Vehicle Dynamics
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Lecture – 32
Random Process and Conclusion [contd.]

In this class, we will sort of revise and wrap up what all we have been doing in random vibration. We will go through what all we did; some of them at least and then move to the complete use of this results of random vibrations for determining the tolerance of a human being for the vibrations that exist in a car, maybe have you have already done a course on ergonomics.

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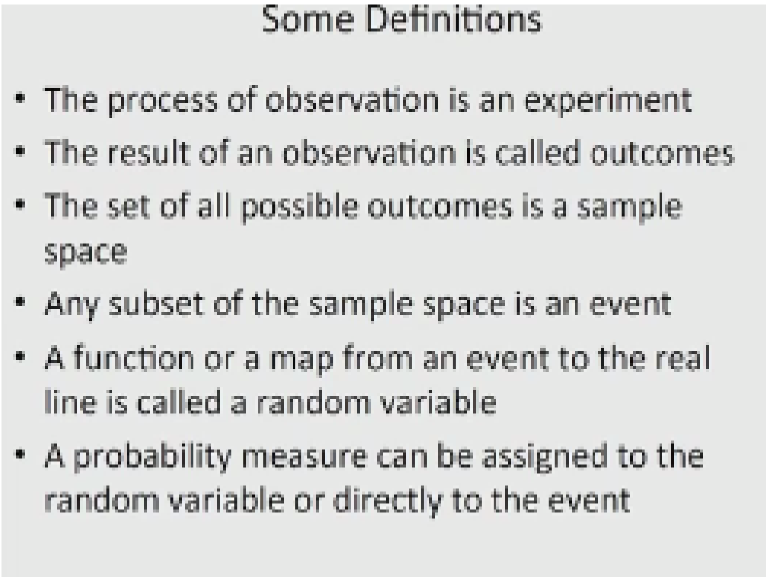

I am sure that you know human factors, vibration and what can be tolerated and so on okay, so we will not go into those details as I discovered in another course but we will understand this random vibration a bit more and see how we actually calculate or what is that calculations that we do in order that; we determine the levels that the human beings can tolerate. So, we will go through this; the whole of these things in one go.

So, we were looking at what is called as an observation, so in any; in a probability sense, observation is an experiment and the result of this observation is the outcome. In our case, we are looking at the roughness of the road as we had seen. The set of all possible outcomes is what we called as sample space, so a subset of the sample space is an event; we explained it with simple example.

And then the function or a map from an event to the real line is what we call as a random variable. So, there is an event, there is a mapping to the real line which is called as a random variable and the map from this random variable to a probability measure can be done or we can directly give this probability measure to the event okay. So, this in a nutshell is; are the definitions that we need to know.

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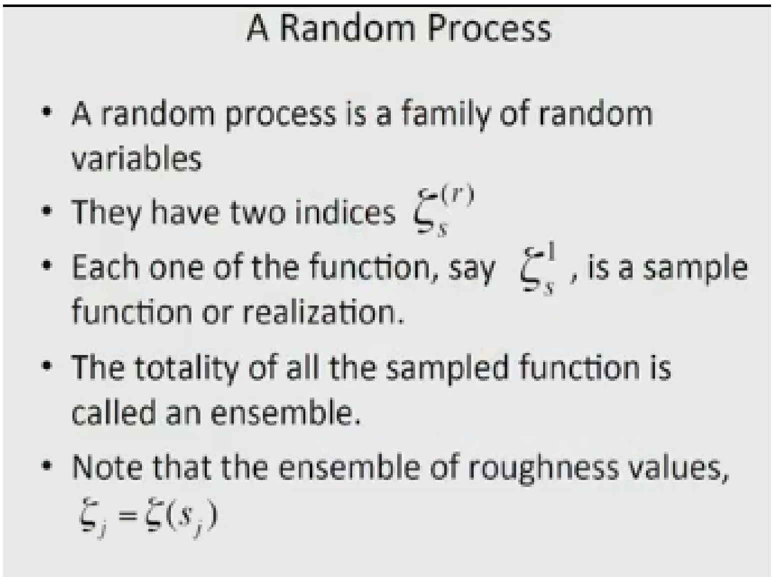

We already saw that the random process is a family of random variables.

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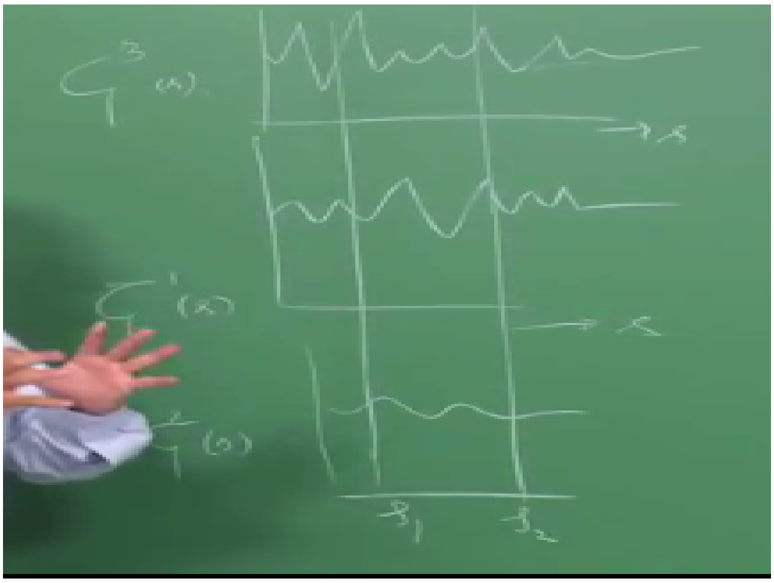

Remember that we did this in the case of the roughness of the road, okay as a random variable, so we said that we can take a number of measurements, so this family of measurements okay is now going to be used and is; we can call this as the ensemble. Remember that the random variables have 2 indices, I put one in the bracket; in other words this R can vary from 1,2,3,4 and so on.

And s; is this distance, s is the distance and this indices 1 of s, then zeta 2 of s and so on are the independent realizations as they are called okay, our sample functions okay, so then we have a number of such sample functions maybe s and that we can call us zeta s and so on. So, all these things; this roughness is form what we call as the ensemble of roughness values okay. Now, there are 2 ways in which we are going to look at this.

We can look at it at a particular value of sn, of course, this sn can be or s can be replaced by time okay and you can define the whole thing in terms of time. Note that, we can look at this at s1 and look at another one s2 and so on. So, we have now at s1, a number of what are called as random variables okay or the measurements that we do at s1 and we do similar measurements at s2 and so on.

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Gaussian Probability Density Function

$$p(\zeta_j) = \frac{1}{\sigma_j\sqrt{2\pi}}\exp\left[-\frac{(\zeta_j - m_j)^2}{2\sigma_j^2}\right]$$

$$\Pr(a \le \zeta_j \le b) = \int_a^b p(\zeta_j)d\zeta_j$$

Joint Probability Density function is defined as

$$p(x,y) = \lim_{\substack{\Delta x \to 0 \\ \Delta y \to 0}}\left[\frac{\Pr[x < x(k) \le x + \Delta x \ and \ y < y(k) \le \Delta y}{\Delta x \Delta y}\right]$$

For Gaussian joint normal probability distribution:

$$p(x_1, x_2) = \frac{\exp\left\{\left[\frac{-1}{2(1-\rho_{12}^2)}\right]\left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho_{12}\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right) + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right]\right\}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho_{12}^2}}$$

Now, this one for example; I can say zeta1 or zeta2, zeta3 and so on has a probability density function, the most common amongst them being a Gaussian or a normal distribution. The Gaussian or the normal distribution function, all of you know and that is the Gaussian or the normal distribution function. Note that the difference between the probability density function and the distribution function.

Probability distribution comes out of integration of this probability density function or the probability density function comes from the differentiation of the probability distribution function. So, for example if I have to calculate the probability of this zeta, what is the probability that zeta is between a and b? What I need to do is to just integrate this between the limits a and b okay, p zeta gives the probability density function.

And if that is integrated, you get what is the probability that this variable is between a and b. We are also interested in joint probability okay of 2 variables x1, x2, s1 s2 and so on. So, the joint probability is given by a Gaussian joint normal distribution function okay. Joint probability between 2 variables x and y is that; look at that closely, probability that x lies between or x of k lies between x and x + delta x and y lies in a range okay.

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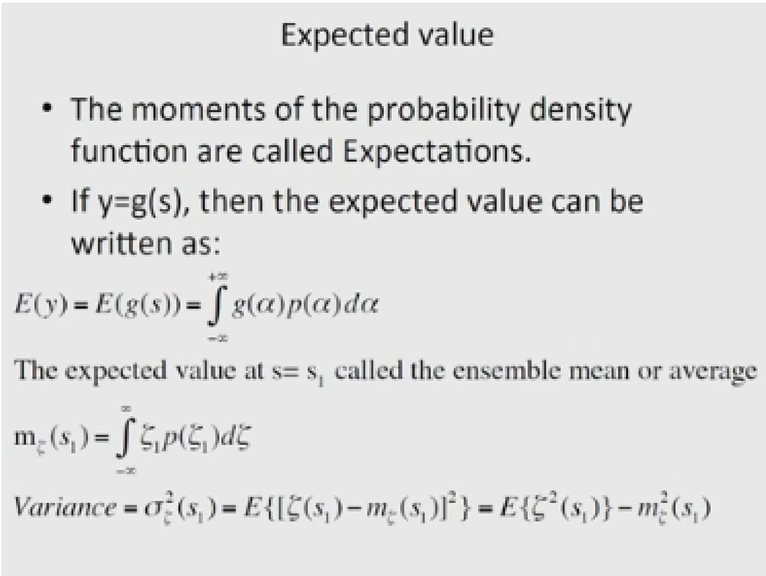

So, in other words it is the second derivative or px, y can be written as tau/ dou i of dou p /dou x or dou square P/ dou x dou y, where p is the probability distribution function. The joint probability can be written as shown here and rho is what is called as correlation we will see that in a minute. We talk about; when we talk about this ensemble, okay we talk about what are called as expectations okay; expectations.

So, in other words, it is the ensemble average, is written in terms of expectation. Note that from a sheer definition point of view, there is a difference between the time averaging and the averaging across or averaging of the whole in sample, okay. Time averaging simply means that I am averaging across here, okay. So, expected value or in other words, ensemble gives us what is called as the ensemble mean or ensemble average.

And that is given by okay, this we had seen in a couple of classes back that it is given by mean is equal to the value zeta1 into multiplied by the probability into dz okay. Now, remember that we had seen, so the key factor here is this equation, which gives you the expected value of say; of function y is given by the expected value of y okay it is a function of x or y = g of s; a function of s is given by g of alpha, p of alpha into d alpha.

So now, I can replace g of alpha; g of alpha by any function okay, so I can replace that for example, if I have; I am looking at the expected value of the; you know what is called as the variance then we can determine; we can determine the expected value of the variance by substituting that difference zeta s1 - the mean okay, that can be substituted; that squared can be substituted instead of g and calculate what is called as the variance.

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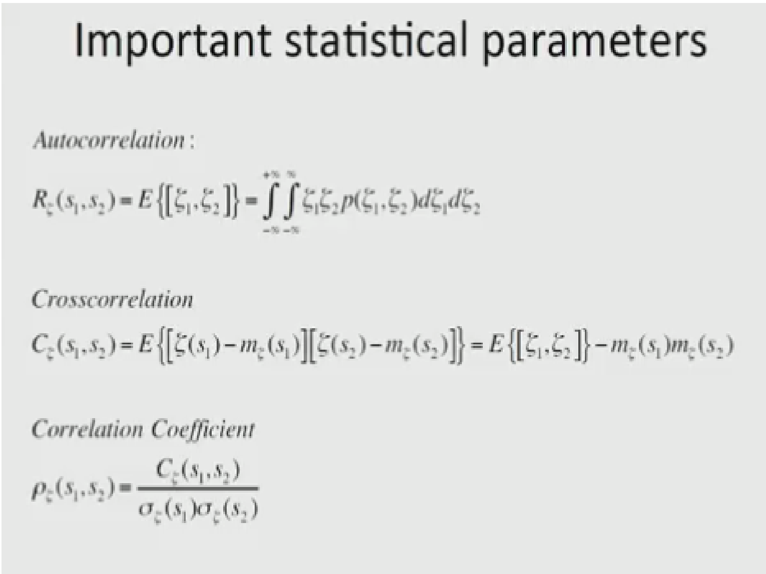

So, expected value of what is in this bracket okay, that square of this term okay is called as the variance. You can operate the integral as you know it and substitute it, expand it and that is the right hand side is what you get. Let us now, so the 2 important parameters; the first 2 important parameters are what are called as the mean and the variance okay. They belong to one set say, they calculated at s1 or calculated at s2 and so on.

So, when I say it is mean or the expected value, it is across this line; vertical line okay. So, if I talk about horizontal line though its distance I would talk about time averages okay; loosely talking it is a time average. So, average can be taken along the vertical line or the average can be taken along the horizontal line, so those are the 2 things. The first is respect to one variable s1.

You can then look at how 2 guys, who are separated by a distance say, tau or time okay, how they are related? There are 2 things or 2 definitions which now become or characterizes this connection between the 2 random, I should not call 2 random variable, but random variable at s1 and s2 okay because we have to be very careful in using this terms, this whole thing is one random process, please note that this whole thing is one random process.

The road is characterized as one random process okay. So, I can say in that way, it is not this is not a different random process, it belongs to the same random process and so they are the random variable. For example, if I look at the acceleration levels in the seat okay then I can call that as another random process. As we go along in these roads what is the acceleration level at the seat if I calculate okay that is called as another; you know random process.

So, I have another set of these values there as well, then I will combine both, we will see that a bit later. So, here we are looking at the same random process, so autocorrelation is the connection between those measurements that are done at s1 and the measurements that are done at s2, so that is the expected value okay. It is an expected value of zeta1, zeta2, so you can see what is that; that definition is there okay; zeta1, zeta2.

We are now replacing zeta1, zeta2 instead of g and so, the probability now, we see that there is a joint probability of zeta1, zeta2 multiplied by okay; dzeta1, dzeta2. So, the same thing; **"Professor - student conversation starts"** Yeah! That is the; sorry that the second one is auto covariance or there is a small; I mean there is a mistake there, it is actually auto; then we have the what is called as variance okay, auto covariance.

So, that is subtracting the mean and I think there is a small mistake in that as it is not cross correlation okay it is again the auto covariance between the 2, so the second line that is the one okay. **"Professor - student conversation ends."** Now, we can actually calculate what is called as the; so, what is the difference between the 2? The difference between this the same as the difference between a mean and the variance.

One talks about how much the variable is connected okay, the other talks about how much the distance away from the mean okay, how is that connected. So, both of them you look at one without mean and one with mean. Usually, in most of these processes we would make sure that the mean value that is by adjusting the measurement that is being taken, we can make the mean value to be 0, in which case both these cases; both these things would become 0.

Or in the other words, that m would become 0 okay. So, that is the; so the second one is actually the auto covariance, so that not cross correlation; auto covariance and auto correlation and auto covariance is given by these 2 equations okay. We define what is called as a correlation coefficient? What is the connection? You know, why are we doing this? What is the relationship between the 2?

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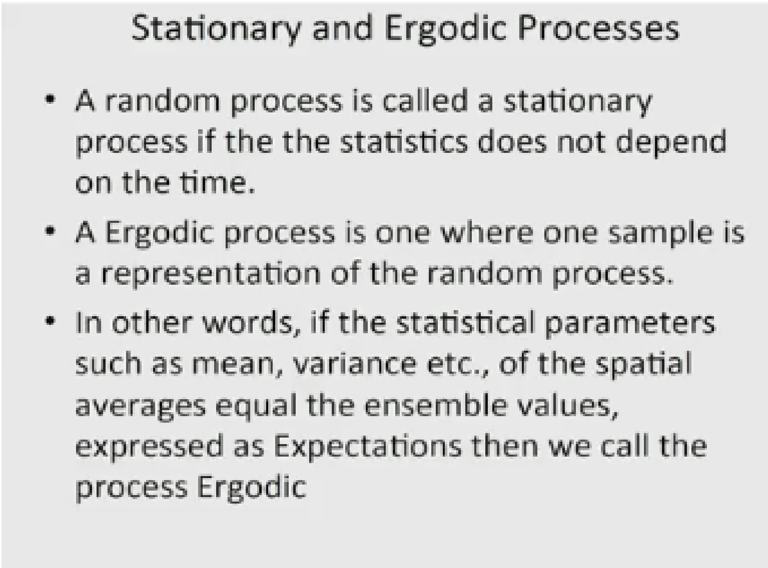

Once we finish this; once we finish a few more definition, we will look at what this actually means okay. So, I define another one, then I just normalize it, normalize the variance okay, this auto covariance by means of the mean and I get what is called as the correlation coefficient. We will come back to this correlation coefficient in a minute, now we let us look at; we will now narrow down this process.

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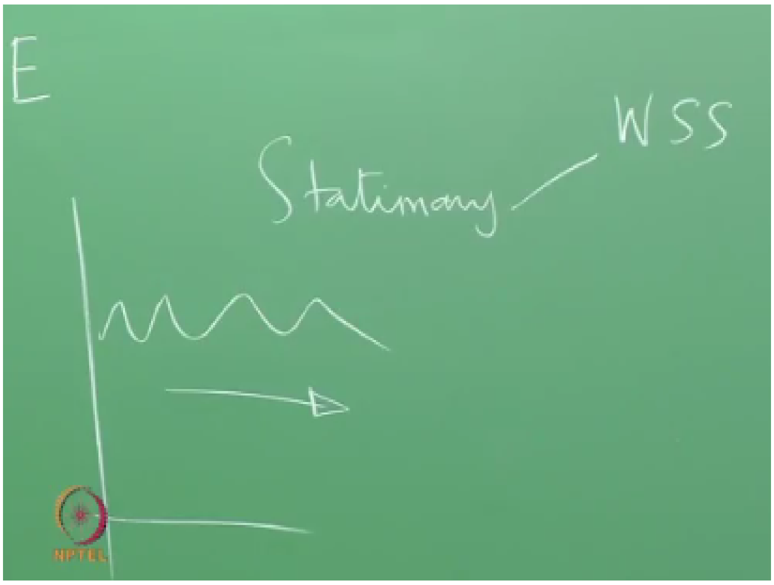

Now, if it so happens now, looks at this, I am defining s1 and s2. If it so happens that the statistics what we are defining okay does not depend on the particular value of s1 and s2 but depends only on; say for example, the difference in distance or in other words time okay does not and depends only on the time okay then we call this process as a stationary process. So, the stationary process in fact, can be classified into weakly stationary processes.

Or we can call them as strongly stationary okay, so but we will not go into that detail because we will then it depends upon whether all of them; all the statistical measures or in other words, all the moments; higher order moments all of them they are the same okay or they do not depend upon where I put this my line does not depend upon where I put the line but depends only upon the separating distance or separating time okay, all the higher order moments okay.

That is very strongly stationary; if the first 2 moments depend upon this time tau then we call this as a weakly stationary process okay. We will just go to ergodic, then we will understand this whole thing you know what we are talking about. Please note again I am looking at it like this, the vertical segment okay, so it is independent of time in other words, that vertical segment if I take and determine and do all my expected values and so on.

Or I take this vertical segment, they are the same or I take 2 other vertical segments and then separated by that same distance tau, then all the statistical parameters that I calculate they are the same okay. So, this distance does not come into picture and that is what we call as a stationary process okay. As far as, we are concerned we are one interested in the first 2 and so that is the first definition of stationary process but a more interesting process because of the limitations and other things.

It is called as an ergodic process, it is a special case of stationary process where one sample represents the whole of the random process, so here I have to go to a different road and determine this okay. So, one sample determines the whole of this process, so I take a sample, if that sample determines all the statistics then we call this okay as ergodic process. More importantly, the ergodic process, we will see how it is.

Whatever time averaging I do along this; the regular time averaging I do along this that reduces to the ensemble or expected values, so the time averaging gives me the complete statistical parameter of the whole of the random process. **"Professor - student conversation starts"** Yeah! We will come to that. So, one of the conditions of course is that the length of this sample we say that the length of the sample has to be big.

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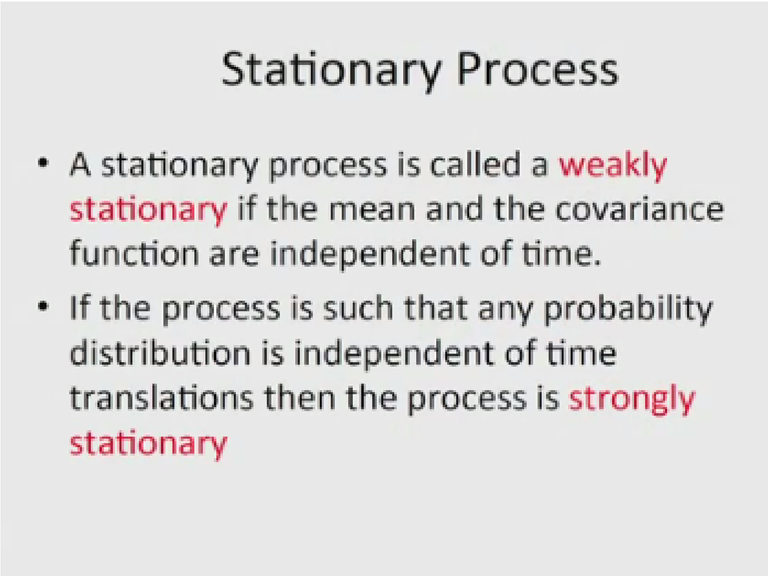

We will come to that in a minute, what should be the length of the sample. **"Professor - student conversation ends"**. So, we call this we already said that if the mean and the covariance you know, function or independent of time and we also defined what is called as this strongly stationary process.

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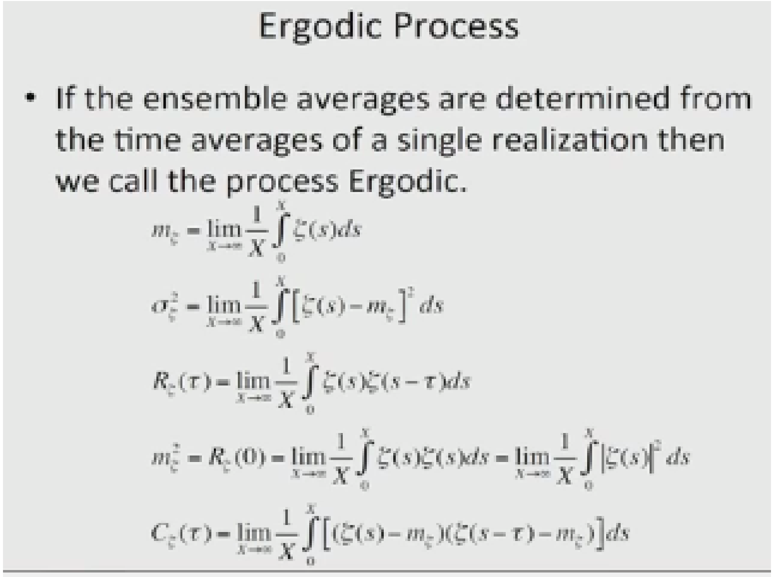

So, we said that if the ensemble averages are determined from the time averages of a single process we said that is ergodic. Let us write down all the equations corresponding to that, what am i doing? I am only okay defining; you can also write this as an expected value, it does not matter but when you write this in terms of time the calculation becomes quite simple. So, the first is the mean definition, look at that x tends to infinity okay.

So, I integrate over a very large length, it is very important that the lengths are large okay, so that is the mean across that is horizontal side, so the mean variance and the third how look at; how it reduces, so we will we look at this from a; they probably; the density function will be Gaussian and the third is what we called as, what is that? Autocorrelation okay; autocorrelation and the 4th is mean square value and lastly we look at auto covariance okay.

So, what essentially has happened is that, d zeta1, d zeta2 which we had put earlier okay that is now we have replaced it okay that time averaging and so it becomes very simple. Now, it is possible to get the estimate of all these things we will not cover that in this course, what are called as estimators? We can do that estimators by looking at discrete values that are taken okay and then summing up the discrete values, we will not look at what are called estimators for these things okay.

We will leave this as a continuous expression and maybe we will cover that in one of the later courses. So, these are a bunch of definitions which I am sure you understand if there is any question, we will take it okay. Now, what does these things mean, you know what is the; what is autocorrelation mean? What does the auto covariance mean and so on? It simply tells you how far is a correlation you know as the term indicates how far the signal that is happening at s1, okay; the road roughness is.

How far does it affect something that happens at s2, in other words what is the memory of a signal. This is not only for a roughness but it is also for any signal. So, it looks at; what is the memory of the signal, if this is white noise which means that say; we will talk about the spectrum or power spectrum density which is distributed throughout. In other words, it is purely a signal which is such that even the next guys are; in other words delta T okay.

If this happens at time T what happens at T + delta T, he was also is not related to T, in other words a white noise is where; this is stick standing at tau = 0, okay you can, if you go and substitute tau = 0 in the third expression, you would notice that is nothing but that is nothing but if I substitute tau = 0 in the third expression this is nothing but the 4th one R of zeta is 0, okay.

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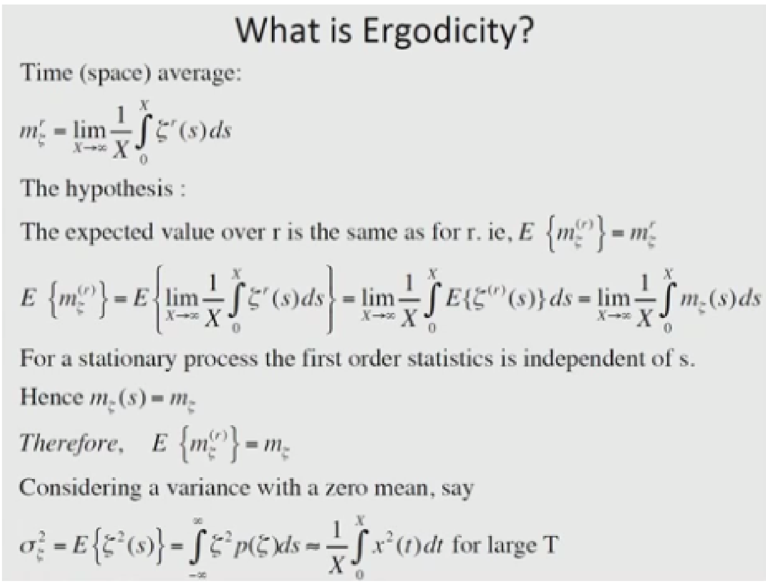

It is nothing but the mean square value of this variable okay which means that they will just be a stick and it would not be connected at all. If I mean; of course this varies depending upon what are the frequency contents and so on okay. What is actually an Ergodicity? It is a very big question, it cannot be covered in this course, there are what are called a ergodic theorem and so on.

We will just follow a very simple method to just understand, what this Ergodicity is, so what we said is that we have a time or a space average, let us stick to that word time because that is what is used in most text books okay. What we are saying is that the ensemble average that is the expected value; that is the expected value is equal to the time average. How does this come about? Okay, let us look at this carefully.

Now, the expected value of the ensemble or the ensemble average given by that expected value okay, we will substitute that okay; expected value of you know in terms of what we had measured here, what substituted here in this expression, switch the expectation, after all expectation is the integration, you can shift it okay and ultimately you would see that okay when I substitute that the ensemble average okay reduces to what is called as the line average or the time average, right.

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Fourier Transform

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$f(t) = \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$$

So, what is important to understand is that, for a stationary process; the ergodic process is also a stationary process, so this very; I mean this expression is very simple expression, you can see that the simple expression where I just took the expected value inside shows that the ensemble average can be calculated in terms of the time average or in terms of the distance s. Now, so these are the 5 things so, which are important to us that is what we had defined okay?

Let us define what is called as a 4ier transform we know that already what 4ier transform is. Note that by 1/2pi can be interchanged it can be root of that and so on okay depending upon the textbook, so it is only a scaling quantity. So, let us look at the 2 values okay the; what is called as the 4ier transform and the inverse 4ier transform okay. These are well known; this comes from your 4ier series okay you know you know that already.

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Power Spectral Density

$$S_\zeta(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_\zeta(\tau)e^{-j\omega\tau}\,d\tau$$

$$R_\zeta(\tau) = \int_{-\infty}^{\infty} S_\zeta(\omega)e^{j\omega\tau}\,d\omega$$

Remember,

$$m_\zeta^2 = R_\zeta(0) = \lim_{X\to\infty}\frac{1}{X}\int_0^X \zeta(s)\zeta(s)\,ds = \lim_{X\to\infty}\frac{1}{X}\int_0^X \left|\zeta(s)\right|^2 ds$$

ie,

$$m_\zeta^2 = \int_{-\infty}^{\infty} S_\zeta(\omega)\,d\omega$$

We will spend some more time in one of the later courses and all these things. So, we define what is called as the power spectral density. Ultimately, we come to a quantity which is called power spectral; this is the 6th, right; 6th important quantity look at mean, variance, auto correlation, auto covariance, correlation coefficient which is nothing but normalized by means of the variance, a square of the variance is this okay.

And then we come to what is called as the power spectral density. Power spectral density is defined as the 4ier transform of the autocorrelation function. So, in other words what is that we are trying to do, we are moving from the time domain to the frequency domain okay, we are moving from the time domain to the frequency domain, the autocorrelation function is in terms of time domain and now we are taking the 4ier transform of it.

So, we have now moved to the frequency domain, right. So, these are the 2 4ier transform on the inverse 4ier transform or do an inverse 4ier transform of the power spectral density we get what is called as the autocorrelation function okay. Substitute tau in that second expression you would get the meaning of what is power spectral density. In order to understand what this power spectral density is; what is this power or spectral density or sometimes called okay spectrum as it is loosely called okay all this and what does this really mean?

The expressions, which I had written there; they are very straightforward okay they are just that the mean square value okay which is the autocorrelation at tau = 0 is what is defined in the third line that is this one we had seen this before. If I now substitute tau = 0 here it becomes that the mean square value is nothing but s omega d omega. So, in other words the mean square value or the power okay is now determined in terms of the power spectral density.

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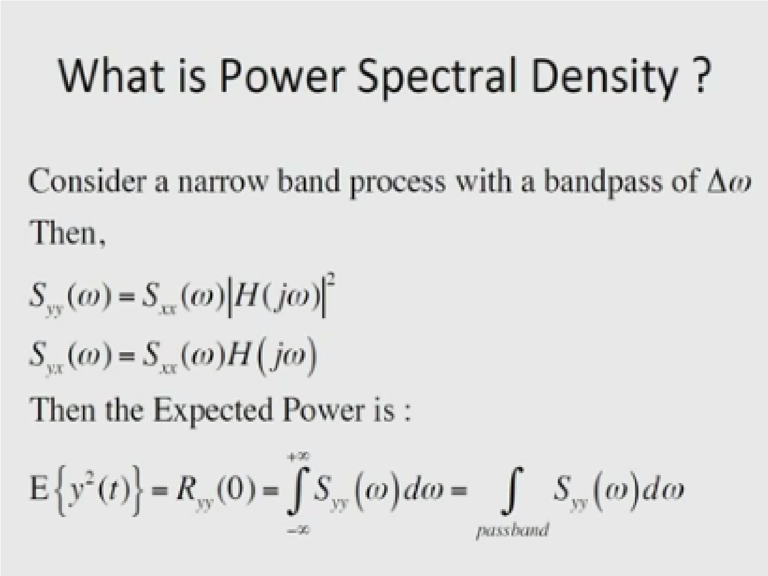

Now, let us understand this more carefully. Let us for a moment take that second expression for granted.

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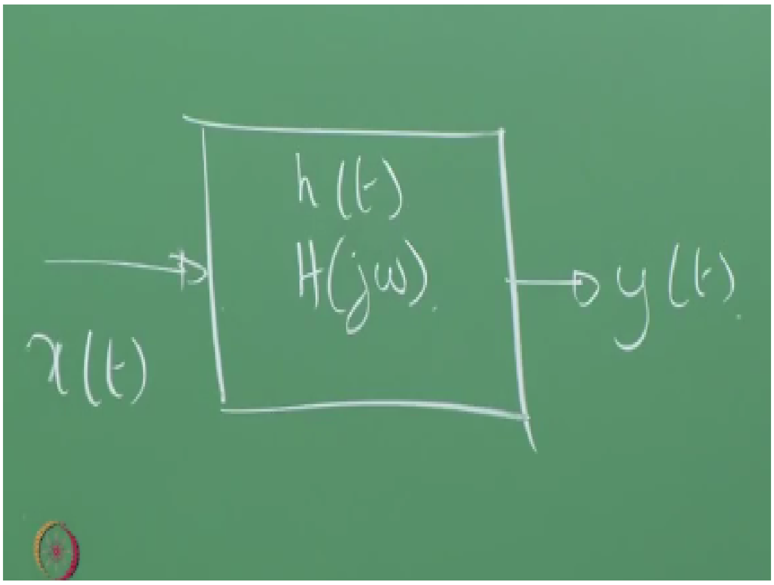

Let us say that I have a process okay where I send in an input and I get an output okay, let us say that the frequency response function of this process is H of j omega and the impulse response of this is say, H of t, right. Let us consider that this process as a band pass filter okay with a narrow band. In other words, this is a filter which has the frequency response function such a fashion that it allows only a small band of frequency to pass through, going to filter off the rest of them.

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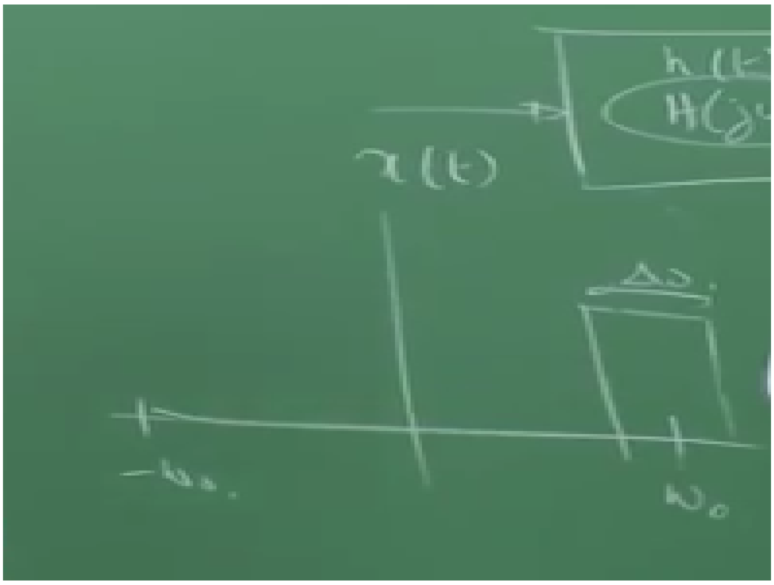

So, if I now; in other words, let us say that centred about around omega0 okay usually practice to do both, so it is going to allow only a small narrow band okay. Now, this pass band; this is what is passed okay, this narrow band of frequency is what is passed okay, which I would also call this pass band, if you do not understand this filter okay, let us now calculate the expected value of the square of this output, okay.

The expected value of the square of the output, in other words mean square value of y, usually the square of this; any these quantities, the electrical engineers are fond to call them as power, it comes from I squared R, so usually the square of a quantity is called as the power okay. So, the mean square value which we saw in the last slide is now, in other words this slide we saw it there as the last equation.

Now, since this narrow band is only passed, this band pass is only a narrow band is passed okay, let us say that is delta omega, that integral from minus infinity to plus infinity is now reduced okay; reduced because the rest of the places it is zero, so it becomes this is the integral limits now, okay. So, that pass band is the power spectral density multiplied by d omega okay and you can ask the band becomes smaller, you can understand that the power that is passed okay in a very small range is nothing but is given by the power spectral density.

Sometimes, you know some of the softwares, which are used in mechanical engineering, they call that sy omega*delta omega as auto power okay, so power spectral density because the word density is used because when you multiply by delta mega, so it gives you the power, so that is why the density term is added. Many of the MVH softwares okay, you would see that they use the term auto power in; what they simply means is that they multiplied this with a band of delta omega okay.

So, that multiplication in other words, what is inside the integral is what they call as auto power okay. So, in other words the spectral density or the spectrum gives you the energy content of the signal and we are now looking at it in the frequency domain, the energy content at various frequencies is what is determined okay by the spectrum okay. One of the key factors though we will not be deriving it completely in this course, take that is granted is the relationship between what comes out here and what goes in.

The relationship is given by the first equation, in other words the power spectral density is a very key equation; the power spectral density of an output okay, output determined from a linear time invariant system okay which is characterized by a frequency response function okay that is given by the square of the magnitude of this function, multiplied by the input power spectral density.

So, in other words, if I have a vehicle say for example we looked at a quarter car model and we had looked at various frequency response functions at various places now, if I want to say that I want to find out what is the power spectral density say at the automobile what we call as the sprung mass, say the base of the car okay, the sprung mass; the floor of the car okay. If I want to find out that we already know what the frequency response function is, we had written that okay.

Now, that frequency response function, the magnitude square of the frequency response function, if I now take; note that it can be complex, so we are looking at the magnitude of frequency response function, square of that multiplied by the input power spectral density gives me the output power spectral density. So, if I want to find out at the seat I put one more okay what is the frequency response function between the road and the seat I write it.

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Properties of The Statistical Parameters

Autocorrelation function is an even function of τ

$R_{\zeta}(-\tau) = R_{\zeta}(\tau)$

$S_{\zeta\zeta}(-\omega) = S_{\zeta\zeta}^{*} = S_{\zeta\zeta}(\omega)$

That means the power spectral density is a real valued function.

And then I can find out what is the power spectral density at the seat location okay that brings us to a very important thing or the properties of these functions. The auto correlation function by the shear definition; remember that there was a xt and xt – tau, so from its sheer definition okay is such that it is a even function or in other words, auto correlation function of –tau = the function of tau okay from the shear definition, so that is the first one.

The second equation again comes from the definition go back and look at the definition okay, you would notice that in e power –j omega t, substitute - omega okay, by I mean by omega by – omega, so you would notice that the power spectral density in other words, what is this, this is nothing but the complex conjugate, complex conjugate okay becomes the power spectral density of omega.

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Two Random Process

Consider to random processes, ζ and η

We define Cross Correlation function as

$$R_{\zeta\eta}(\tau) = \lim_{X \to \infty} \frac{1}{X} \int_{0}^{\infty} \zeta(s)\eta(s+\tau)d\tau$$

$$C_{\zeta\eta}(\tau) = \lim_{X \to \infty} \frac{1}{X} \int_{0}^{\infty} (\zeta(s) - m_{\zeta})(\eta(s+\tau) - m_{\eta})d\tau$$

$$S_{\zeta\eta}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{\zeta\eta}(\tau)e^{-j\omega\tau}d\tau$$

Note that cross spectral density is a complex valued function

$S_{\zeta\eta}(-\omega) = S_{\zeta\eta}^{*}(\omega) = S_{\eta\zeta}(\omega)$

Cross Correlation function is neither odd or even, but satisfies the function:

$R_{\zeta\eta}(-\tau) = R_{\eta\zeta}(\tau)$

In other words, the power spectral density is a real valued function okay, power spectral density is a real valued function and that is what is okay important here that the output power spectral density and the input power spectral density both of them are related by the power spectral density which is a real valued function okay. We can also look at cross correlation functions; these are the cross correlation functions okay between 2 different random processes.

This is very important to understand okay what, we are looking at this whole thing is a random process. Though, we call this as one random variable and another random variable they belong to the same okay that is why we use the term here when we calculate it is auto okay, so auto is what we used because it belongs to the same random processes. So, we can connect 2 different okay, the cross correlation between them.

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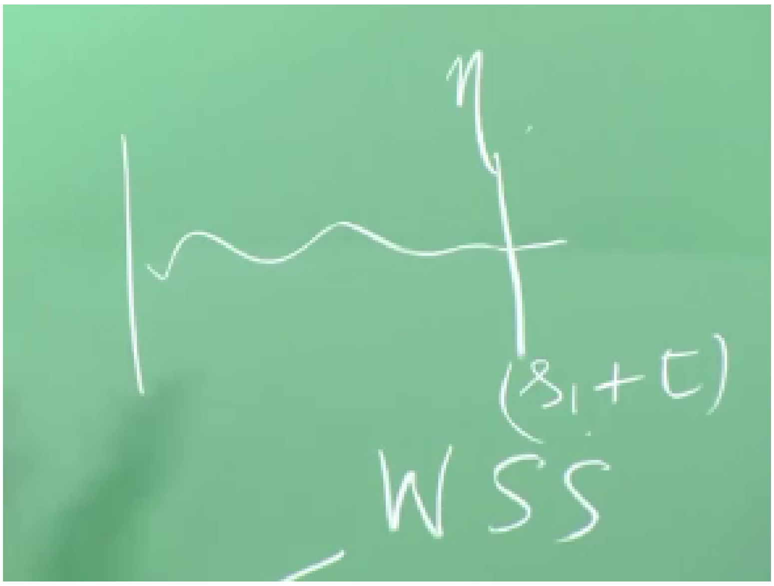

What happens when you give an input here? How is this correlated with another random process which is at a time? So, in other words if I have another random process see; I have a random process and I have 2 random processes okay, the other random processes I will let me call that as eta, it can be anything, you know whatever it can be maybe what happens at the seat location okay or something else.

Now, the cross correlation is the connection; the statistical connection between say for example; that road input at s1, okay to what happens the statistical connection between that and at s1+say tau okay because of the fact that the distances do not now matter because we are looking at something else then, so s has to be okay though we define the road in terms of distance, it is proper that we come back to time rather than stick to only distances.

So, please note that though I used it interchangeably, note that it is always better to look at that in terms of time. So, we define what is called as a cross correlation, cross covariance and cross spectral density okay, so cross spectral density. Note the difference between okay s; the power spectral density eta eta to s zeta eta, okay. So, note that difference, the difference is at the same we are looking at when we look at the YY, you know that is at this place what happens?

Now, there is a delay factor okay, a tau which connects the cross correlation factors okay. Again through the definition, we can see that the cross spectral density is a complex valued function obviously, it is a complex valued function, we also have to look at the phase, because we are looking at 2 different points okay and this is the frequency domain, so we look at the phase as well.

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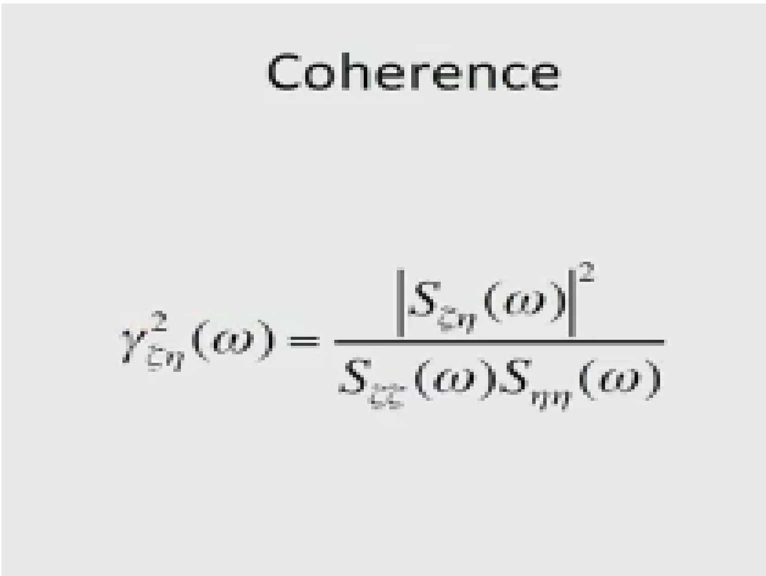

And the cross correlation function is either odd or even and again go back and put that in the definition, you would see that is the expression you would see. One of the quantities see; we saw the cross correlation coefficient okay, the correlation coefficient rather sorry; correlation coefficient, one of the quantities that is in the time domain okay and one of the quantities of interest to us in the frequency domain a corresponding in the quantity in the frequency domain is called as the coherence, it is called as the coherence.

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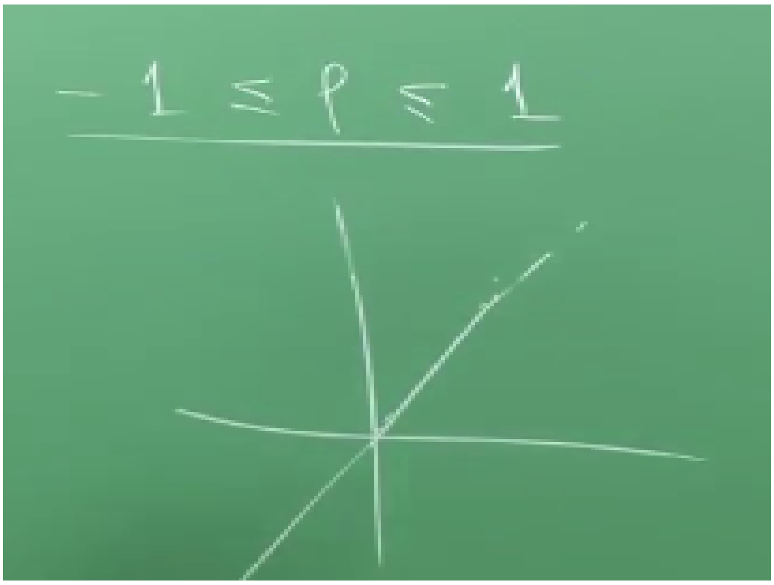

So, all of you know what is a correlation coefficient okay, the correlation coefficient all of you know varies from -1 to +1 okay, rho is between -1 to +1, that is because we had normalized it by means of that variance okay, so it is simple probability, so which means that if there is a perfect correlation then you know that you can plot that the 2 variables and maybe all the points will draw will fall in a straight line.

And if there is a negative correlation you would also know that okay, when one is positive the other is negative and so on okay. So, the negative correlation means that one something increases, the other factor decreases and so on. So, that is the correlation which we defined okay in the time domain. What is the correlation in the frequency domain? You should be able to now imagine time and frequency in a very very similar fashion, the x axis is time. Now, the x axis becomes frequency, it does not matter now, x axis just becomes frequency okay.

So, frequency domain where we look at this whole thing in terms of that f, after we do the 4ier transform. In fact, today there are techniques that are available in order to look at say fatigue life in frequency domain, so you do that in time domain or you can do that in frequency domain. So, coherence which is extensively used to look at, how I mean; look at the input and the output connection in many of the experiments.

Say, for example you would do some experiments in the vehicle dynamics lab in which case you would look at coherence all the time and look at how whether output what you measure is pure noise or whether it is correlated English word, correlated with respect to the input okay that you look at it in the frequency domain. If the coherence is not near one then okay suppose, it is say 0.9 to 1, then you know that there is a good coherence between what you gave.

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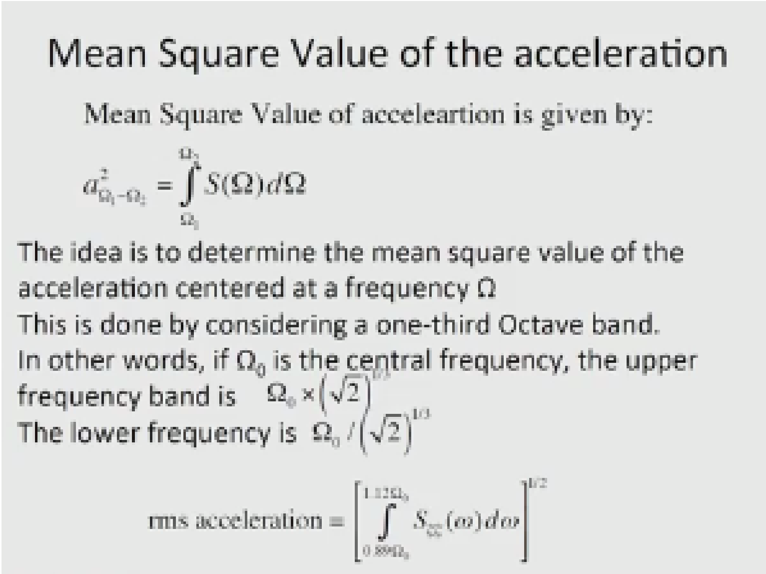

Or in other words, what they are measuring is due to what you gave as an input. If there is no coherence then you would see that what you are measuring is noise and has nothing to do with what you have given as an input. So, coherence is an important quantity especially you would be using that in the experimental work. One of the quantities of interest is what is called as the mean square value of the acceleration.

So, why are we doing all these things? Okay ultimately, I measure or I calculate, what is the power spectral density at the seat? Okay, we know that. Now, we have defined quantities, we know the connection between power spectral density at the road, power spectral density at the seat, we know that it is everything is done, now okay. Why am i doing this, what is it that I am doing this, I mean; what is it that I am going to get out of it?

One of the simplest thing you would notice is that the acceleration levels okay what you; say for example, feel when you do a drilling or something where you are subjected to accelerations are very important input of vibration to our body okay. In other words, our tolerance levels depend upon the mean square value of the acceleration okay. Now, what is the mean square value of acceleration?

Remember; Y squared the same thing okay mean square value of this now instead of Y we say that it is the mean square value of the acceleration, we know that this is nothing but the integral of the power spectral density, this is exactly what we did in this problem, so power spectral density in a band omega 1 to Omega 2 okay. In other words, we have a central frequency like this and we take this band from omega 1 to Omega 2 okay.

These band omega 1 and omega 2, it is called as an 1/3 octave band, it is called as an 1 /3, there are 1/6 octave band and so on, 1 /3 octave band, if the upper frequency or upper bound, this point is given by root 2; 1/3 of I mean power of root 2 or the lower frequency is given by as you see divided by third, the cube root of 2 and so on. When you calculate that that becomes 0.89 to 1.12 omega0.

Octave has a very important parameter and we will see that okay again, next goes on noise; the noise and vibrations. So, the RMS acceleration is now calculated okay based on the power spectral density, this RMS acceleration is related to our ability to withstand vibrations okay. Now, we have various; so I can calculate for various frequencies what is the RMS acceleration from that equation okay.

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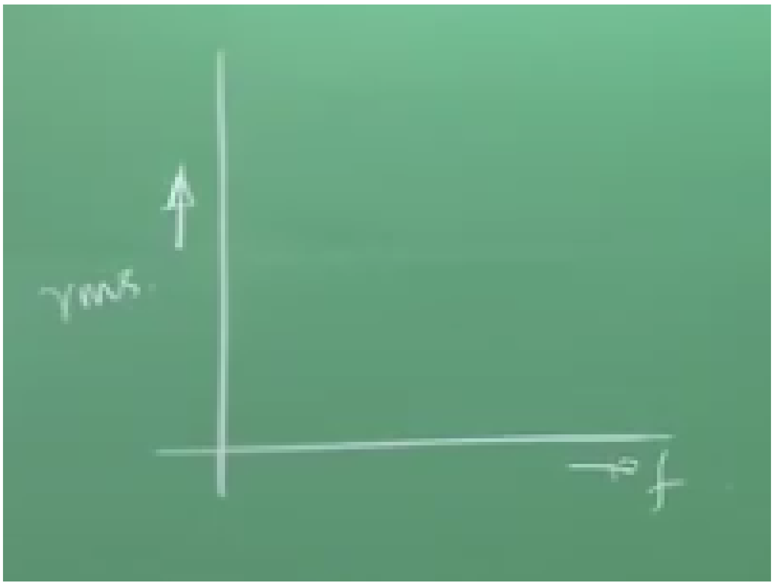

So, in other words I can have a plot of frequency versus the; 1 minute, versus the RMS acceleration. Yeah! **“Professor - student conversation starts”** No, No it does not matter, I am looking at it anywhere power spectral density, so you can take that power spectral, it is a general expression, so we are talking about power spectral density at the seat. So s, okay; No, No, this expression here; this expression here is now substituted in terms of the seat okay.

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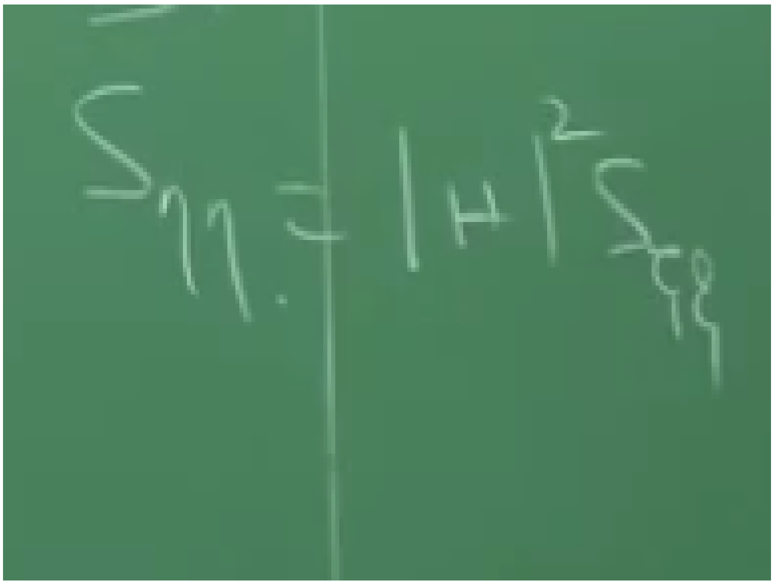

If you want to call this as eta eta, you call this s eta eta, okay, so suppose this is the road input okay and this is the seat, why is the seat output and then if I calculate the power spectral density at the seat and call that as say; s eta eta or s eta whatever it is okay, which is nothing but remember that H squared value right, so I am talking about this value, so take it to the seat okay that is what goes inside that integral okay.

And take; one minute, let me explain this process and take 1 omega okay and then put that as a band and calculate 1/3 octave band at that value, so I get one value for a mid frequency of omega by integrating that value, that is the RMS acceleration okay for a frequency; that frequency. So, like that I will have RMS acceleration at various frequencies acceleration; accleration with function of frequency; yes root mean square value of the seat position.

But note that I am calculating or I am measuring, what I am measuring as an output okay that power spectral density of the acceleration say; for example if I measure the power spectral density of this output okay is what I measure is s, so if I can measure anything I can do, I can measure whatever I want. Suppose, I put an accelerometer usually what I do is to put an accelerometer and measure it.

And so what I get out is the power spectral density of this acceleration measured from the accelerometer and that is what I am going to use okay. It does not matter; the simplest and the best method is to put only an accelerometer and measure it okay. Now, that is the; so this is called as the RMS acceleration, right. **"Professor - student conversation ends"**. So, usually you can put an accelerometer at the seat, you can put an accelerometer at the back okay.

And you can put an accelerometer at the floor and so on. Now, a lot of tests have been done from the point of view of human tolerance human vibration tolerance okay and it has been found that there is a difference about you know the tolerance level of a human being whether we are subjected to say a seat vertical acceleration or horizontal acceleration also. I am sure you have been; this has been done to you.

And we have talked about a very famous standard 2631 okay in your earlier course on ergonomics because that is what essentially the whole of ergonomics is all about that standard 2631, so huge standard, we do not have time here to cover it but nevertheless once I know how to calculate that RMS acceleration you have what are called as weightage factors and these weighted factors are applied which are different.

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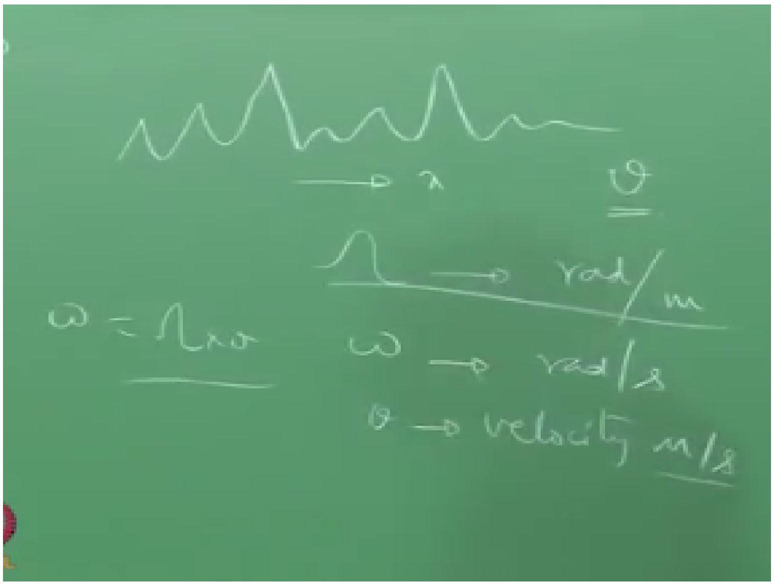

Whether it is a vertical vibration; whether it is a horizontal vibration and so on. So, with these weightage factors, we will understand what the tolerable limit is whether we are going to be comfortable or not okay. We will talk about that in the next class. The last of the topics that we need to cover which we will do in the next ten minutes is what is called as the Road roughness. Of course, you had already seen what power spectral density is.

You already saw that power spectral density is very important input to this whole problem and that road roughness is actually the input, remember that many of the equations you can either solve it in the frequency domain or in time domain okay but nevertheless if I have to give the road as an input as a power spectral density, it is in the spatial domain in the words; when you look at when you go and measure what is called as Road profile okay.

This would be in terms of distance x and the actual power spectral density okay from which you can get the correlation; auto correlation and so on, is actually measured in terms of time, so we have to have a relationship between the spatial frequency and the temporal frequency. In other words, when the velocity of the vehicle is v, obviously the frequency with which they the vehicle gets excited would be a function of v.

So, in other words, if omega is the; what is called as; we will call this as a spatial frequency which means that this is expressed in terms of radians per meter okay and for the problems that you do, you require the same thing in terms of say; radians per second, so I have to convert radians per meter which is road characteristics that has to be converted into characteristics as an input into the system into the vehicle.

And that obviously comes from v the velocity of the vehicle, which is expressed in meters per seconds. So, obviously omega which is the radians per second is given by capital omega, spatial frequency multiplied by omega, okay. So, this becomes omega is radians per meter per second, so this becomes radians per second okay. So, this is the first thing that we need to know understand when we convert the spatial frequency or spatial power spectral density into a power spectral density which goes as an input.

I do not want to call this as temporal and confuse you, just that as an input when it power spectral density when it goes inside all our calculations, then it should be in terms of the; the omega should be in terms of what is called as the temporal frequency that is meters per second okay. Now, so this is a very good nice way of distinguishing between the road and the vehicle. So, that brings us to the topic of how do you specify a road roughness.

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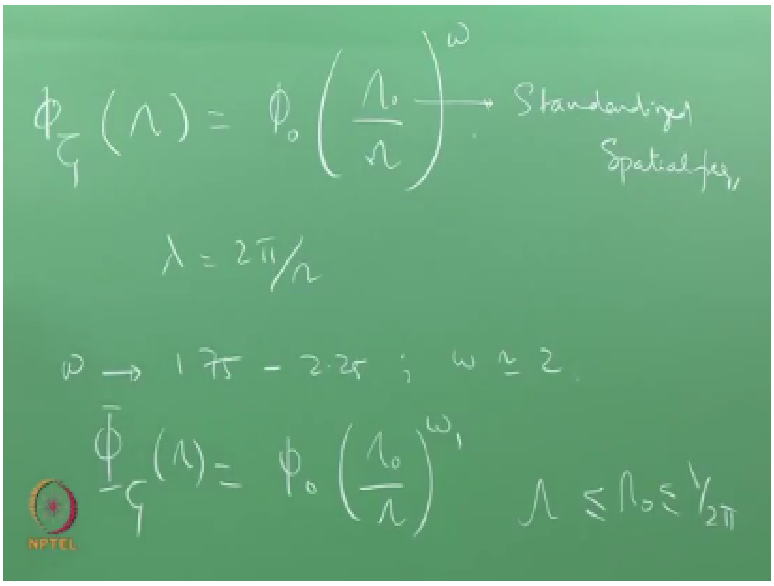

In other words, how do you characterize the road? There are a number of ways in which you can write down the equation for the power spectral density of the road okay.   Let me call that as the power spectral density of the road. The power spectral density of the road is written in terms of capital Omega which is means that it is written in terms of radians per meter is given by a characteristic power spectral density phi0 multiplied by omega0 / omega whole power w.

The W actually indicates the waviness and w varies from okay of course, you have the wavelength to be 2pi/omega obviously, you know that and w actually varies from 1.75 to 2.25 practice is to put w=2, omega0 specifies a characterized in a frequency a standardized spatial frequency okay, it is a standardized spatial frequency, we will see in the next; you know equation how we are going to characterize the standardized special frequency.

This is one of the first equations that were written, the equations were further modified later okay and there is an ISO specified power spectral density of the road, so you want calculate the power spectral density for in terms of the temporal frequencies meters sorry; radians per second, we will do small manipulation to that. Now, the ISO came up with a small modification to this.

And they said that if you plot it in a log log plot then there are actually 2 different; you know slopes in a log log plot of the frequency versus the power spectral density. So, they gave this actually, this is a log good to plot it in the log log plot because of the type of equation you have so, the equation that you get is something like this and for omega greater than omega0, the equation is equal to the same similar type.

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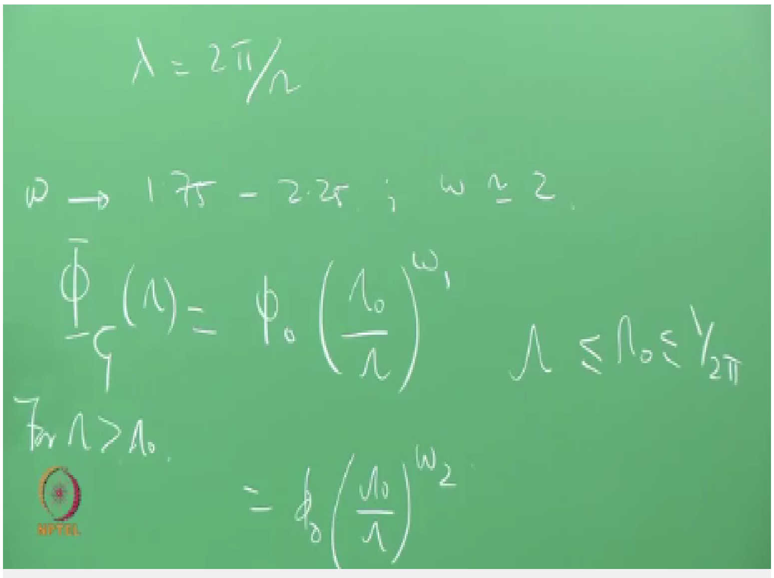

Only thing is that omega to the power w2, so in other words there are 2 slopes okay and they are characterized by 2 equations or 2 w; w1 and w2. Now, the good roads and the bad roads are characterized by these w1 and w2, so if you really look at that.

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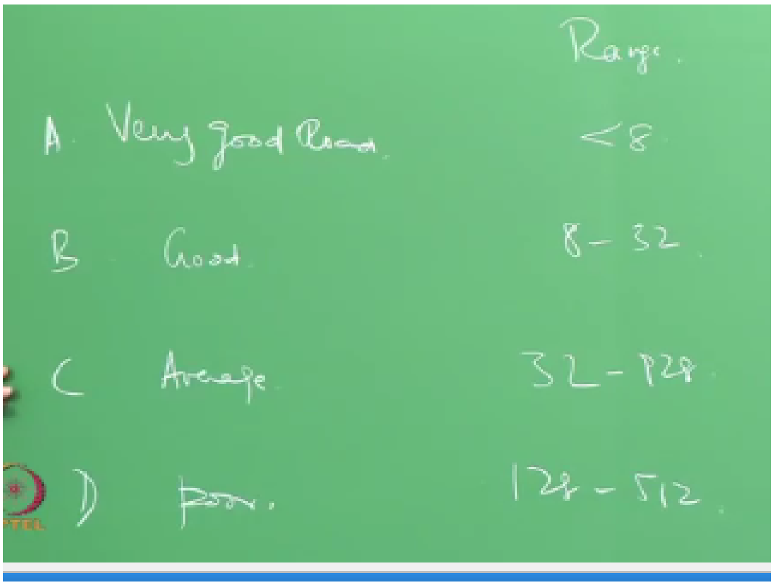

For example; for a very good road these standards are something like this, for a very good road the range of this w1, w2 is that is less than 8, which means at 0 to 8, the geometric mean is 4 and this is called A class road and for a B class road which is a good road, this range is 8 to 32 okay and that is the finite value; the finite value okay and then C which is an average road, the values vary from 32 to 128 and so on okay.

There is a poor road which is 128 to 552; 128 to sorry; 512 and so on. The usual practice is to have w1 to be 2 and w2 to be 1.5. So, in other words, the lower bound for an average road if you want to do an analysis, the lower bound for an average road, you replace this phi0/32, okay that gives you the lower bound okay, of course omega0 is 2pi okay and phi0 is 32 and for a upper bound, you replace that phi0/128 okay and put that here right.

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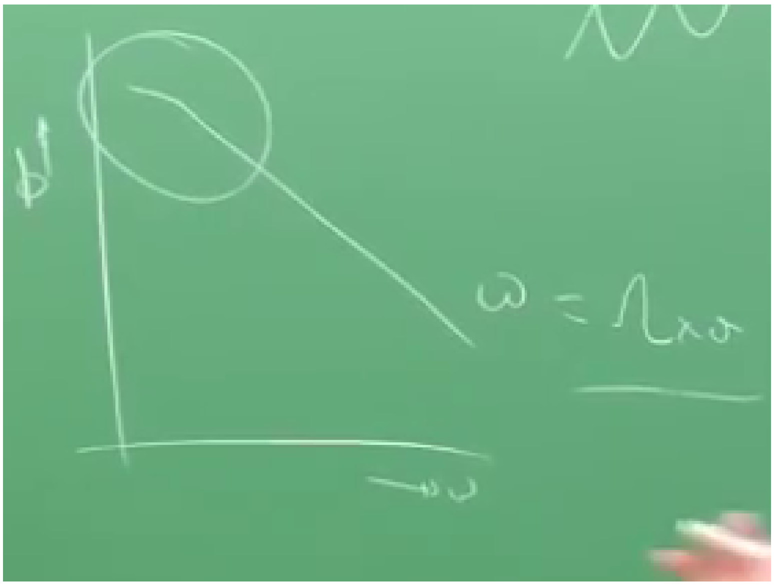

So, you can have for an average road, you cannot; I can give you one figure okay it varies from 32* 1/2 pi omega whole power 2 and then again the same thing whole power 1.5, when omega0 changes. This is a very standard; ISO standard but people in recent times have issues with this; the issue is that when omega, the frequency okay, if you really look at a graph; the graph would be something like that with respect to omega versus this phi.

There have been issues with this equation; the issue with this equation is that when that Omega; the Omega value when it goes as it tends towards 0, the power spectral density now shoots up okay and goes to infinity the variance reaches infinity at omega =0, when omega =0. So, that gives us a very unrealistic value, when you reach omega okay, when you move omega to be closer to 0.

In other words, this would start this part the error is very large okay because of that fact that as omega tends to o okay, this one tends to infinity. So, in order to avoid this in fact, just one statement is very important that essentially what is that you do you? You put down an equation, so if you want to like for example, our road if you really want to model your vehicle in this road then you have to do a profile measurement.

And then you have to fit a curve okay which is given by an equation which you are going to write down; which you have written down. So, it is very important that you identify a road with an equation of this form or an equation which we are going to put down because of this difficulty okay. There are a number of issues for example; this gives what is called white noise but the roads are coloured noise where when there are a number of parameters.

We are not going to cover all that in this course but there will be a course on NVH, if any of you want to take it, maybe next semester this, after this 2 semesters from ends, this is the January session where the first part of the course will cover a lot more on random vibration as well as on signal processing and there we will be covering more detailed analysis of the road as well as how you know what is white noise, what is coloured noise, what is shape filter.

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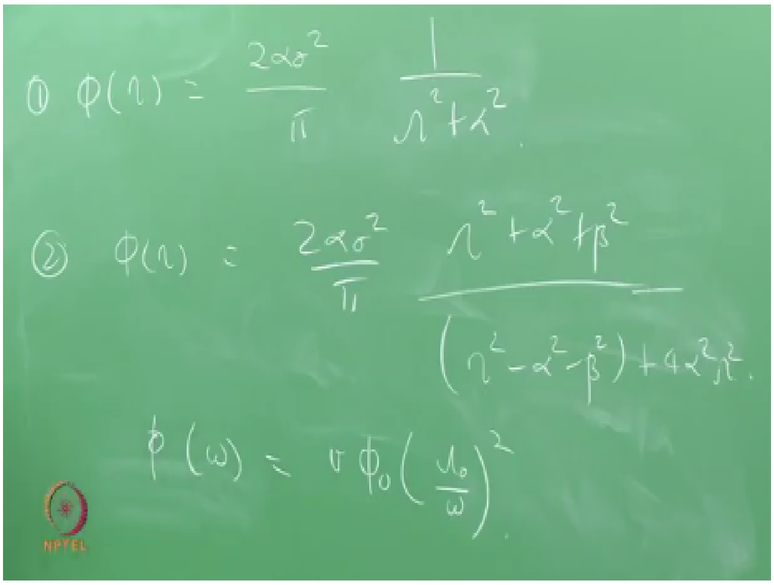

What the shape filter which converts white noise into coloured noise all those things will be covered there. So, the only thing I am going to write down before i close this topic that there has been changes in the way the power spectral density is written because of this difficulty that omega makes this omega, omega tends to 0, the power spectral density goes to infinity, so in order to avoid that they have been other; this kind of unrealistic behaviour is avoided by using other types of equations.

Or that is one type of equation, the other type of equation that is written and so on, so these are other types of equation this which gives you a more realistic picture of the actual road measurements especially, as you move towards the omega to be small, you give you get much better in the picture of this and of course, when you want to convert it into omega, substituting for v and so on.

So, you would notice that we can write down v phi0*omega0 say for example, if it w seems to be 2, you can write down that is the equation. There are lot more issues in in random vibration because of lack of time in this being the last class, I do not want to go into further details, people who are interested in some more details can slightly more information again look at (()) (01:06:24).

Others who want more details on shape filter, white noise, coloured noise can look at the paper by (()) (01:06:39) in the journal Sadhana okay. We will stop here and this course will end with this and we will continue that in the next course okay. Thank you.