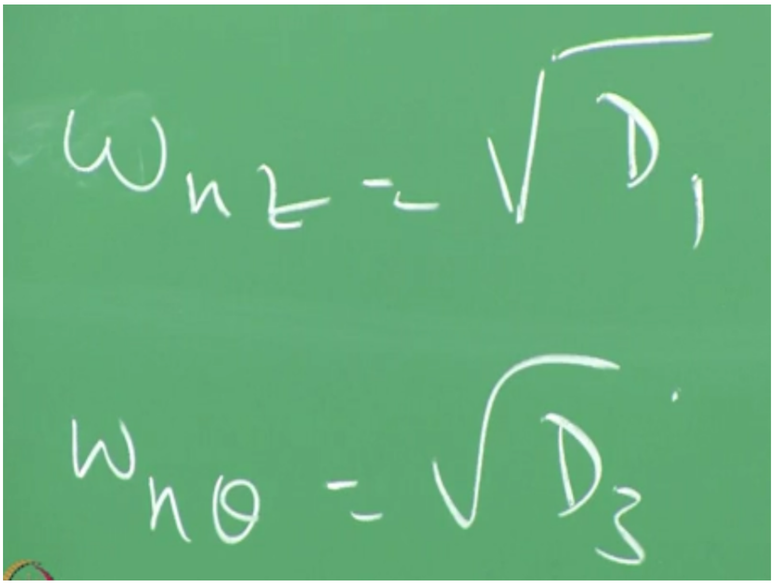

For example, for the uncoupled case, you can write down 2 natural frequencies to be, root of D1. So that is the 2 matching frequency. Please do not confuse between this stiffness, a was multiplied in one of earlier classes with another surface stiffness. C alpha, do not get confused between the 2. This is a different k, this is the stiffness in the vertical direction, nothing to do with C alpha. Okay. So do not get confused with that, right?

So that is the uncoupled case, as I said not a (()) (11:49). In fact, if you want to calculate, you can calculate like that, of course D2=0. Now let us look at how to calculate the natural frequencies for this and all of you know it. I am not going to write down too much on that. I can assume a solution of the form theta is equal to theta a*e power i omega and t and z=z a e power i omega n t mm.

I am just going to substitute that into my 1 and 2, this is the thing and then write down, the equation of the form D1 - omega n squared, c a + t2 beta a=0, substituted you will know that when I differentiate, this is 2 times I get that omega n, - omega m square. That is the reason why I have - omega m squared here and D2/Ry z + D3 - omega n square, theta a=0, okay? So what is the next step?

If I have to, nothing difficult, so what is the next step? I want to find out the natural frequencies. What is the next step? Yes, take the determinant of this. Okay, write down characteristic equation write it down in terms of omega 4 and then take the 2 routes for omega 1 square, and omega 2 squared. So that is straight forward and I am going to write down the final result for this 2, omega n square and omega.

I will call this as omega n1 squared, omega n 2 square, the 2 natural frequencies, right? Any questions? It is pretty standard stuff.

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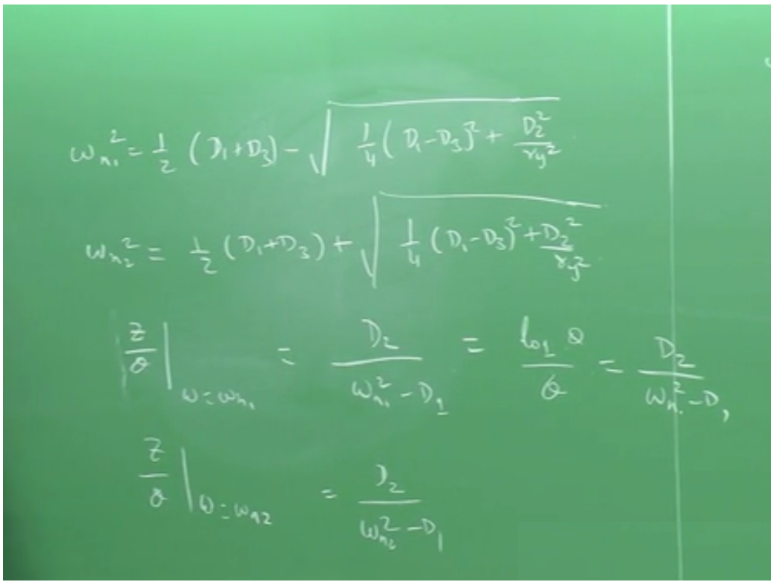

So I will write down omega n1 squared, half*D1 + D3 - route of 1/4*D1-D3 squared, + D2 squared divided by ry squared omega n 2 square, is equal to 1/2*D1 + D3, + route of 1/ 4*D1 - D3 square, + D2 square/ry square, right? Any questions? So these are the 2 eigenvalues. So the corresponding, eigen vectors (()) (14:57) shapes for these 2 natural frequencies.

Can be calculated by substituting it and determining, you said and theta are the 2 cases. Interestingly this is what is the result you get, if I know get omega=omega n1, is equal to D2/omega 1 squared, - D1 and z/theta as omega=omega N2 is equal to D2, of course divided by only by omega square - D1. So in other words this gives that the ratio between the bounce and the pitch mode itself, bounce and the pitch that gives the ratio.

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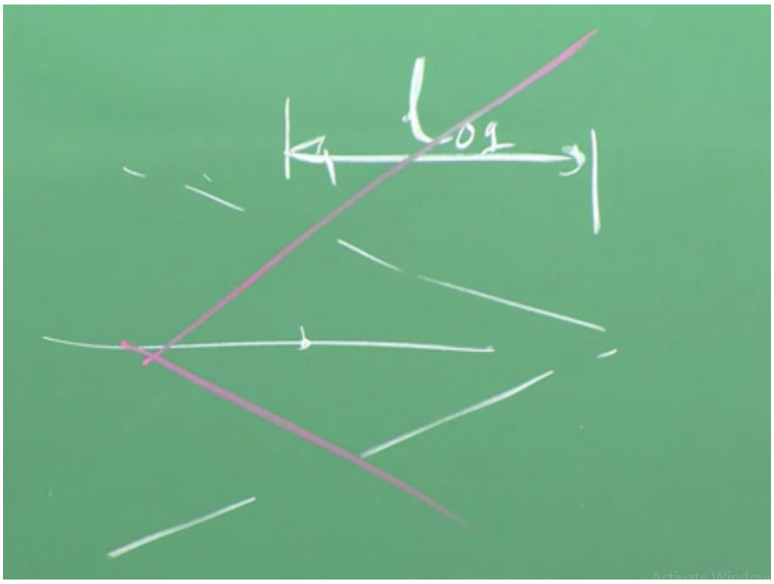

In other words, if I now just represents this by a straight line, my vehicle, the sprung mass and this happens to be the center of (()) (16:27) location. Then the oscillations would be about a node for the first natural frequency, something like this. Let us say that this distance, is L01, that is the distance, okay? Where we have what is called as the node, where the point does not move. So the oscillation is about this. How do I find out this point?

I have already written down the (()) (17:11). I have substituted very standard to degree of freedom problem, all of you have done this, vibration , I am not going to repeat it. So how do you find it out? Exactly. So this I can substitute body. This is equal to L01 multiplied by, so approximately theta divided by theta=D2/omega N11 squared, right? From this you can find out L01, right. The same fashion you can get L02 by substituting it in the next one and in which case actually the oscillation is going to be something about another point.

That oscillation is going to be about a point which is at the rear, okay? That is for omega N2. So this if I can keep it in such a fashion that the 2 nodes, you know this would get excited when the vehicle goes, the rear wheel goes over a bump and the first one gets, the second one gets excited when the front wheel goes over a bump and so on. So if I can keep in such a fashion that the front wheel bump does not affect the rear guy and vice versa then I have to have these nodes at the seat locations properly, right?

So you can further extend it. You can simplify it. You can express this in terms of ROA squared and so on. I leave that to you because we have lot more ot cover in this subject. But this is the essential aspect of looking at it in a very simple terms from the point of view of a half car model, okay? Right. Any questions? I am sure that you have done a very similar problem in your vibration courses, that is the reason why I am not going to continue.

Maybe the same problem you did. Okay, now with all this you have a number of questions, I know, few of y ou ask me. The question is that what is the excitation. You are talking about the natural frequency, you are talking about other things, how do I now characterize the excitation for this vehicle. Okay, till now what you are doing is essentially extending your knowledge and vibration. Regular your one on one course on vibration.

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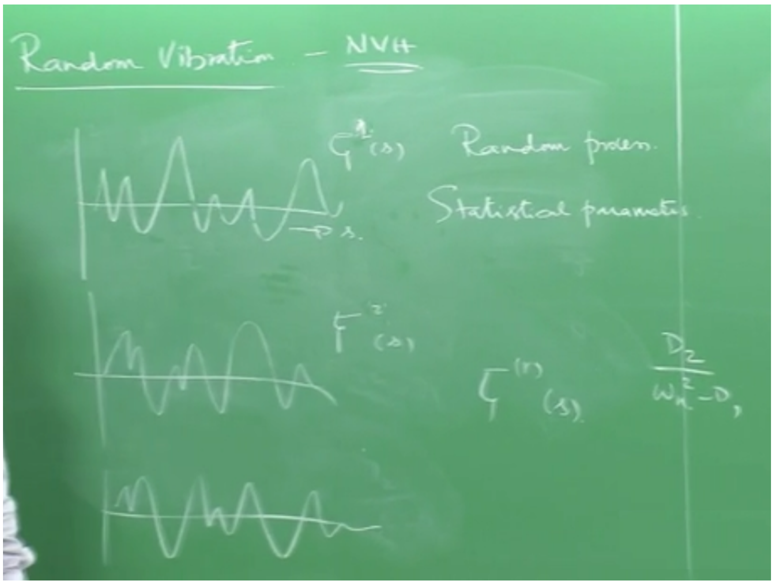

But we will shift and we will see and enter into a realm of what is called as random vibrations and actually that forms the basis for understanding the noise vibration NVH of the vehicle. So we have to go through what is called as the random vibration analysis and that is what we are going to do now. I am going to pick some of the concepts which probably you would have studied in your random vibration course.

Sorry, I do not think, there is an elective but I do not think you would have done a random vibration course, so maybe some probability course which you have done, okay you would have learned some of the things that I am going to pick up, right? So what is that we are trying to study now. Assume that you do an experiment, you have a profilometer.

You go and look at the road profiles across say a number of sections. Say for example you go to one of the high ways and then measure the road profile for a small length, some length possible length and then go to another place, measure it, go to another road measure it and so on. So when you start measuring these things. Then you would notice that obviously suppose a plot, say the heights of what you have very accurately measures by say a laser profilometer or whatever it is.

So you will have for the first measurement it would be something like this. The second measurement would be something like this and so on. So maybe a third measurement would be something like this. In other words what essentially from the language of probability you have done, is do an experiment and you are in the process of what is called as observation, right?

So how do I in other words, the key factor here is, how do I express this surfaces from a random processes perspective. In other words, you have to understand what is a random process? How do I express this as a random process? And more importantly what are the statistical parameters which govern the random process?

"Professor - student conversation starts" Sir why is this actually happening in these cases, the road itself is like a -- Yeah, but no 2 roads can ever be the same because the gravel that is used, the stones that have used, are not of the same side. Everything is not the same. Any surface, for example whatever be the surface, you have tally surf, for example if you want to measure the surface roughness, you will get something like this.

You are not going to get a smooth one like. Of course depends upon what is the magnification you go to, right? That is why I have said that we have a laser profilometer which is used and get this. So you will never get a straight line because of so many undulations which are there which are randomly distributed. The reason for studying this is to see the effect of uneven. Good question. Why am I studying this?

Because this is going to be my input. Okay, this is going to be my input. **Professor - student conversation ends"** Certain statistical parameters which I am now going to explain will form the basis of input to my system which I would call as the linear time invariant system. You would notice that we are going to develop certain statistical parameters and that statistical parameters of the input which is the road and the output for example, the seat location accelerometer, okay.

Acceleration of the seat location for example would be related through a very simple frequency response functions. So in order to do that I have to study what is the input. In simple words what it simply means is that, you would have done that in ergonomics course, all of you took an ergonomics course, right?

And you will see that there is a tolerance level or perception of tolerance to a human being or the perception of fatigue and so on, is proportional to the acceleration levels that you are given, right, and depending on the acceleration you can withstand that for a longer time, shorter time.

We will go into all those details a bit later. So in other words, if I have to look at right comfort, ultimately that is my area of interest, if I have to look at right comfort, I have to look at this right comfort from the point of view of these statistical parameters which will form an input into my linear time invariant system and what is my output, they are the acceleration levels at various points in the car and that would be related to my well being and that is how we are going to study the right comfort.

So in other words, you would have noticed that if you drive in a very, what we call as a Kacha road, then the vibration levels are going to be higher, they are going to feel, you are not going to travel in that road for example 4 hours, for example from here to Bangalore you have a road which is a total kacha road. By the time you end up there, your bones are all not intact. You feel so tired.

On the other hand, the beautiful road that you are travelling you can give urinary one stop, anywhere you can travel for the next 5 hours in order to reach Bangalore if the traffic is quite benevolent to you. So in other words this input is going to affect your right comfort and the whole idea of this design of suspension system for example is to see to it that you isolate to great extent or make your life more comfortable.

We will not be able to go through the complete process in this course because as I said this forms the basis for NVH. A lot more needs to be done. It is not only just the vibration levels, but also noise which is generated by the roads. The road contribution for the noise is very high. So 25 decibels of the total 60 decibels or something like that maybe from the road and in other words, the road contribution is extremely high. It is 25 decibels and have a Ph.D student who works on noise.

And his major aim is to find out for example what is the vibration level at the (()) (28:23) position and related to what is the noise that one hears inside. So these things are not only going to affect the vibration but also the noise. Most importantly tyre is going to interact with the road and that is going to create noise. Tyre noise is a major issue, major major issue, apart from the engine, the tyre, these 2 are the culprits for noise and how this is going to have an effect on the tyre and so on.

All these things become important. In other words, unless I characterize the input, I will not be able to study the output. So that is what I am going to do. So we will go into a very short introduction into random vibration or random processes before we look at the other things. Okay, let us now define certain quantities which are common in probability from where we will take off into random processes.

We will be slightly more formal. I can take off from here because you have done a course in probability, so I will be slightly more formal in this. We had already defines what is an experiment, what is an observation.

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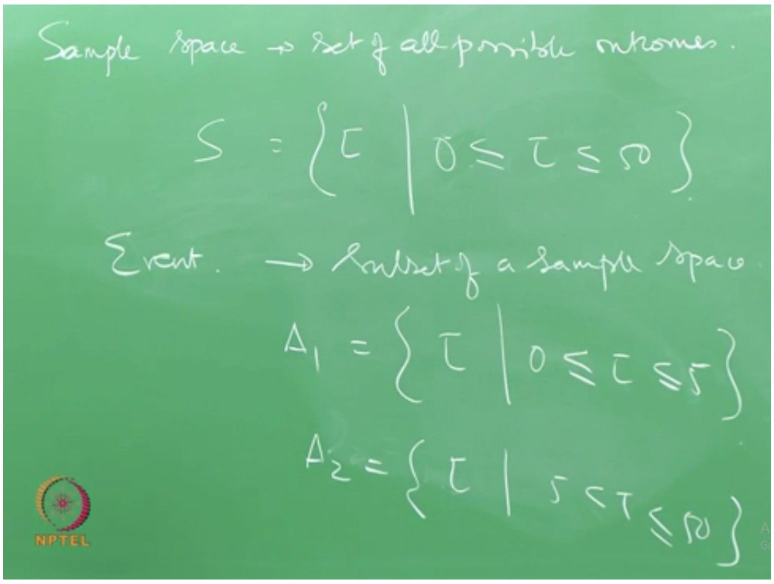

The result of an observation is what is called as the outcome. The set of all possible outcomes is what is called as the sample space. So I do an experiment, okay. I observe and then I look at an outcome. The set of all possible outcomes is the sample space. Let us call the sample space as s and it can be continuous or it can be discrete. Let us define a very simple continuous space, sample space yes. Let us say that we start the class at t=0, okay, so the students starts coming in.

They are free to come in for 15 minutes, not in this class but nevertheless usually people walk in from for 15 minutes. So I am going to now observe and my outcome is the time that people start coming in. So let me write down that. So from that let me write down that sample space. Sample space now consists of all possible outcomes where tau is assuming that the students do not come before the start of the class which has never happened. 0, to 15.

Okay, so this is actually the sample space. This is a continuous sample space. We call as event a subset of the sample space. An event is the subset of the sample space, yes. For example, I can say that A1 is an even where A1 covers tau which is less than or equal to 5 minutes. So A1 is an event. All that is within the first 5 minutes. I can define A to be tau which is 5<tau <= or > tau is >5 and <=50.

So these are events. So events can be this continuously as tau itself. Now a random variable. Note this carefully. Random variable is a function. Random variable is not a variable in the strictest sense but is a function which actually maps these events, let me call that as the event space.

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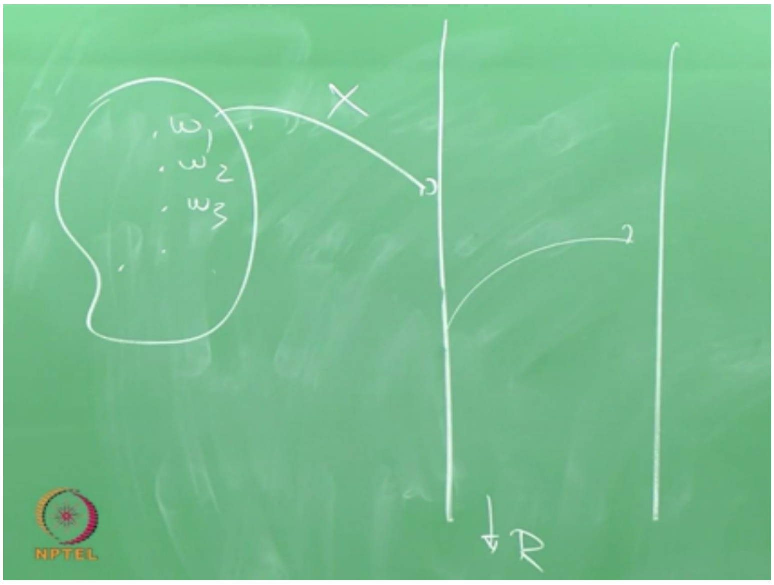

There are number of events and so on. Random variable is one which maps this event to a real line. So basically random variable is a function but though we loosely later use random variable to be a variable but actually in stricter sense of the word, random variable is a function. Now we can assign the next step is we can assign a probability to this random variable in other words from here I can assign a probability.

Many times probability, look at this, there are 3 steps, one is an event, event to a random variable, then from here to a probability. So I can say that the probability of this event happening is 0.5 and this even happening is 0.5. So the total probability of course is equal to one. So random variable is that function and from here I can map it through a probability function. Instead of that it is also possible to look at a distribution.

Instead of defining it through that, you can also put probability directly to the event as well, right. So now let us extend this and see how we are going to deal with the profiles that we have measured in the road. The reason why we did this is because we can apply the rich random processes, the mathematics behind the random processes to road profiles and understand random vibrations from a very mathematical front. Let us say that I have taken a number of readings.

Let me call this readings by zeta, say r, where s denotes the distance from the origins. I start a strip, I take readings from the strip, that is the s and I take another strip and another strip and so on. And I take a number of such readings. So that this I would call as zeta one yes, this I would call as zeta 2 yes. Let me not put that in the bracket, zeta 2s and so on. So like that there can be infinite such zetas which have 2 things, 2 notations, one is r and other is s, okay.

So this is what is called as a random process. In other words random process is a family of random variables.. It is a family of random variables and are usually like in this case, have 2 indices, r and s, right?

"Professor - student conversation starts" Sir (()) (37:33) Stand for? Length. It can be time for example. You can look at it as a time. But you can look at it as **"Professor - student conversation ends"** This actually gives the roughness of the road, okay. And note that if I take any, let us say that is s1 and that is s2 and so on. Just because I measure at one location the surface roughness.

Say for example if I know s1, it is not possible to predict or say that this would be the same as S2 or it is 4 times, or if I measure here, I cannot say that this would be some 4 times s1 or the same as s1 and so on. So I have to now bring a statistical basis for this what I am measuring at this position. This is usually called, the psi what I am measuring is what is usually called as a random variable. What I am measuring here is what I call as the random variable. Okay?

So now let us just put this together. Let us just develop this. So in other words, though random variable is a function we are going to call what I measure as the heights at various positions as the random variable so that there is a mapping in the sense, I make an experiment I make the observations and then I call this height as the random variable. The random process is the collection of all these things, the ensample of all that is possible to be done.

Okay, let me put this together and let us see what we mean by each of these terms and how we are going to statistically understand this process, okay?

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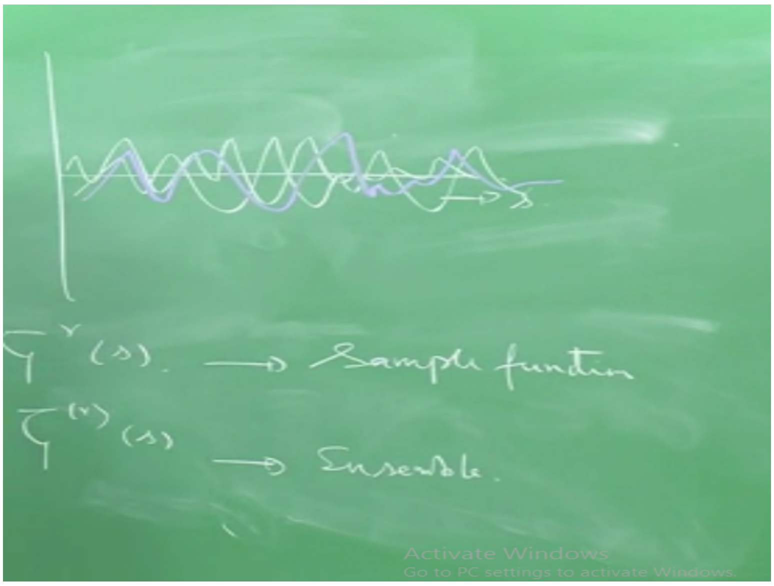

Let us now put this in one go, in other words, one s, right? One of them would be like this. First one. Second one would be like this. The third one would be like this and so on, okay? Those are the zeta one, zeta 2, zeta 3 and so on, clear? So let us give zeta rs, r=1.2 and 3 is what is called as the sample function. It is a sample function. When I put this into a bracket, r, that means it is the ensample, all of them.

Some time it is called also as the realization of sample function or realization. The collection of all these things is what is called collection of all those r's is what is called as the ensample.

“Professor - student conversation starts” When we say, also when we are looking at variations across the road and variation between roads? Yes, absolutely. The beauty of that, that is what, we are coming to that. These heights vary along the road, in other words in one of the sample function, they are not constant, they are varying. So for example that shows how it varies. And also varies across roads. **“Professor - student conversation ends”**

Now, how do I study this, that is the whole process, the random process. So what are the statistical parameters that I can use it. Let us now put down, let us give some notations. So right now the most important thing is random process, random process is the collection of all these things. And the sample function is one of them. And that ensample is complete collection of whatever that is possible is called as the ensample.

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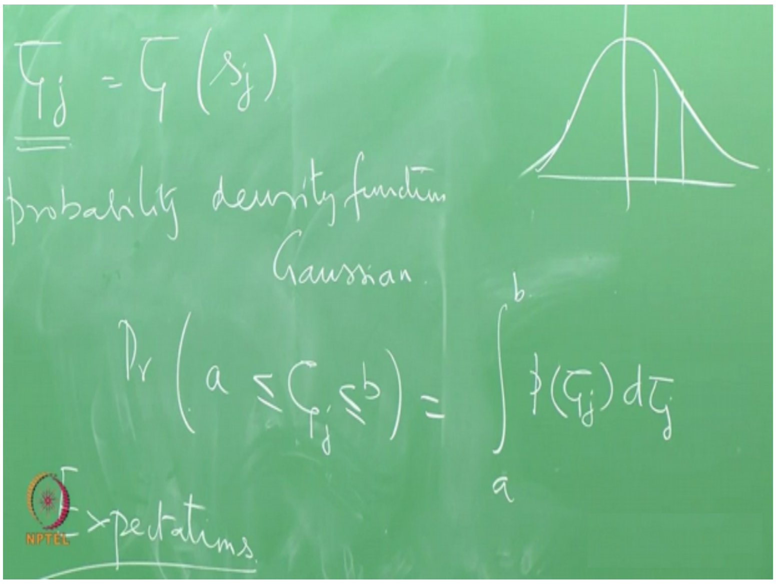

Now let us say that let zeta j indicate the value of this roughness, say in terms of height for a particular distance, sj. So as you rightly said, the question is what is sj, for example it can be s1. The question is how is the roughness distributed over this, at every point in whole ensample, okay? So in other words that variable, has a statistical properties, number one. Number 2, the other thing the statistical property I am interested in is, as I vary yes, how does this heights, this roughness changes with that length.

So these are the 2 things I am going to study, right? I am also going to look at certain important characteristics of random processes, like stationary, whether it is a stationary random process and whether it is going to be what is called as (()) (44:04) right? Let us now introduce what is called as probability density function. All of you have heard about this, density function which I am going to use in order to characterize this random variable which I called as zeta at the position.

A number of statistical or probability density function is possible, one of the most popular probability density function is the Gaussian function. Going back to the definition of the probably density function, the probability that this roughness value which I would now start calling as the random variable is between the values of a and b is given by how do we write this, is given by this. What am I looking at? What is the probability that it is between say 3 mm and 5 mm, right?

So it is given by integrating the probability density function, okay? In that range and finding out what is the area under b and a. So in other words, I can plot the probability density function. Say for example as a normal or a Gaussian distribution, okay, normal Gaussian distribution whatever it is, I can plot this as a Gaussian distribution. We will put down equations in a minute. One of the major things, is that clear?

It is quite straight forward. So probably in other words, this is nothing but the area under the curve from a to b, you see the probability. Probability density function gives you the probability versus the tau j and you take that area between say a and b and then say that what should be the, what is the total probability and from which you get the probability distribution function as you keep integrating. Suppose I integrate t his value between a and b, between - infinity to + infinity.

I get the total probability to be equal to one. So integrating this is what you would get a probability. Distribution function, you can also look at probability density function as the differential of the probability distribution function. We will not go into too much of necessities because we have one more to look at. We are going to look at this whole thing, this whole from the point of view as what is called as expectations. Okay. Now in order to understand this expectation.

Let us do a small experiment for next one minute and then we will extent this to what is called as the expectations. We cannot complete a probability discussion without the dice and a coin, can you? You cannot. Suppose I take a dice and then I am going to throw the dice, okay, say maybe 6 times, what is the, in the general language, what is the average value that you would get. Like I through this, first time Ih row it is 2, second time it is 1 and so on.

How would you calculate now what is the expected value as it is called average of the mean value. How do you do that? What is the probability that in the dice, it is an unbiased dice and throw it, what is the probability that I will get 1, 1/6. 2.3.4 is 1/6/ So the probability that I get one of these numbers is one/6. Okay. Tell me what is that value we get? How do I calculate it?

"Professor - student conversation starts" 1*1/6+2*1/6. Exactly, sum it up and you will get the expected. **"Professor - student conversation ends"** I am going to use that in order to define that expectations. And I am going to distinguish between what you have learned earlier, say for example as an average, mean or in other words sample in the population. We will start from expectations in the next class.