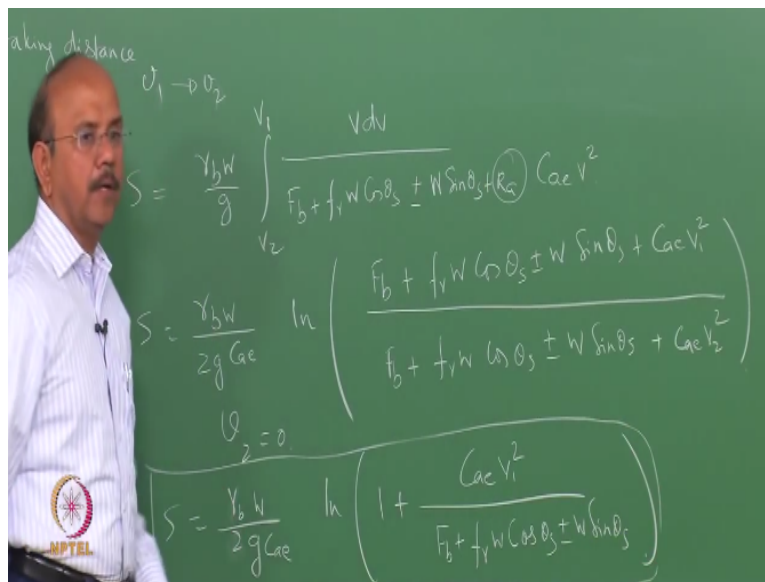


**Vehicle Dynamics**  
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**Lecture – 05**  
**Semi Trailer**

In the last class, we were talking about the breaking distance, right, we had looked that how to calculate the breaking distance.

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We had derived from a very simple dynamics relationship. What should be the distance that the vehicle would travel and we go from velocity  $v_1$  to velocity  $v_2$  or  $v_2$  to  $v_1$  rather. Now what we are going to do is to in order to calculate the breaking distance, we would now integrate it or rather  $v_2$  to  $v_1$  or I think that is the order we wrote. We would like to at what would be the braking distance when the velocity or the final velocity = 0 obviously 0.

So, now you can right down the integration what we did and then you can calculate what is the distance that you would travel before we break the vehicle, okay. Now this is the formula which we wrote, if I remember right, this would be, I think this is how we had given so that should be  $v_1$  to  $v_2$ , I think this is how we wrote,  $VDv/Fv+FrW$ . We wrote also  $\cos \theta S$ ; you can make  $\cos \theta S = 1 +/- W \sin \theta S + Ra$ .

We said RA is the aerodynamic forces and we just replaced this with the  $C_{ae} \cdot v^2$  and we said that  $v^2$  being there that will have an effect on integration and you can integrate this straight forward, there is nothing very difficult about it,  $C_g$  to  $C_{ae}$  (()) (2:42)  $F_v + F_r$   $W \cos \theta$   $S \pm W \sin \theta$   $S + C_{ae} \cdot v_1^2 / F_b + F_r$   $W \cos \theta$   $S$ ,  $\sin \theta$   $S + C_{ae} v_2^2$  square, okay.

Just quickly, I am not going to spend time you can do that, then this is the distance travelled when the velocity is vary by  $v_1$  to  $v_2$  or  $v_1$  goes to  $v_2$ . Now putting  $v_2=0$  which is actually the stopping distance when the final velocity = 0. This equation can, you know you can write that equation to be, okay. So that gives you the stopping distance that you would get, right. Remember why we wrote that  $\gamma$ , okay, that is for rotational inertias and other things.

And we also define what is called as the efficiency of braking remember that we said the efficiency of braking is simply that all the  $F$  that is to be spent goes to braking the vehicle or in another words we said  $m a$  or  $m \cdot$  the force that is the deceleration should be = or  $w/g$ , let write that,  $w/g \cdot d$  should be =, if you remember  $w$  that is the normal load that is acting that is the weight multiplied by  $\mu$ , if you remember  $= d/g$  and we said that, that is the maximum efficiency.


Right, that is one of the things that we wrote. You can also say that the maximum efficiency is when the braking force such that it has already compensated for all the rotating inertias. In other words,  $\gamma B=1$ , okay that is compensated for it and the rest of it is available for in order to break the linear case. So in which case, we can also write down the minimum breaking distance, I will call this as minimum breaking distance, where we have  $F_b$  replace by the  $\mu \cdot w$  that is the breaking force.

So this can minimum breaking, let me write it here,  $S = \gamma b w / 2g C_{ae}$ , natural log  $1 + C_{ae} \cdot v_1^2$  square,  $\mu \cdot w$  because that is the minimum, all the forces go to,  $+F_r$   $w$ , break the vehicle  $\pm w \sin \theta$   $S$ .

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$$\frac{W}{g} d = W \mu.$$

Min. braking distance.

$$S = \frac{\gamma_b W}{2g C_{ae}} \ln \left( 1 + \frac{C_{ae} V_i^2}{\mu W + f_r W C_{nd} \left( \frac{W}{S} + W \right)} \right)$$


So that gives things minimum braking, I am just coping it, it is nothing very difficult. If there is any questions on this all of them are just forces, okay, just did not want to leave any of the things and that is the reason why I am carefully writing it, but concept wise they all are very simple. So that is the minimum braking distance.

One of the things that is important, where now lots of research going on, is on the time lack between a driver applies the break and the force is realized, okay, the breaking force is realized, this become very important because there is always be a time lack. This time lack is due to right from the application to the point when the forces felt at he breaks and the breaks start acting.

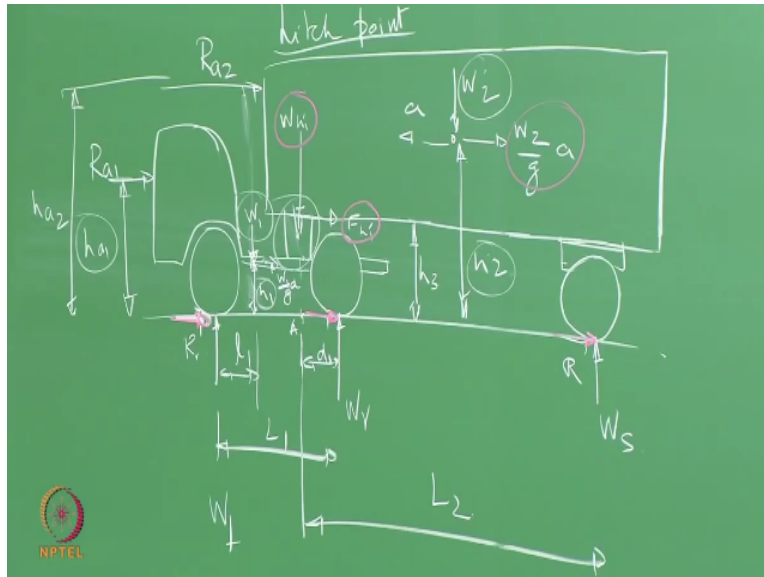
Delays are due to so many reasons here right from the paddle motion to the hydraulics that is involved or the fluid that is involved and so on. So, I just want to point out that, that time lack need to be understood and taken into account, if you are looking at breaking distance. So what we are taking here is the breaking distance when the force is felt but it is not from the time the vehicle or the driver applies the breaks and then starts breaking.

So usually that is the factor, should not go into details, I just want to point out that. So we have covered quit of bit. We now know about the acceleration breaking in a passenger car. We will slightly deviate now, not that we are going to go away from the longitude of dynamics but we

have to learn lot more in longitude dynamics. We will talk about tires. We will talk about aerodynamic forces so on.

But before we go there, let us complete the topic by looking at tractor trailer.

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Okay, this is very important vehicle today. Specially in country like India becoming very popular, off course, the number of axles increase. When the number of axles increases, I would not be able to do the kind of calculation that I do here, because the problem starts statically determinate. So, we are looking at it only from the point of deriving a simple equation on forces acting, Hence, we would restrict ourselves to a tractor trailer, where you will see the trailer has 2 axles.

And that tractor on the trailers are connected by means of what is called a hitch point. There are number of terminologies for this. Sometimes peoples call this as fifth wheel and so on. That is the hitch point. Let us understand what are the forces that can act on this and as I said in the last class the reference for this is theory of ground vehicle by (()) (10:37). So we will move away from this after this class to look at, I see it is a tire dynamics. So let us look at this.

So let us look at what are the forces that are opposing exactly the same thing, there is no difference in the concept, absolutely the same but only thing is that when I take moments what

are not more forces that come into the picture okay. So, let us just recapitulate what are the forces that are acting for accelerating vehicle.

One of our great interests is to find out what are the reactions that take place in these axes. What we would call as front axle, what I would call as real axle, I have to be consistent, I there is any confusion on the terminology or on the notation please let me know, I will be as consistent as possible, Ws. Now, what are the forces there are reacting, off course, the aerodynamic forces. There are 2 aerodynamic forces that we take into account.

One, that is acting on the tractor. There is tendency in the industry to call this also as the horse, they call this as a horse, because it is something like horse drawn carriage, so when they say horse driving a trailer, do not be carried away by it, because it is a loose terms or the local term that they used, that is called a hoarse. So, that is, the  $R_{a1}$  is the aerodynamic forces that is acting on the tractor. The similar aerodynamic force acts here and that is  $R_{a2}$ .

The other force that are reacting, you know it. The same is rolling resistance force. So, that acts everywhere, all the 3, rolling resistance force. The D'Alembert's force acceleration, for deceleration it will just be opposite, same things. The other force which is important are the hitch point forces, okay, simple mechanics. The hitch point forces direction had to be careful, okay, you can look at this from 2 perspectives.

One from the tractor perspectives and the other from the trailer perspective. So, there are very simple mechanics. You can understand that, so you be careful in putting down the hitch point forces as they are called. Remember that, we are considering the weights and it is D'Alembert's forces of the tractor and the trailer separately. So that  $s$  becomes the weight of the tractor and this becomes the weight of the trailer and the CG locations are written as  $h_1$ ,  $h_2$  and so on, right.

We will make some simplifying assumptions, not that it is necessary. So I need not write the equation, the full equation on the board, I will make some assumptions. Here again, the concepts are simple, equations are going to be long, I am not going to spend much time, I am going to

wind this up as soon as possible. So we will go to the next topic, okay. So any questions, ask me. So, I am going to just write down the final equations, telling you how I got it.

You can derive it yourself, straightforward. So the first task for us is to find out what are the reaction forces that are acting onto this vehicle. Once we finished this, we will go to the breaking which is very interesting and that is the next step we are going to do. You can easily write down; why do not you start writing down what would be the reaction forces that would be the result of this kind of arrangement.

Note that I put a point a, it is customary to take the movement about that point, many times in order to find out the forces. So let us do that, let us say that  $W_s \cdot L_2$ . I would take  $W_s \cdot L_2$ . Please note what I am going to do is to separate out, in your mind you can build this up, separate out the tractor and the trailer and then put the free body diagram, this is just combined, I just put it like this. As I told you this is important.

Separate out the tractor and the trailer. When it is accelerating off course  $F_{hi}$  for the trailer will be in the other direction, okay,  $W_{hi}$  would not change and vice versa. So in another words, there will be support here for the trailer. So note that and accordingly change it. In another words when I am considering the trailer, part of it, this will be in the another direction, this will be in the opposite direction and vice versa.


So well set very simple. I am going to write down only the final equation for  $W_s$ , okay, this will be a good exercise for you, verify it.

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$$h_2 = h_3 = h_2$$

$$W_s = \frac{d_2}{L_2} W_2 + \frac{h_2}{L_2} \left( R_{a2} + \frac{a W_2}{g} + W_2 \sin \theta_5 - f_{r1} \right)$$

$$F_{h1} = R_{a2} + \frac{a W_2}{g} + W_2 \sin \theta_5 + f_{r1} W_s$$

$$W_s = \frac{W_2 d_2}{L_2 + f_{r1} h_2}$$


Okay, I am going to, just make as small assumption so that I do not need to write down a lot of things. So, I will write down  $h_3$ ,  $h_2$ ,  $l_2 \cdot W_2 + h_2$ ,  $L_2$ . Note I am taking all the movement about the point a, which I have defined. The hitch point forces, that is the force which actually pulls the trailer. So, that will be opposed by of course all the other forces. So that the hitch point force can be written as, be very careful with this + and -, I do not want to every time give an explanation and confuse you but you would know how to write that.

So, I am not going to do that but be very careful, when you put + and when you put a -. One of the easiest ways doing it is that when it is going uphill, accelerating, it is acting in the same direction is that of the  $R_a$  toward the accelerating forces. When it is coming down hill, the forces will be in the opposite directions. So, the direction of forces determine whether it is + or a -.

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$$-W_s = \left(1 - \frac{d_2}{L_2 + f_r h_2}\right) W_2 \quad (1)$$

$$= W_1 l_1 + R_{a1} h_{a1} + l_1 \left(\frac{W_1}{g} + W_1 \sin \theta_s\right) h_1 + F_{hi} h_3 + (L - d_1) W_{hi} \quad (2)$$

$$L_1$$

$$F = R_{a1} + a \frac{W_1}{g} + W_1 \sin \theta_s + f_r (W_1 + W_{hi}) + F_{hi}$$

Now using these 2 equations and substituting for whatever is in the brackets here, I can write down  $W_s$  to be  $W_2 \cdot d_2 / L_2 + f_r \cdot h_2$ . That is the reactions that is happen in the trailer, the rear wheel of the trailer. Note that these vehicles or rear wheel driven or in other words, that is that wheel which has the drive and breaking happens in all 3 wheels. So, breaking forces are applied to all 3 wheels.

In another words, that the driven wheel is this for the breaking wheel, is all 3s. So there is the distribution of the breaking forces between these 3 wheels. So the load of the hitch point off course all of you know, how to calculate that, that is the  $W_2 - W_s$ , because as I said the trailer is supported now by the rear as well as the hitch point and so can be written as  $W_2 - W_s$  and that will do  $1 - d_2 / L_2 + f_r \cdot h_2 \cdot W_2$ .

So maybe you can call that equation as 1, that equation as 2, that equation as 3, that equation as 4 and this equation as 5 and so on. We can write down this as a coefficient and call that as  $C_{\chi}$ , we follow theory of ground vehicles and just write that down as  $C_{\chi}$ . So, now we shift to the tractor. We consider tractor as a free body and draw the free body diagram for the tractor to exactly the same thing and find out the reaction forces in the front and the rear, again by taking movements and so on.



So  $W_r \cdot L_1$  is what I am going to take, so let me write the final result  $W_1 \cdot L_1 + R_{a1} \cdot h_{a1} + h_{1a}$   
 $W_1/g \pm W_1 \cdot h_1 \cdot \sin \theta$ ,  $F_{hi} \cdot h_3 + L_1 - d_1$ ,  $W_{hi}/L_1$ . That is  $W_r$  again you can write down, you  
can apply equation 1. Let me call that as 6, then you can simplify this equation. Now the  
technique is the same, so what is the next step, we will simply this by looking at the longitudinal  
forces and hence the longitudinal force  $F$ , the balance of the longitudinal force, what is the  
longitudinal force here, traction force.

So the longitudinal force which is the traction force has to now equilibrate all of the forces that  
are reacting which is very again very straightforward. You can say that, that is  $R_{a1}$ , I am not  
going to write down that equation, is very simple and then it is that is the next force. Then,  $W_1$ ,  
that is the next force, plus off course the rolling resistance force. Remember that we had  
something called draw bar pull and we removed it.

But now it become very important and this is very similar to draw bar pull which we had used in  
the last class, in one of the earlier classes. So that is the force that you have, you can substitute it  
and you can determine what would be the force, we can calculate what should be the maximum  
force. Substituting into this expression, the previous expression, I can simplify this equation,  
okay, rearrange them and so on.

I am going to skip that rearranging, writing down and so on and I am going to only write this.

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$$F_{max} = \mu W_r$$

$$F_{max} = \mu \left[ \frac{l_1 w_1 - h_1 f_r (w_1 + \chi w_2) + (L_1 - d_1) \chi w_2}{L_1} \right]$$

$$(1 - \mu h_1 / L_1)$$

The maximum force that can be supported by the tractive effort or by the rear wheel is obviously is equal to like what we have written, we are lumping all the effects of the tyre into one number call  $\mu * W_r$ , so the maximum is given by  $F_{max} = \mu W_r$  substituting  $W_r$ , simplifying all these equations. I can write  $F_{max}$ , I am writing the final form, okay,  $= \mu * l_1 w_1 - h_1 Fr W_1 + \chi W_2 + h_1 - d_1 * \chi W_2 * L_1 * 1 - \mu h_1 / L_1$ .

So that gives you the maximum force that can be supported by a tractor semitrailer, technique is exactly is the same. **“Professor - student conversation starts”** Any questions? We are assuming the  $h_3 = h_2$  right? We are assuming all hitches to be the same which is not correct, okay I agree. **“Professor - student conversation ends.”** This is just to write down laziness of writing down the equation that is all.

You can write down that more elaborately putting down  $h_a$ , to obviously  $h_{a2}$  is not going to be  $h_{a1}$ ,  $h_3$  is going to be very different from  $h_2$ , is going to be different from  $h_1$  and so on. I agree. Just to write it down, this is not correct. You need not make that assumption, there is nothing that assumption, you can substitute it and write down the final form. The equation will become long, concepts are simple, nothing will change.

You are absolutely right, that kind of assumptions, is only for a classroom deriving the equations and in practice that would not be correct. So, that is for traction, we will quickly finish breaking

in this class. So that we will be move forward because it is longitudinal dynamics, we have to move, it accelerate. So, we will look at breaking here. Any other questions, anything, okay. Again very simple dynamics is what we are going to use for breaking.

What we are going to do now is to divide this into 3 categories. We are going to look at tractor, we are going to look at trailer and we are going to look at the tractor-trailer together. So that is the first thing we are going to do. In other words, we are going to get 3 sets equations by considering these 3 systems or these 3 scenarios. That is the number 1. The number 2 is that we are going to look at  $\sum F_x$ ,  $\sum F_y$  and the movement about an axis which is perpendicular to the board.

So, in another words, for each of this scenarios, we are going to get 3 sets of equations. So, we are going to get 9 equations. Then, we will see how we can simplify it, so that we can find out what are the loads that are reacting or the normal reactions at the 3 wheels. So, this is the strategy. Why we are interested ultimately or what is our goal? Our goal is to find out what would be the reactions into wheels.

Our goal is to find out these 3. Why are we finding this out, because we want to make sure there is a locking sequence. So the most important thing in breaking is what happens when these wheels lock. So, I have to find out when these wheels lock. So I have to find out what these are? So that is my first step. That is why I did all these things or I am going to do all those things. Please note that when I look at all these 3  $W_s$ ,  $W_f$ ,  $W_r$  and  $W_s$ .

I will multiply this with  $\mu$  in order to look at the maximum breaking force that is possible before the wheel locks. So in another words, derivations is pretty simple, in another words what we are ultimately interested in is whether the front wheel locks, rear wheel locks or the trailer wheel locks. These are the 3 things. So in another words, what can be the sequence at which we can have locking would determine how we are going to distribute the braking forces, clear?

Now, what happens when the front wheel locks, the situations is exactly similar to what we had before, we are going to lose directional stability or directional control. So, we are going to lose

directional control and the vehicle is going to go forward. What happens when, remember the scenario, it is exactly similar, when the rear wheel locks, when the rear wheel locks, remember that the whole force goes to the front and use (()) (31:23) so directional stability is lost.

Now, what happens, here in this case when we have this wheel, it locks. Here in this case what is going to happen is very interesting, the second one, let me come back to this. When this locks, let us see what happens. We saw that is directional stability, but here, there is going to be small difference. Let us see what happens, I am going to break this and there is going to be, this guy is now moving forward, because if he has to be stopped that has to be stopped and so on.

So, when this guy locks more important effects is what is called as jack-knifing because this kind of yo-yoing which is in 2-wheeler has to be looked at more closely. Because there are 2 forces that may be acting on either side. So may or may not happen, we do not know but when this locks, there is going to be jack-knifing, one going into the other that would be the case and that would be a more important or dangerous situation that would happen.

But we cannot talk about stability because that is for 2 axles, passenger car it was very straightforward. So, when this locks, there is going to be jack-knifing, there will be a difference in the deceleration between the 2. So, one drives into the other. What happen this locks, when this locks, there is going to be, please note that at the hitch point, this is something like supported here and this has a hinge there and so when this locks, this is going to sway and that is going to be dangerously swaying, perpendicular to the board.

So, the effects are such that each one of them has is important off course. One is more dangerous than the other 2. Of course, I said that locking the front is the least of this danger, why because, the driver will be able to find out that is going to lock or his wheel is going to lock, so he has a feel. So, he will be able to correct it. So that is the least of the problems. Jack-knifing is the worst.

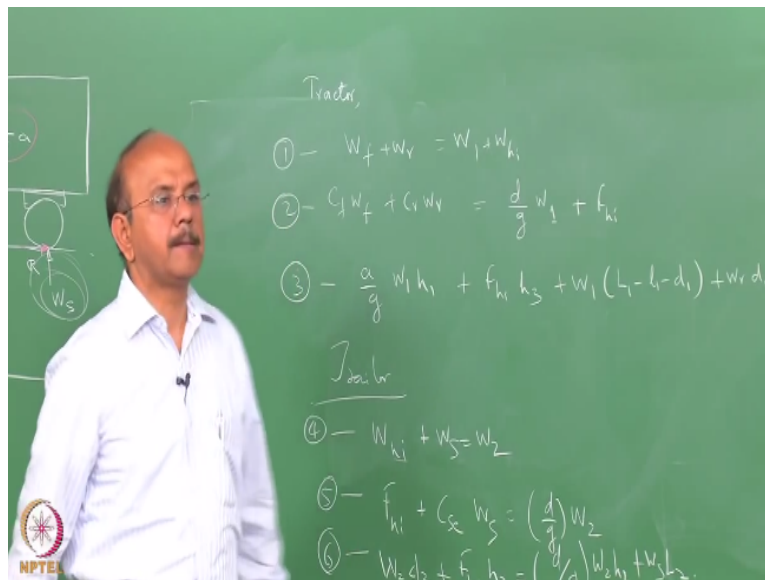
And this is also very dangerous especially for vehicle which are coming or which are in the road, other vehicles, because of the whole trailers is going to sway. So, this is, it is very important that

these concepts are taken into account, so which locks first. So I would not allow this to lock, this would first lock, no issues. Why we are taking this because we want to distribute the total braking forces, so this is fine.

Because I can find out what is happening and then this locks and then finally this locks would be a situation which would be useful in order to decide what should be the braking forces. Of course, this is only to understand the concepts. If you look at ABS, which we will quickly run through after understanding the behaviour of the tyres, you would appreciate the ABS in vehicles.

Because, this kind of situations, locking situations, how the ABS handles this kind of locking situations becomes important and the concepts will be very well understood at that point. So now what is my goal, I just want to write down the 3 sets of equations. I will do that right now and then write down what would be the Ws that would happen in these 3 axes. So, I am considering first the tractor, so let me consider the tractor first.

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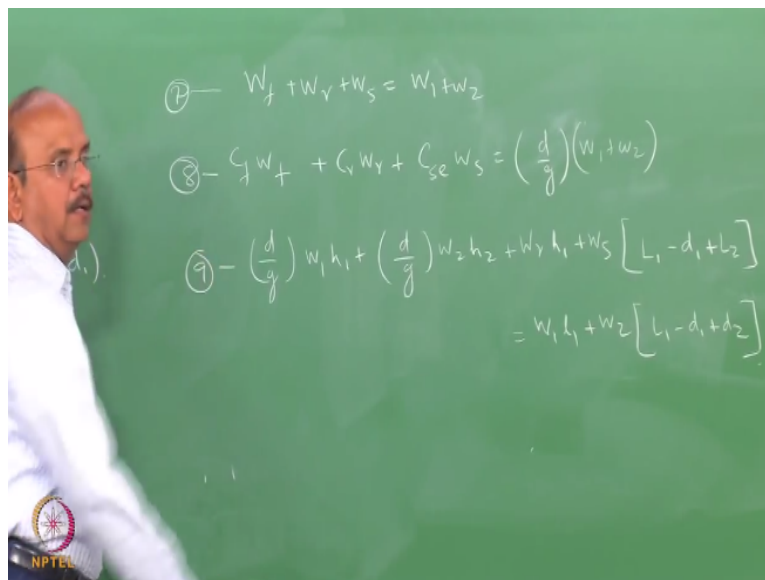
The tractor  $W_f + W_r$  this is the, I said the sigma forces = 0, this is the vertical forces that are acting. So  $W_f + W_r$ , they are 2 vertical forces. The weight plus the hitch point. Then, we will consider the braking forces, the braking force which acts in the, we will consider that as the x

as usual and the other one as the z direction and that as the y. So, this is the x direction forces, the breaking forces. So  $C_r \cdot W_f + C_r \cdot W_r$ , that is the coefficient of breaking,  $z = d/g \cdot W_1$ .

Please note that in breaking  $f$  is in the opposite direction, so it should be  $+ \cdot F_{hi}$ . So that is the second equation and the third equation that I am going to write is the movement equation in the y direction. So, that is the set of equations for 1, 2 and 3 for the tractor. We have similar set of equation for the trailer. Sometimes called as semitrailer because there is one wheel there so, anyway I will call this as trailer and again the same set or the same directions is what I am looking at, nothing difficult, the 2 directions that is the 4, 5 and 6.

Then, I said that I going to take the tractor trailer combination and write down another 3 sets of equations.

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That is in the z direction,  $W_f + W_r + W_s = W_1 + W_2$  and then the total breaking force that is acting, those are what I would call as breaking co-efficient—that is my  $F = Ma$  equation. Off course you know that  $d$  is the deceleration. So the signs are taken into account once I say that, that is the deceleration. So, that is 7, 8 and 9. So, these equations are now you know solved in all to get  $W_f$  and  $W_r$  as I told you I am going to write down only the final expression.

So the tractor, so I have to find out  $W_f$ ,  $W_r$  and  $W_s$ , and I am going to just write down final equation for this. Okay, you can derive that.

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$$W_f = \frac{w_1 \left[ L_1 - \left(\frac{d}{g}\right)h_1 + \left(\frac{d}{g} - C_f\right)h_3 \right]}{L_1 + (C_r - C_f)h_3} + \frac{w_2 \left[ (L_2 - d_2) + \left(C_{se} - \frac{d}{g}\right)h_3 + \left(\frac{d}{g}\right)h_2 \right] \left[ (L_1 - d_1) - C_f h_3 \right]}{(L_2 + C_{se}h_3) \left[ L_1 + (C_r - C_f)h_3 \right]}$$

$$W_s = \frac{w_2 \left[ d_2 + (h_3 - h_2) \left(\frac{d}{g}\right) \right]}{C_{se}h_3 + L_2}$$

The final equation for  $W_f$ , it is a long equation, multiplying, substituting and all that doing all this, okay. I promise that, that is going to be a long equation and it happens to be so. Now we will write down  $W_r$ , so obtain from these equations. So, we need to off course know  $C_{se}$  and  $C_f$  and  $C_r$  so on in order to solve the equations, the key equation is this. Once you know this, you can go back to those nine equations, okay, in order to find out  $W_f$  and  $W_r$  as well.

So you need not go through this. You can as well you know from this equation, once you know  $W_s$ , you can go to one of those 9 equations and you can determine  $W_f$  and  $W_r$ , right. So we will stop here, just to assimilate this, we will quickly spend 5 to 10 minutes on the distribution of the breaking force in the next class and then we will move to tire dynamics, right, so we will stop here for today.