Vehicle Dynamics
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Lecture – 04
Brake Force Distribution, Braking Efficiency and Braking Distance

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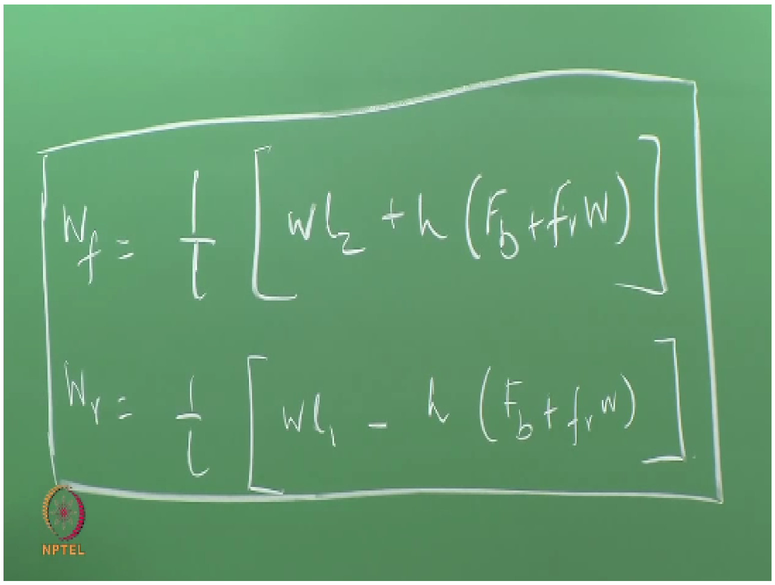

So, in the last class, we were looking at braking, maybe stopped just sort of this or maybe we did this derivation. A very simple derivation. There is nothing much in this. We looked at the moment equilibrium and then we derived this, and we also made a comment that there is load transfer in braking where the front forces are high or the front reactions are high as the load gets transferred to the front. Unlike the acceleration part where the load gets transferred to the rear, this is what we saw.

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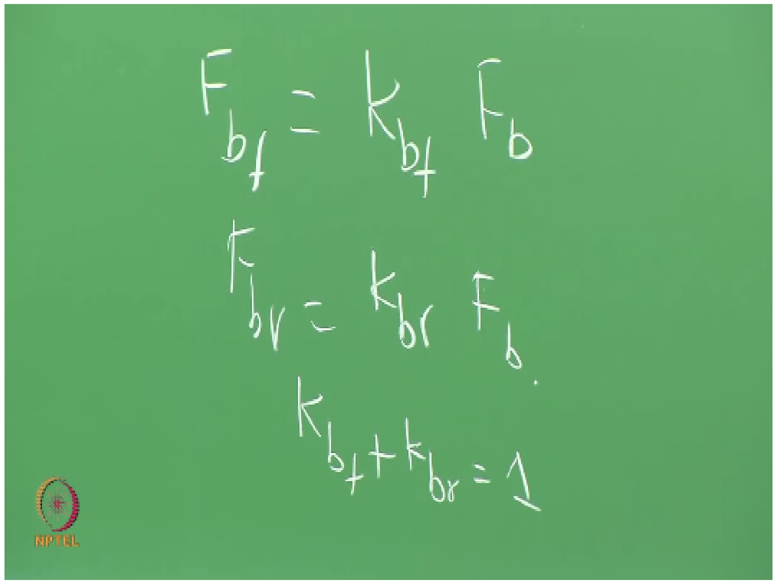

We also said we went to the next step and we said that we can dump the whole of the effect of tyre on to the road, then the whole concept of this interaction into a factor called mu which is very similar to your well-known friction condition, the Coulomb’s friction and said that W*mu will give me a limiting condition for the wheel to lock. So, in other words, the maximum force that it would take front or rear would be given by this condition.

We can put that the maximum braking force in the front is given by Wf and the rear is given by Wr. We also said that the total braking force is split into the front and the rear, unlike we have cars which is front wheel driven, we have some cars which is rear wheel driven, we have vehicles which are all four wheels being the driving wheels and so on, braking is distributed in the front and rear. In other words, obviously you know that you have front and the rear brakes, okay.

So, the total braking force is now distributed between the front and the rear and let us call this fraction of the braking force which is distributed to the front and Kbf. So, the braking force in the front can be written in terms of into Fb. In same fashion, you can write it for Kr, Kbr and braking force at the rear=Kbr*Fb. So, obviously it is a fraction. So, Kbf+Kbr=1, fine. Now, let us look at the physics.

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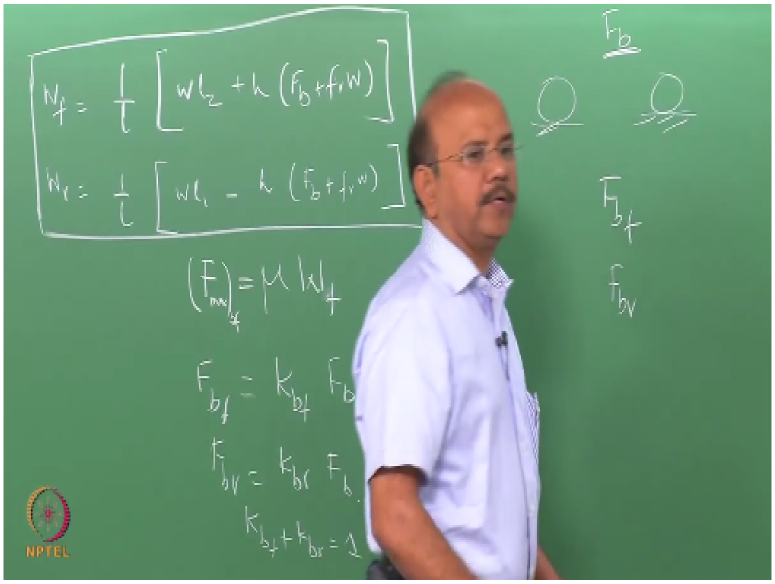

We have 2 wheels that are braking, i.e., 2 sets of wheels rather, front and the rear wheels which are braking. Now, the front wheel brakes when the condition that the force generated in the front wheel is equal to mu*Wf, obviously and the rear wheel brakes in the same fashion. So, assume for a moment that they are randomly distributed. Kbf and Kbr are randomly distributed or the Fbf and Fbr are randomly distributed, you do not know, just one value, okay.

What would happen, one of the wheels will break first, the front or the rear wheel, okay. So, what is this condition. The front wheel breaks, okay, which means that it starts giving. Under one circumstances when you given more force to the front, okay. So, look at it this way that the wheel there is no skidding. The rear wheel, okay, is not locked. So, it still has some force that is unused for braking. Hence, optimum condition is one where you use the complete force Fb to brake the vehicle.

So, when will this happen when both the front and the rear wheel they lock together. Only, when they lock together, you do not lose any braking force, okay. So, that is an ideal condition where you distribute the forces in such a fashion that both of them start skidding at the same time. When would this happen, when you distribute this braking force, whether the front braking force or the rear braking force.

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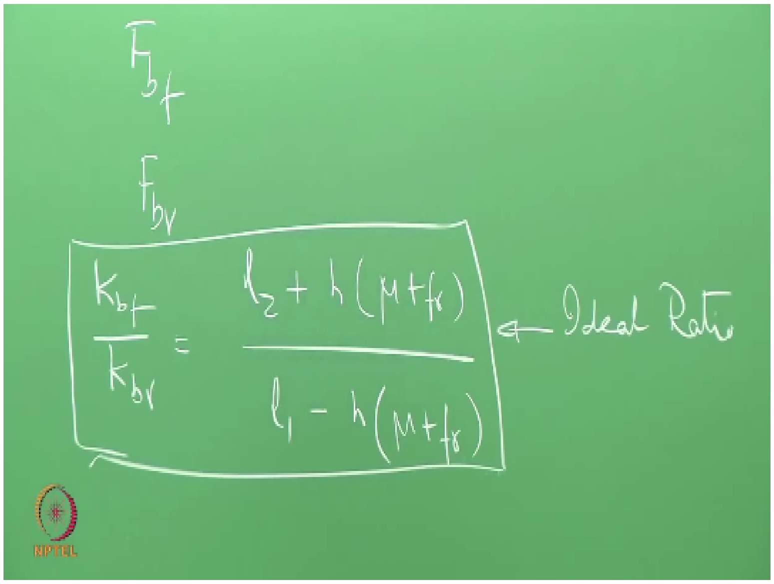

You will distribute it in such as a fashion that it is proportional to, when it is locked, when it reaches mu*Wf. So, how would you now distribute both of them propositional to Wf and Wr. So, in other words, if you have the braking forces to be distributed, okay or the maximum braking force develops at the same time means that they are distributed proportional to the front Waf and the rear force or the rear reaction Wr.

So, what is that condition. We simply say that Kbf/Kbr should be equal to, okay both of them is proportional to Wf and Wr, okay. So, this in a very simple fashion can be written as L2+H/L okay and I am going to replace Fb, the braking force by means of the deceleration quantities or you can say in terms of mu*Wf. So, you can write this as +H*mu+Fr and Wr you can write that as L1-H*mu+Fr, okay.

So, if the 2 forces are distributed proportional to Wf which I have just rearranged it, so in this fashion that is an ideal braking where both the front and the rear would brake at the same time. Typical values for this would vary from one position to the other or one vehicle to the other, and even for the same vehicle it would vary. So, though we can that this is the ideal ratio, it is not a constant even for a vehicle and definitely it would vary from one vehicle to the other, right.

Why would it vary for the same vehicle for example, because when you load the vehicle, it is not necessary that L1 and L2 which are the distances from the CG locations are the same, okay it may vary. For a truck, it would vary a lot more than car obviously. So, when you load for example a truck, then definitely the CG locations can shift to the rear.

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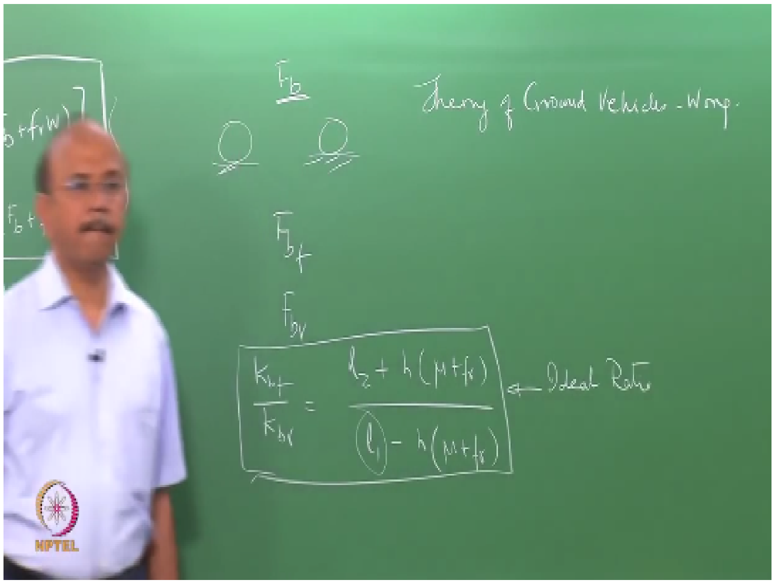

Incidentally, L1 in many of the textbooks and this we are following theory of ground vehicle from all this part, one of the books I think I forgot to mention by Wong, okay. We will shift from one book to the other and our own experiences. We will look at this course. So, one of the things I just want to point out is that, instead of L1 and L2, people also use A and B. A and B is very common. The notations A and B, A being the distance of the CG from the front, B being the distance of the CG from the rear, okay.

So, I used L, l, you know interchangeably both of them means that it is just the distance between the 2 wheels, okay. So, this is the ideal ratio. So, why would it vary within a vehicle or for a vehicle. Obviously, when the road conditions are different, mu would be different. Mu would not be the same. So, mu would change, okay. So, for the same vehicle, A and B or L1 and L2 are not a constant. I did not use A here because you may confuse it with acceleration.

That is the reason why I have not used A and B here, okay. I used L1 and L2 at this place. So, this is a problem. So, you do not have an ideal, that is why you have so many other, you know, devices like ABS and other things, okay in order that we do not get into conditions which we would discuss probably later in the course, okay. Now, that is the first thing. Then, what is usually used. What is the ratio that you use, varies. Many of the truck companies today use 50% to 50%, okay.

Some of the car companies use 47, 53 and so on, right. So, it is a very difficult ratio to arrive at because the same vehicle there would be a change and hence, we have this problem, okay of fixing a ratio, right. Now, why are we worried about locking. What happens during locking, okay. Now let us understand that.

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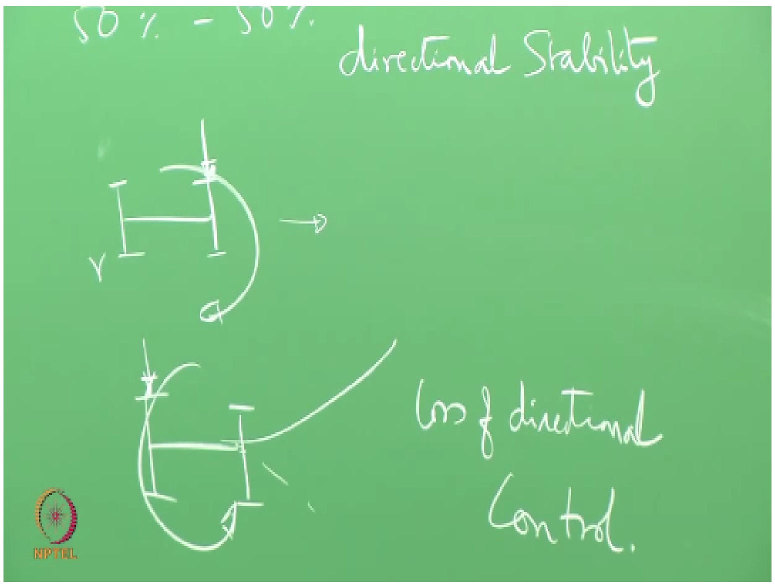

Let us say that I have a vehicle, just to schematic diagram of vehicle, okay which is traveling in this direction, of course and let us say that one of the wheels locked. Let us say that rear wheel is locked. What is meant by locking physically or from a physical principle. Locking simple means that all the mu that is available for that tyre has been completely consumed, right. So, no more that mu factor, the friction factor is available for any other force that axe the wheel, right.

Let us assume that the rear wheel have locked, all mu has been consumed. Now, there is a small perturbation to the vehicle. There is a small perturbation. It is only in theory that the vehicle would run straight. There would be a camber of the road or there can be a cross wind or there can be a small unrelation or a small bump, whatever it is. There are perturbations in the roads and you cannot always drive straight and due to so many reasons, your vehicle has to develop a lateral force, right.

So, it has to develop a lateral force. This guy here who is the rear, okay, he has lost the ability to support any lateral force and so, only the front wheel would develop the lateral force. So, when the front wheel alone is capable of developing a lateral force, that is the situation, okay. All of the lateral force that is required will be developed by the front wheel. So what happens, the vehicle starts spinning or in other words, the vehicle loses what we would call as a directional stability.

So, it would spin. In fact, it can spin so much, depending upon the perturbation, it can just spin and even you know go 90 degrees or more. So, when the rear wheel locks, the vehicle loses direction and stability and would start rotating or yo-yoing as it is called. On the other hand, let us consider what happens when the front wheel locks.

When the front wheel locks, obviously it loses the ability to take further forces and so, all the force that is required to cater to the needs of the changing orientation of the vehicle, the lateral force is at the rear. So, the vehicle would now lose directional control. A small change this side would not happen because the vehicle would now be yo-yoed in the other direction, so would now start going like this, okay.

So, we call this as loss of directional control, clear, okay. Now, if you are a driver you have all the feel for the front wheel because your steering would give you the feedback. So, when the front wheel locks or front wheel is about to lock, you know that something is going wrong and it is possible by an expert driver to get back that kind of loss of feel, okay. In other words, to steady the vehicle.

On other hand, if the rear wheel locks, even an expert driver cannot feel it ad by the time it realises, everything is over. So, we need special devices in other to avoid this kind of locking and loss of direction or direction stability and directional control, it that clear. Now, let us go into details of this braking further. One of the key questions which we are going to ask is what is the deceleration that can be achieved, okay by a given braking conditions.

What is the maximum deceleration that I can achieve? A very important question, because ultimately brakes are also judged by deceleration and more importantly the braking distance, okay. So, in order to understand this, we need to understand what would be the deceleration level before the brakes start locking, right. I am going to take 2 equations and then just equate them and then get the what is called as the maximum deceleration when the vehicle loss.

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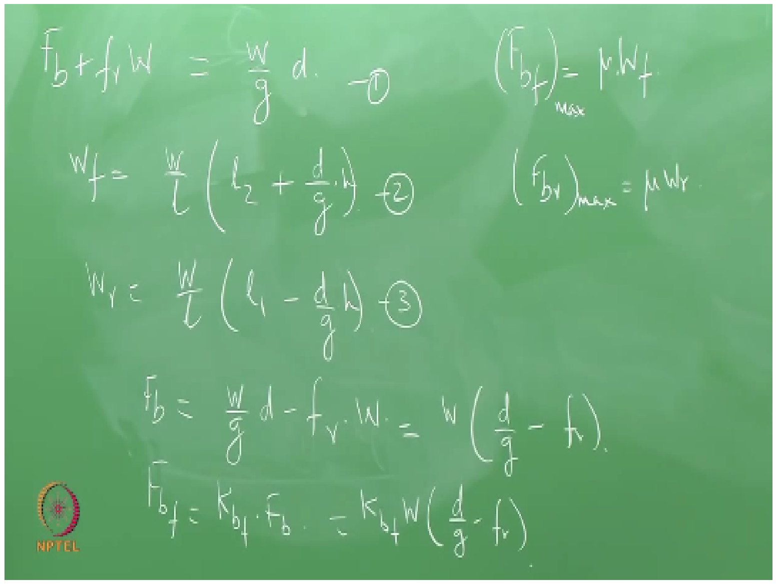

Remember that our good old friend rolling assistance is going to help braking, okay. So, one of the conditions that we already wrote neglecting all other forces, of course. Let us say we neglect all other forces like aerodynamics. So, you can add them, that can be an exercise to you just to bring out the physics. Let us look at only these 2 forces. So, this will be equal to, note that we have replaced –A by D and hence all the directions are now, so D is actually the deceleration.

This is one equation. We also have an equation for Wf and Wr and that can be written as, how did we write it. We took W out. What is the first term for Wf L2+, what was the term which was inside Fb+Frw. So, what can you do from this equation. You can replace that what was in the bracket in the previous case by this and can I now write this as the d/g. So, in the same fashion, I can write down Wr=W/l l1-d/g.

Okay, what was in the bracket, I just replaced it by the first equation. Now, let me write down Fb from the first equation, call that as first equation for this, second equation, third equation. Fb=W/g-Fr*W. I also know that the front braking force=Kbf*Fb, okay and the rear braking force to be Kbr*Fb, right. So, which is 1-Kbf*W and so on. I also know that Fbf will lock, i.e., the front force is maximum is equal to mu*Wf, okay.

The maximum before it locks is that and Fbr max=mu*Wr. So, now what I am going to do is, so let me just re-write it a bit, W/d/g-Fr, okay. So, this would be Kbf=W d/g-Fb, right, okay. Now, what I am going to do is, under the maximum condition. Note, that this is valid only for maximum condition, okay. I can equal this with this, okay and rearrange the term.

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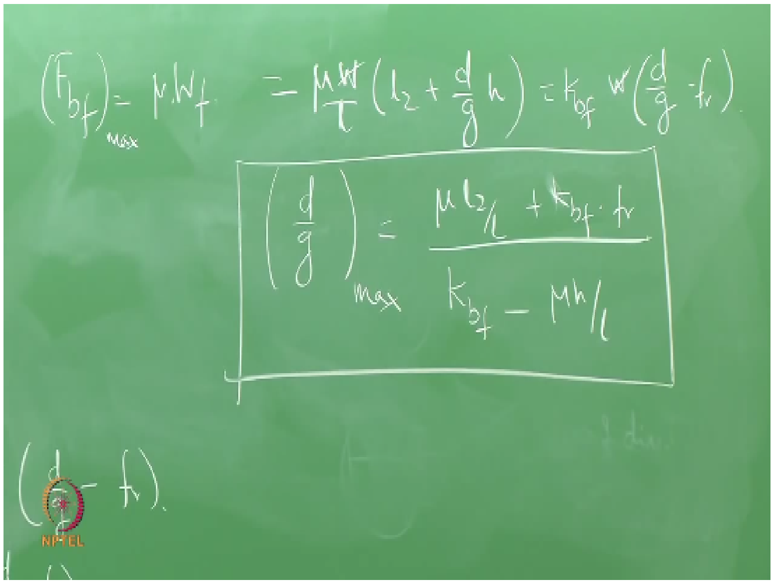

So, substituting from equation 2, I can write this as mu*W/l l2+d/g*h, okay. That is equal to Kbf=W d/g-Fr. Let us for a moment remove that. Let us only look at this first, okay. This is when, what is this condition, when we reach the maximum level before it starts locking. So, rearrange the term and then I get this d/g or in other words the deceleration, normal x /g. We will talk about it in a minute, just rearrange it and tell me what can it be, okay. Take it to the other side. So, bring it to this, not very difficult. Remove that W.

Mu l2/l+Kbf*Fr/Kbf-mu h/l. So, that is the maximum, maximum means what, when it locks. Please note that we have normalised with G and this is the language vehicle dynamics use. Whenever they talk about acceleration, this will not talk about meter per certain square and so on. They would usually normalised to G and talk in terms of this normalised quantities. They would say that it is 0.3g, 0.2g, 1g and so on. Hence, this expression normalised with g.

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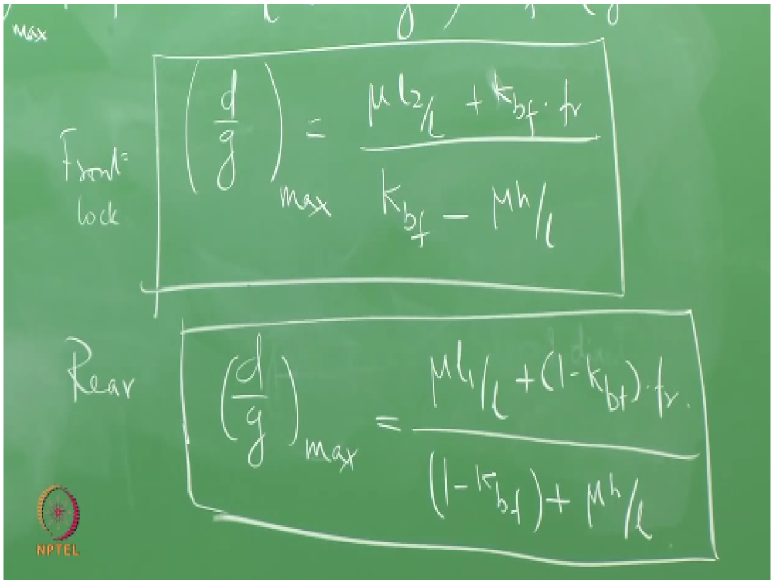

Derive the expression for, when is this condition, this is front wheel locks, okay. The maximum deceleration that is possible when the front wheel locks, okay. Now, derive what should this be when the rear wheel locks. What is the condition, just put this, substitute it, and rearrange it. See that you get this, this would be the condition. The same thing, I have 2 versions of the same equation. I am equating it, rearranging the terms, and I am getting this, okay.

So, front lock, let me call that and that is the rear lock, right. You have to be very careful in interpreting this equation. For example, Kbf=0. Let us say, possible, I have all the braking to the rear wheel. Kbf=0. What happens to this equation becomes -, is that correct? Kbf=0, it becomes -. Why should it become -? The whole crux of this thing lies in this equation. They are very simple, all of you know it, okay.

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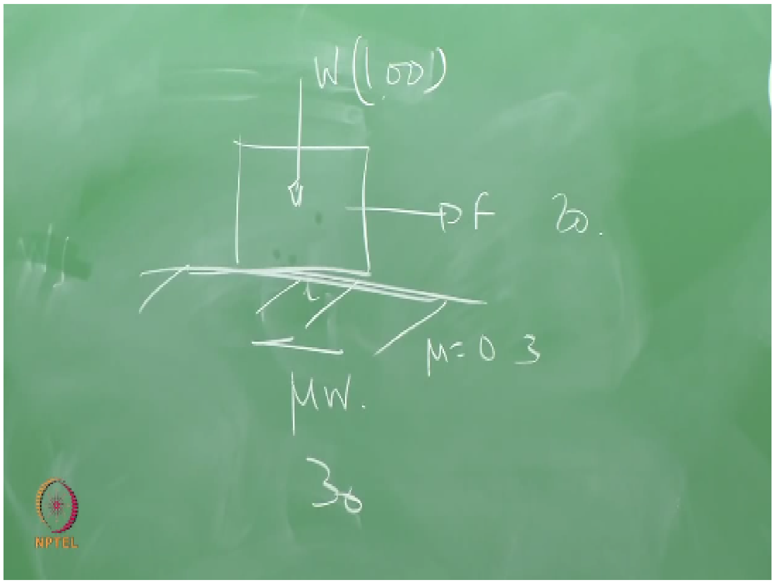

So, it is very important that it is an inequality, okay. So, it is an inequality. It is very dangerous to put, draw for example for friction to put this as W, many books do that and then, say that f, okay and then put this as mu*W, okay. Now, let us say that this is 100, let us say that mu=0.3. So, if you put a diagram like this, what would happen this would become say 30, okay. So, I apply the force as 20, okay.

The equilibrium dictates that actually the block moves in the opposite direction, okay. So, you have to be careful in interpreting it because very correctly as I said, it is an inequality. So, whenever I used equality which means that is the condition under which we overcome the friction or else if you substitute it as you like, you would get into trouble and you will see like here that block would start moving in the opposite direction which is physically not correct.

So, this is one word of caution you know it, sometimes you are stuck when derive it and that is very important that you understand this, right, okay. Just now with that in mind, let us now interpret what happens when I now change Kbf, right.

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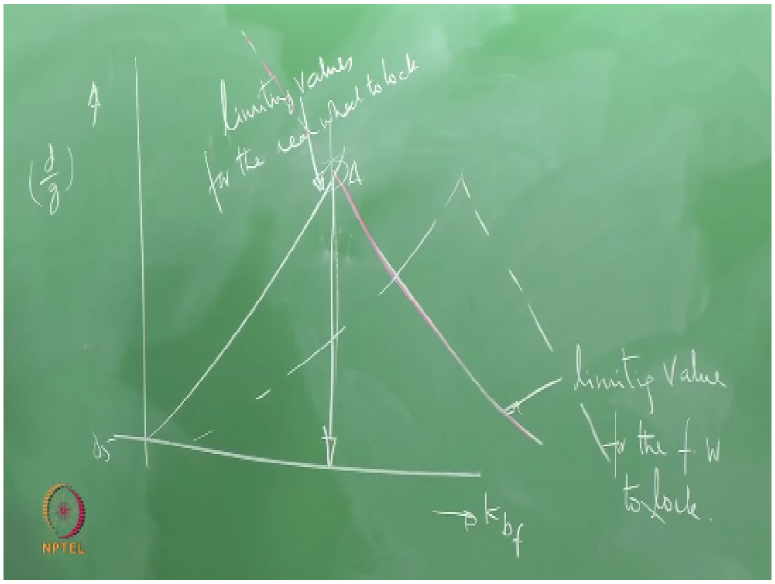

With that in mind, so it is a very important rider, okay. Let us say that I plot Kbf in this axis and I plot my deceleration, okay. So, let say that d/g in the axis. As I keep increasing Kbf, if there is no force in front, so which will lock, the rear only will lock. So, when I keep increasing Kbf, more and more forces would go to the front. So, if I now draw, let us say that is about 0.5. If I now draw a limiting case for the rear wheel to lock, this would be the case, or in other words, the deceleration at which the rear wheel would lock is this, okay.

So, that is the limiting values or the values at which the limiting values for the rear wheel to lock. Why is it increasing because the force of the rear wheel is now increasing, right? That small block which we drew the force is small, okay. Now, when I increase Kbf that means I am giving force to the front, okay. So, actually at some point that would start decreasing. So, that would be the curve at which the limiting value for the front wheel to lock.

Look at that carefully, actually I have 2 curves, let us call this point as A. If you look at the points which is to the left of A who will lock first, the rear wheels. To the right of A, the front wheel will lock. So, at point A what would happen, both wheels will lock. I have used all my braking force optimally when both wheels will lock. So, that would be ratio at which both wheels will lock. Unfortunately, this is not as I told you some time ago. This graph is not a constant.

So, if there is a shift of L2 because its weights are higher or something like that, then it would take maybe something like this, okay. So, this would be depending upon the weight as well as depending upon the mu values, clear, okay. So, that is as far as deceleration is concerned. What is as important as this for us is the braking distance. Though many companies put a lot of importance on braking distance, okay.

If suppose you are a tyre manufacture, one of the things that would be required of you is to meet the braking distance, okay. We spent a few minutes on the braking distances. Though I said braking distance is important, of course distance depend upon deceleration. So, there is no question of removing this and talking only about the braking distance, right. Let us look at the ideal condition.

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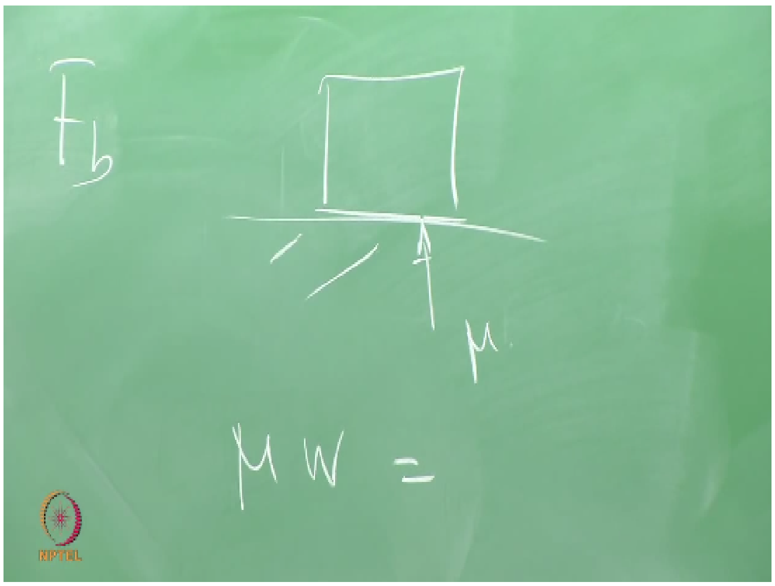

What is an ideal braking? When I apply a force Fb, I look at the car as one unit, just now what we drew. Let us say that it has wheels and blah, blah, blah. So, there is no issues about it. Let us say that mu is the factor. So, what would be the ideal deceleration. Ideal deceleration is mu*W which is the braking force. Note that, I am not going to use as I said inequality, I am just going to use equality, so you will know what is the condition at which I am writing, okay.

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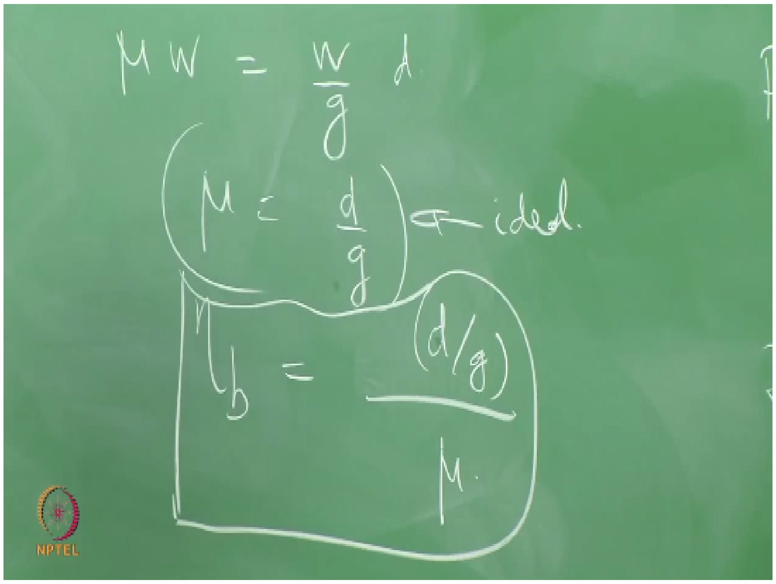

So, this is the force that can be withstood by the tyres total, okay, ideal condition. This should be equal to w/g*d. In other words, mu=d/g. I said we always normalise it with g. So, when g/d=mu, that is the ideal condition which we can achieve. Any other condition that you cannot achieve it, you cannot overtake this condition. You can achieve only this. Hence, we define an efficiency of braking. I have called this is as an ideal or the best condition possible.

So, obviously efficiency depends upon the best condition that you can achieve. So, if I call this as an efficiency of braking, so the efficiency of braking is given what you have achieved with a particular brake in vehicle which is d/g that is normalised with g. Note that d is deceleration, I keep repeating so that you do not get confused, divided by mu, okay. So, that is the condition. Brings out all the simple facts you know that when mu is less.

For example, you are driving in water or ice or whatever it is, the mu dictates the braking force, decelerations, and so on. Now, we are looking at the total force, Fb also goes to decelerate or in other words when there is a deceleration, the deceleration also is for all the revolving components. In fact, this becomes very important in a motorcycle or it is important not to that extent in a car and so on, okay.

So, in other words, since there are rotating components which also have to be controlled or has to come down, okay. Rotation initially has to be taken into account. So, actually the whole braking becomes quite complex if I have to write it. In order to avoid that complexity, what you usually do is to put a factor to the mass.

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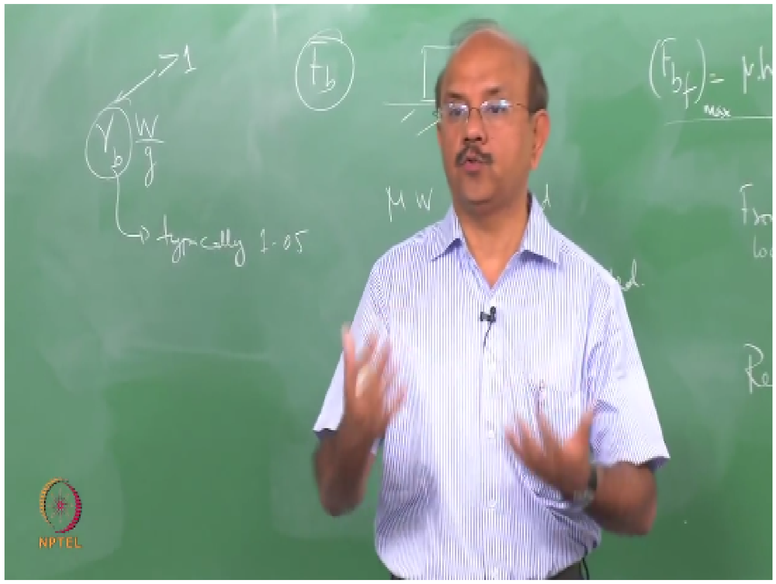

So, you say that if W/g is the mass of the vehicle, then you say that I would multiple it by a factor which I would call as gamma B, okay. So, in other words, since I need to accommodate all those guys who are rotating, I would say that this is as if I am braking a vehicle with a higher mass. So, gamma B>1, of course and it is typically 1.05.

Not set rules, okay, for some of the vehicles and for motorcycles and other things where you can apply similar things, if can go very high, it can be 18%, 20% and so on, okay, 1.15, 1.2 and so on. So, no set rules. I am just saying, I am just giving you a figure. Normally, it is about 1.05. There is a way to calculate this. I do not want to digress right now, okay as to how this is done. But less assume that I know it and I give you a figure and it is 1.05.

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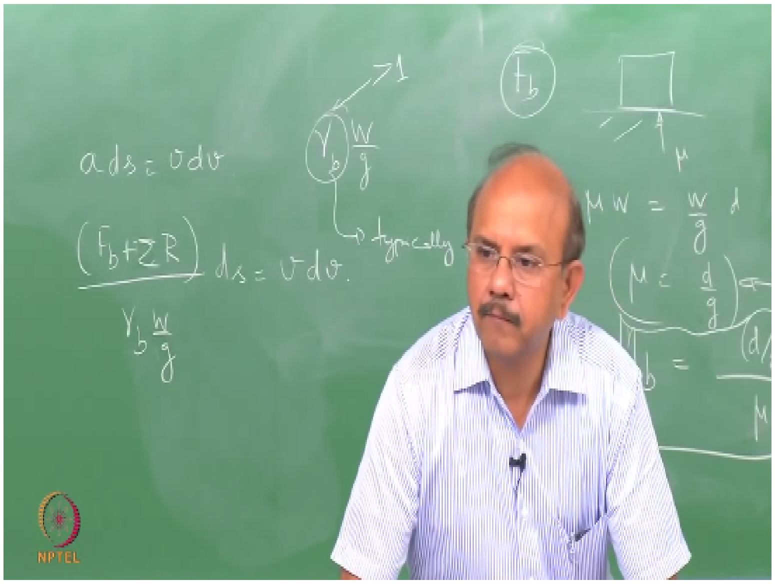

So, my main idea is to calculate deceleration and I am going to use a very simple formula which all of you know ads=vdv. This is high school physics, okay. Now, I am going to write down this for A f/l which I would call that as Fb+sigma of all the resistances. Please remember that in the previous expressions, I had left out certain resistances just to make it easier for you to understand but in actuality I have introduced all the other resistances as well which includes the gradient and which includes the aerodynamic forces, okay.

So, we need to include not only rolling resistance, we have to include aerodynamic forces, we need to include gradient and so on. So, I am going to put all in sigma R. We will expand that in a minute because we are going to get an interesting twist to this, and that is A/M, M I already said it is equal to w/g and that multiplied by ds=vdv, clear. Now, this would have been very simple, but for the fact that there is a term in R which depends upon V.

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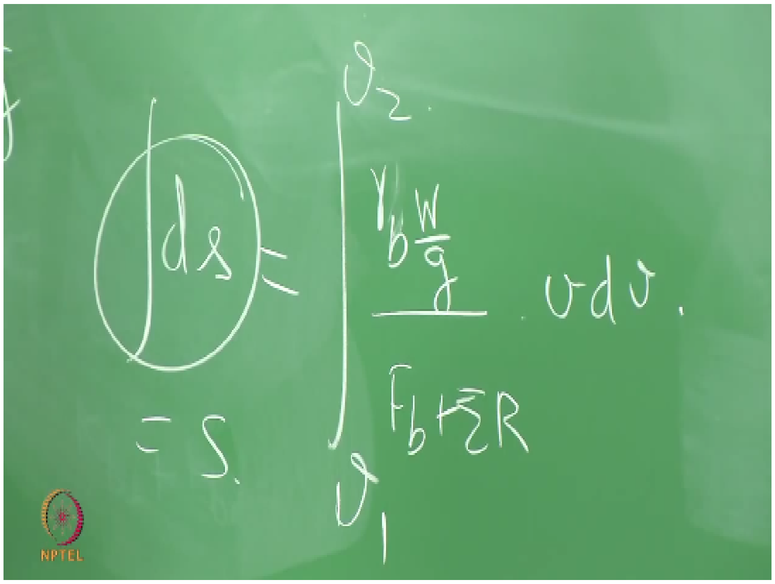

So, in other words, what I am going to simply do is to take this to the right-hand side, rearrange it and write it as gamma Bw/g/Fb+sigma R*vdv and then integrate the left-hand side and the right-hand side, okay. It can be from say v1 to v2 and the corresponding distance we would call when it comes from velocity v1 to v2, we will call it as yes, so that this would become yes. As I told you there is small twist.

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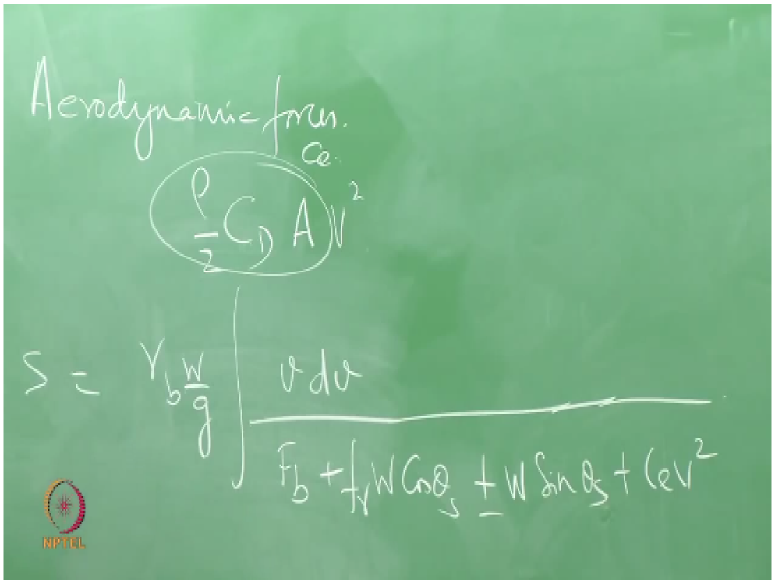

What is the twist, the twist is because the resistance here involves aerodynamics forces. Aerodynamic forces can be written by rho/2 dense to the air, very important co-efficient drag coefficient CD multiplied by the projected area A*v squared. So, since v squared is there, I cannot take that out of the integral, so I have to re-write that integral S to be equal to, I can take out the first term and then write this as integral vdv/Fb+ write down all that you have for r.

If you want to be very correct, then you will write that down into cos theta. Remember that this is what we talked about. Besides we will make an assumption, remove that cos theta but if you want to be correct, you can include that as well as Fr W*cos theta + or - I would say Y+R- we said depending upon we left it to you how we are going to write + or - W*sin theta S whether uphill, downhill and so on +Ra, we will call this to be a factor called C.

Let us say that and dumping everything because we can assume that this is a constant and write this as +C*v square. Whether it is + or - as high school physics, I am not going to go depending to that. So, you have this v square term. I have to now integrate this. So, regular integration d of v square, okay, then 2 will be there. So, I will bring that 2 outside, okay. So, d of v square or you can say that this d of all this + v square also, okay.

And then what would be the formula, what would happen. It is in terms of log to the base e okay, right. Integrate this from v1 to v2. We will continue this integration and further interpretation of this in the next class.