Vehicle Dynamics
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Lecture – 07
Mechanical Properties of Rubber

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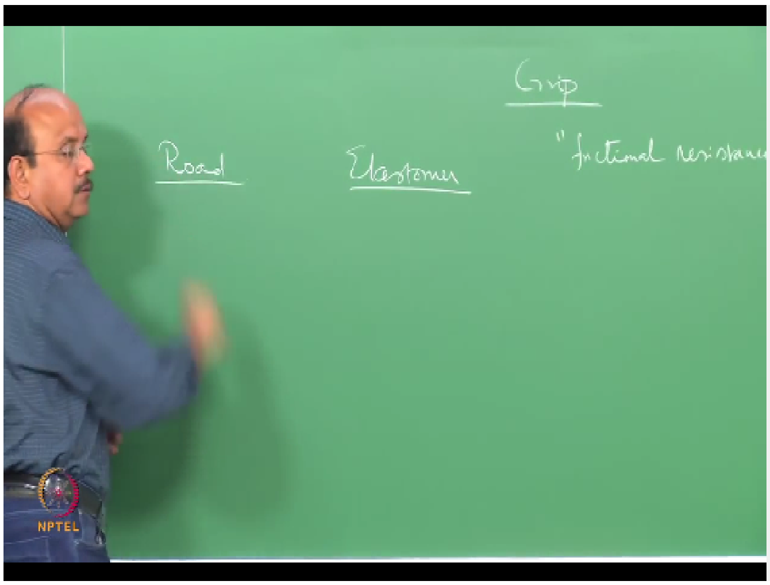

In the last class, we started discussing problem of grip in a tire. The fundamental aspect that gives grip to the tire is its interaction with the road and that it develops what we called as the frictional resistance to its motion. Now let us understand this from 2 perspective, we have started this already. One is from the elastomer perspective, how does elastomer function when it contacts the road and the other from the road roughness perspective or the road perspective.

So there are these 2 things we have to understand because after all it is the interaction between these 2 which results in what we call as grip. Just to summarize what we did about the elastomers. We said that the elastomers are basically elastic viscoelastic material, in simple terms they are viscoelastic materials. We said that there are 2 things that have an effect, one is the frequency and the other is the temperature.

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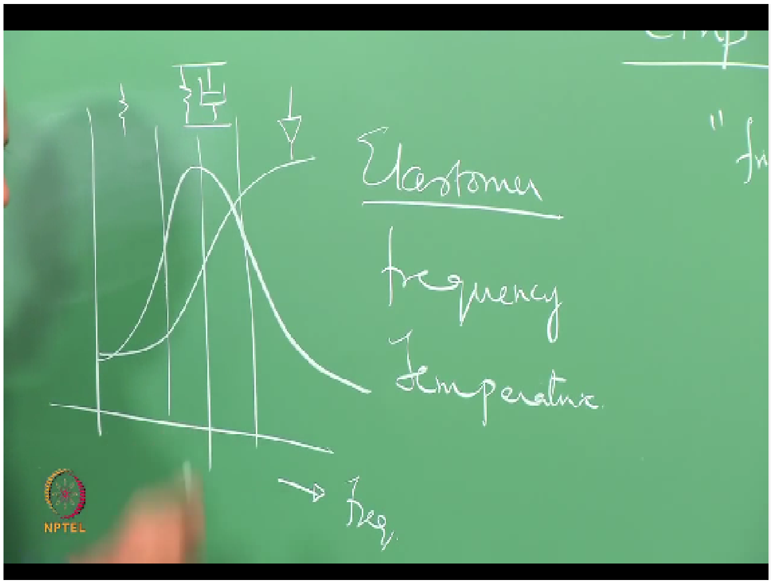

And in fact, we saw that they have an opposite effect and we determined or we explained that the region where say for example an elastomer operates, can be divided into 3 parts and that you have a region where it is, say for example this would be the frequency. We said the region where the frequency is low, the elastomer acts like a spring, right, and in another region where the frequencies are very high, it becomes very rigid, okay, so it becomes something like a very rigid part.

Essentially because it has to slide back to its original position and the sliding back is affected by the interaction of these long chain molecules, macromolecules with others which are present and hence it becomes rigid and between the 2 regions, we have the viscoelastic part. We said that the graph or the modulus goes like that and because there is a viscoelastic part of the centre, if I now plot the energy that is consumed due to deformations then it would go something like that, right.

This we said is the correspondence to the glass transition temperature, okay.

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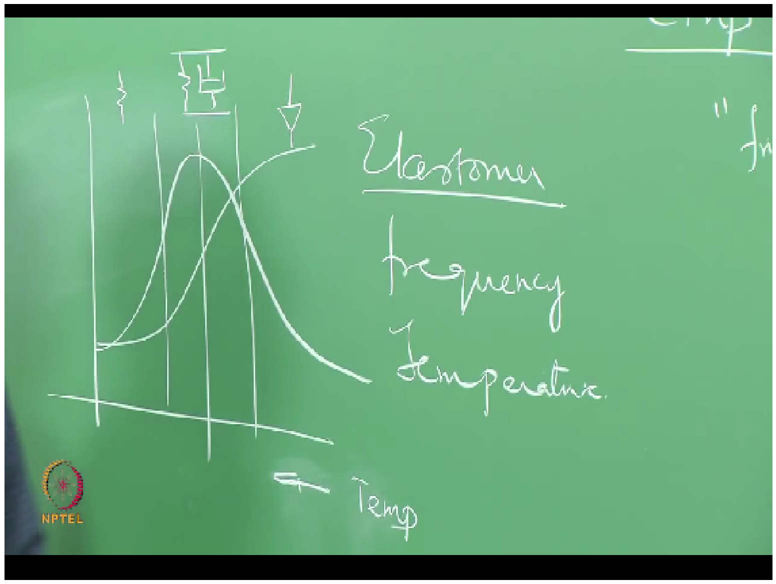

So if I now put the temperature here, we said that this whole graph goes in the opposite direction and that what happens at higher frequencies, happens at lower temperature and so on, okay. So this is what are we said yesterday. We said that there is a relationship between the 2 and is given by Williams, Landel and Ferry, WLF equation.

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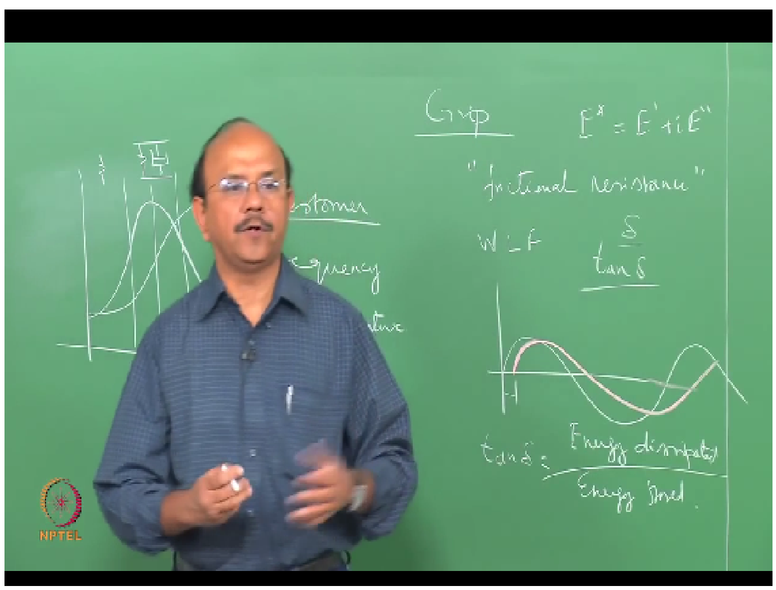

This equation gives a shift for frequency when you shift the temperatures. When there is a shift in temperature, you would see that there is a shift in frequency as well, okay. So we will come to that in a minute, just summarize what all we said yesterday in the last class and we also said that there is a lag between the applied force in the deformation or in other words applied stress and the strain that is developed, okay, there is a lag between the 2.

So if I now plot, for example, the stress and then plot the strain, the strain develops something like this and that there is a phase lag between the stress and the strain given by the equation delta or tan delta, is an important quantity in the study of viscoelastic materials. In your classes on vibration, you would have studied about, or in controls, you would have studied about phase, phase lag and so on, okay, and you would have expressed the quantities of interest in a complex plane, okay.

The complex plane is essentially used in order to bring out this kind of phase information. Hence here we have what is called as a complex modulus for a viscoelastic material which consists of a real part which is called as a storage modulus plus an imaginary part, which is called as the loss modulus. Note that there is nothing imaginary in this.

The only thing is that, we express the phase information through a complex quantity and call that complex quantity into real and imaginary, right. You can say that this tan delta is an important quantity basically because that gives you the amount of energy that is dissipated. So in other words tan delta which is from here you can find out what is delta, tan delta can be written as energy dissipated.

So when there is a loading-unloading, there is an energy that is dissipated and can be looked at as ratio of energy dissipated to the energy stored. So tan delta becomes an important quantity which tells us about the dissipation of energy. So if you now plot, so what does it essentially means.

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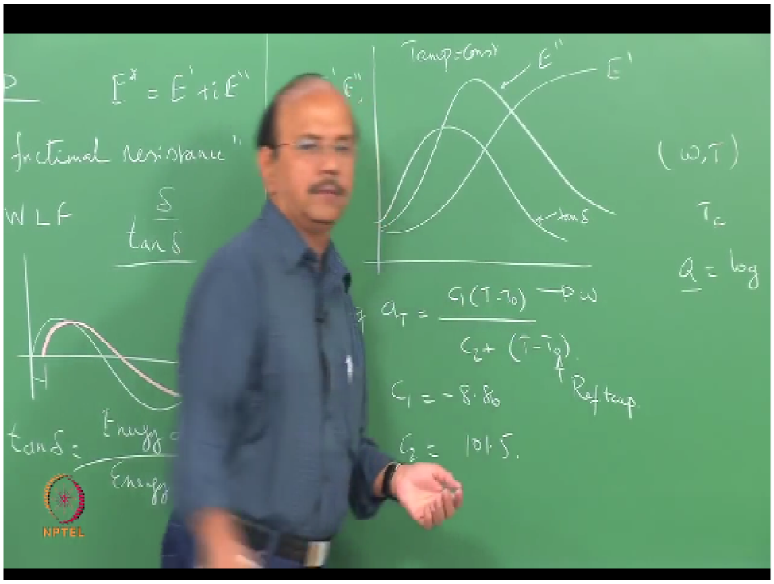

If I now plot, say for example, omega versus all these 3 quantities which is E prime E double prime and tan delta, the graphs would look something like this. We had already seen how E prime would look. So E prime would look like that, that will be the E prime value, E double prime would look something like this, that will be E double prime value and tan delta would be something like that, that will be the tan delta value.

This is at 1 temperature, so this temperature is a constant here. If I want to determine this at any other temperature, it is not necessary that you have to keep on applying the same set of or do with the same set of experiments, that is not necessary, okay. This whole graph gets shifted depending upon the temperature, this whole graph gets shifted. In other words, if I have data at one temperature here with respect to frequency, I can find out the data with another temperature by shifting this graph.

In other words, what happens at omega and at the temperature T, if I now shift the temperature to T+delta T, then what is happening that omega would happen at omega+delta omega. This shift, okay, can be determined from the WLF equation which is given something like this. Log of aT, the shift is given by C1, so we can write this as C1*T1-T0/C2+T-T0, where C1=-8.86 usually, I mean that is how it varies and C2 is 101.5.

T0 is a reference temperature, okay and which is the glass transition temperature plus about 50 degrees, right. Now glass transition temperature itself depends upon the frequency. So when I shift the frequency, that glass transition temperature also shifts. Now how do we use this.

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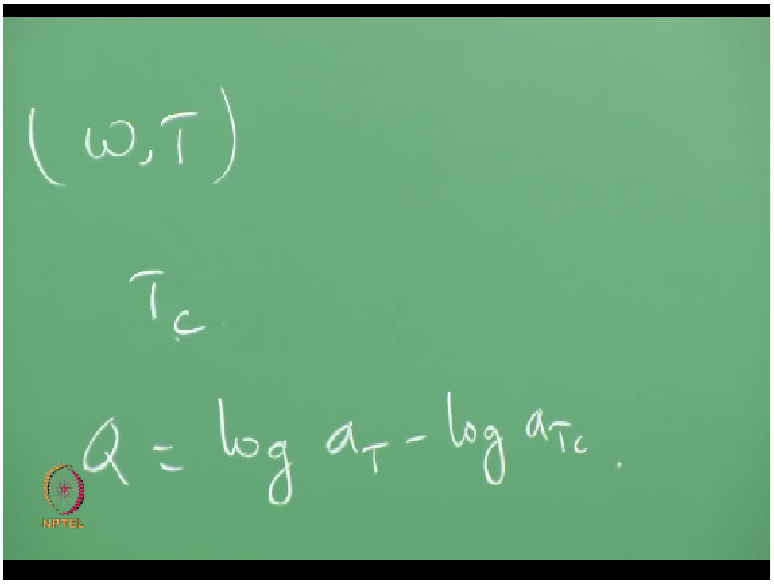

Suppose I have the data to be or the data recorded at omega and T, okay and I want to determine what would be this data, say at a temperature, say Tc, okay. Then what I have to do is to find out the shift, say let me call that as Q which is log of aT-log of aTc and shift the frequency and this is of course given a log, so shift the frequency to that decayed frequency, okay. So each data point that is obtained in omega T, okay that is this experiment, okay.

Now say for an example you are plotting this graph of log omega versus, here if you have, log omega versus the value, say E prime or E double prime and so on, okay and you want to find out what would happen in the new temperature then you shift to this point say for the new temperature by this much amount, by that much amount. So this is the shift factor, in other words, this shift factor is the relationship between the temperature and the frequency.

"Professor - student conversation starts" Any questions? You say if you are increasing the frequency, temperature is (()) (13:16). Yes, yes, yes. So depending upon, that is this relationship. So what happens, what is the relationship between frequency and temperature. So what happens at lower temperature, happens at higher frequency, okay. So that is understood, correct. So this is just a mathematical expression for this shift, right.

So usually 1 decay that is if there is a 1 order of magnitude in the frequency, would result in the shift of, say 6-7 degrees is the shift, you know that is what would happen, okay. That is the first part. What does aT signifies in that. That is the shift. So the shift is calculated by this formula. So WLF gives you actually the relationship of shift or relationship, sorry, relationship between frequency and the temperature, okay.

So that is what I explained now with respect to Q. It is, in other words, what it simply means is that I can do all the experiment at a particular temperature, we call this as a master curve and then I want it at any other temperature, okay. I can determine that not by repeating an experiment, but by using this kind of shift factors. So that is what WLF tells us, okay. That is a property which we will, T or T1? T0 is reference temperature which is usually T1? T1 is the temperature or T rather where the temperature at which I am determining the aT.

Sir there is (()) (15:08). That depends upon, see what is the value of C1 or you know where do you apply, all these things depends upon the elastomer in question, okay. Usually this is applicable for the range of interest for what we have, right. So 10 to the power of say if suppose I have a 10 to the power of 1 the frequency at or 10 to the power of -1, I want 10 to the power of 5, it is possible to do that. **"Professor - student conversation ends."**

Before we go further, let us look at the frequencies of interest, then we will understand how we can apply this. So let us understand what are the frequencies of interest. In order to do that, we have to now look at the road. So this is the elastomer behaviour. We will now shift to the road right, okay. Because this friction is going to be very important and the friction is very different from what you see in metals. So it is important that we understand how it works.

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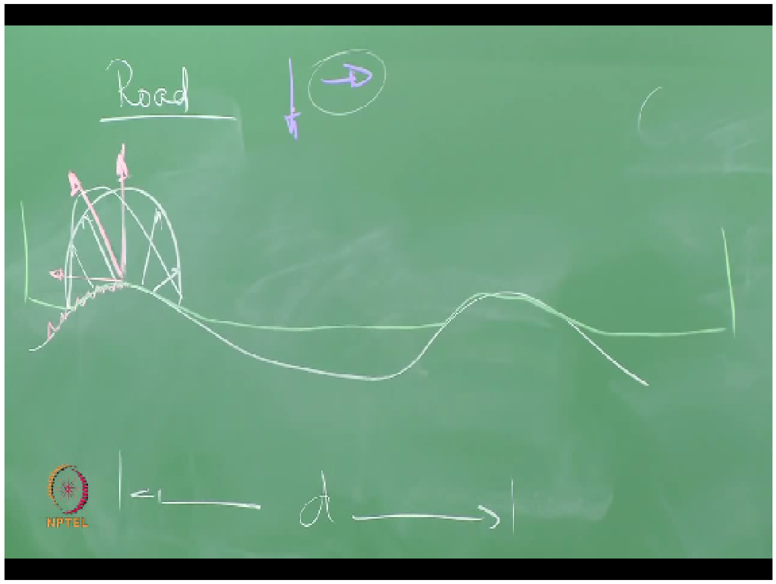

Now we shift to the road. Then we are going to look at the road from 2 perspectives, one in a macro perspective, okay. So the road looks let us say that something like this. It is not that as big as this. Let us say that that is about 1 cm is this, 1, 2, in terms of centimeters. What is this, this is those gravel stones which are sticking out as you see it in the road, okay. So there is a binder between them, okay, the asphalt binder is what binds this kind of stone.

So these are the stones. This is at a macro level. If you now go and look at the stone more carefully, you have also small undulations or small micro roughness which you would call as something like that, micro roughness okay. So these are the 2 things how actually a road can be looked at. A macro which can be in terms of a centimeter or so and micro in terms of a few microns which can at the maximum be 0.1 millimeters, that is this roughens is the order of say 0.1, maximum of 0.1 millimeter, much less 0.01 millimeter and so on.

A few microns to maximum of 100 microns, right, okay. This is how you look at the road. What does rubber do or what is the tire as it contacts the road, what does it do. It actually engulfs the whole road. So you can see that would how say a tread block would look like. So it engulfs the road, right. So because the deformations are quite large, it just goes and sits over this gravel or the stone, right.

Now what happens. 2 forces are exerted; one is the force which is normal force. On the other, let us say that traction, breaking, whatever it is, a tangential force, okay. These are the 2 things that act on this block. Now again there are 2 things that happen. One is the micro-macro, the other is 2 forces and there are 2 phenomena that happens between the rubber, the block and the road.

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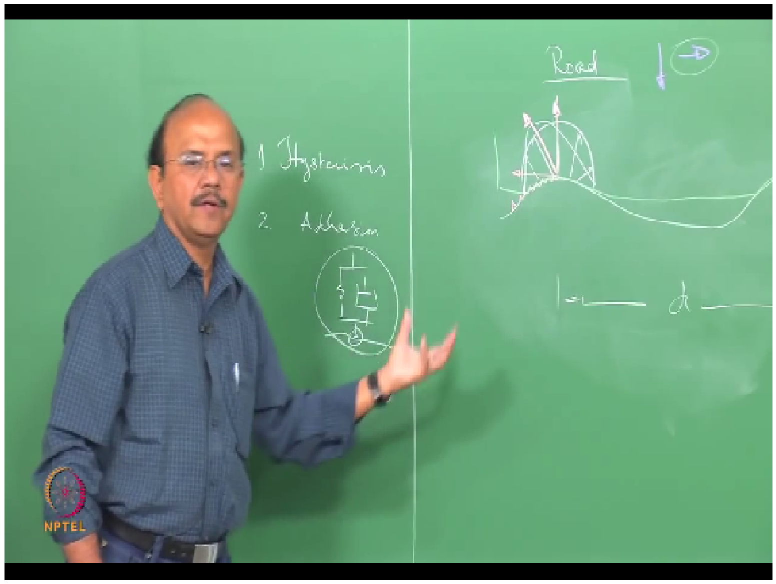

The phenomena can be classified into what is called as hysteresis and molecular adhesion or adhesion. We call this as molecule adhesion, okay. So molecule adhesion again happens at a very small length scales and hysteresis happens at a much larger length scales, right, okay. So let us quickly look at this because then we will go there and understand this. What do we mean by this adhesion?

Actually these long chain molecules which we represented by means of a simple model, 2 spring and dashpot, this long chain molecules get attached to the surface, assuming right now that the surface is dry, it gets attached to the surface through what? Van der Waals forces or Van der Waals bond is what is formed between these long chain molecules and the surface. So they form this bond, let us say that this is a rubber piece, okay.

They form a bond with the surface molecules or the surfaces that exist, right the surface and then when they are subjected to a force, these bonds that are present here, it breaks, so it sticks, breaks, slips, then again sticks, breaks, slips and so on. So at the adhesion, at the molecular adhesion level, there is a sticking, slipping, sticking, slipping and so on. So it is the Van der

Waals bond that forms, breaks, forms, breaks and so on, right.

But what happens here, what is this hysteresis and what happens here. That is a very interesting phenomena and we already know that when a rubber piece is subjected to loading-unloading, it loses energy through this phenomena of hysteresis, okay. Now let us say that, for a moment forget about that. We have normal forces.

Obviously, the normal force would give rise to a contact pressure which would act something like that. Now when there is a lateral force that this block is subjected to, then obviously this kind of symmetric distribution, okay, which equally breaks the normal force, is no more maintained and this pressure distribution becomes unsymmetric because it has to now equilibrate that lateral force as well.

So what was lines like this which was equilibrating this, would also have a force which has to equilibrate this and hence, the total sum of these 2 forces would be something like that and with the result that the pressure distribution now shifts. Superposed this on to this, these 2 phenomena. What really happens at this point is that there is a slip, okay, there is a slip and this whole block can move out of this hill okay and slide down.

In other words, the loaded region, do not look at it as this big, you know we are talking about, this whole distance itself is 1 centimeter, at maximum of 1 centimeter, right. So keep that in mind. So what happens is that this slips, it goes over the hill and slips. So when it goes like that, what was loaded becomes unloaded, okay. So when it slips, it may again get reloaded and so on.

So the frequency of loading-unloading because of this macro, this kind of macro pieces, okay, depends upon the velocity of sliding and is given by simple v/d, the distance between the 2, this I would call this as d, then v/d is that frequency with which this slips. The frequency with which that stick-slip happens, is much less. So if this is around 10 to the power of 4, this would be actually 10 to the power of 8. So that is the frequency at which it slips.

So in other words, there are 2 things that contribute to what we call as friction, the 2 things that contribute to what we call as friction, one is the hysteresis loss and the other is this kind of micro molecular interactions.

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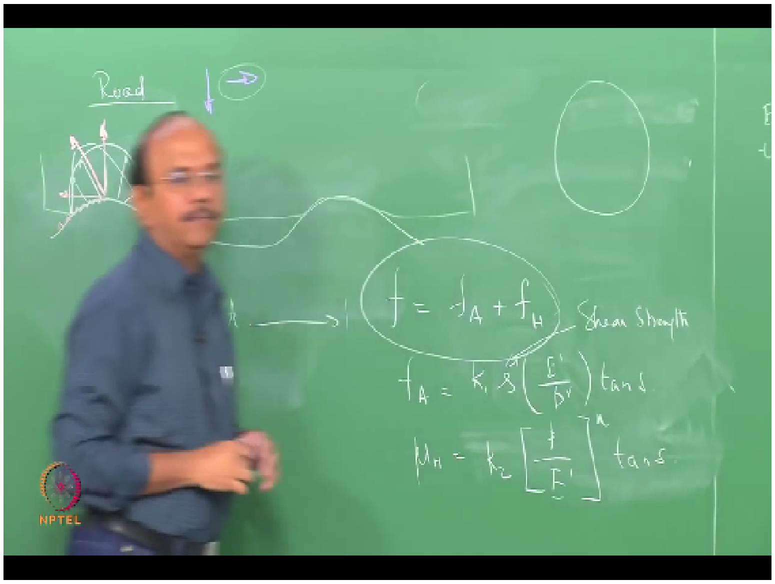

So that energy is what gets converted into a frictional resistance in rubber and can be written in a very simple form as the friction forces or a friction coefficient if you want, can be divided into 2 categories, one due to adhesion and the other due to hysteresis, right. So we can call this or we can quantify this by means of a simple equation, this is by Moore and is very good book on friction of rubber, it is a book on pneumatic tires, very early book, very good book.

So I am taking this from there. So that is the reference. So that is given by addition, let me write down that carefully. K1*s*E prime/p product r*tan delta and mu hysteresis=k2*p/E prime n*tan delta. S being the effective shear strength of the material, p is the pressure that is acting, okay the normal pressure that is acting, effort r is usually about 0.2 and n is >= 1, okay.

So essentially, friction is the result of these 2. In order to put this in proper perspective, we will go a bit back and forth so that we get a macro picture of the tire, whole tire. We are going to micro, we are going to in a molecular level. We will look at the length scales again when we summarize. But what I want you to understand is that, is that this is happening in a rolling tire, that is phenomenal.

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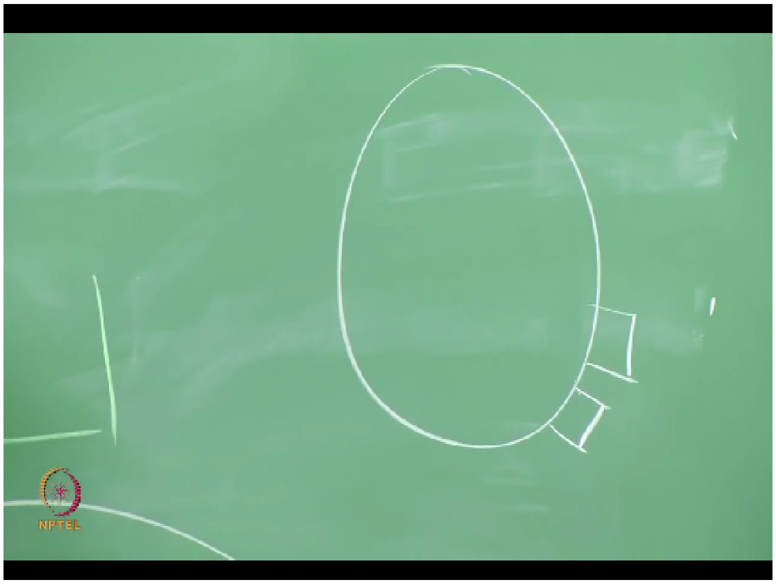

So a rolling tire, okay, is subjected to this kind of phenomenon. So as it rolls, let us say that these are the tread blocks, okay, that is the carcass, okay on which we have what are called tread blocks. We saw that yesterday. So these tread blocks go through that kind of cycle, that kind of you know, engulfs the stones that are there, gravel that are there. They develop a bond between the surfaces, all these things happen when this whole tire rolls. So that is very interesting, you can look at how fast that is going to happen, okay.

So in order to get again a macro picture from a tire perspective, let us see what happens and then again we will come back here, right. So let us see what happens in a tire. Let us now consider a tire again in isolation and then after we have finished that, we will tie-up the concepts that we have explained now with the tire.

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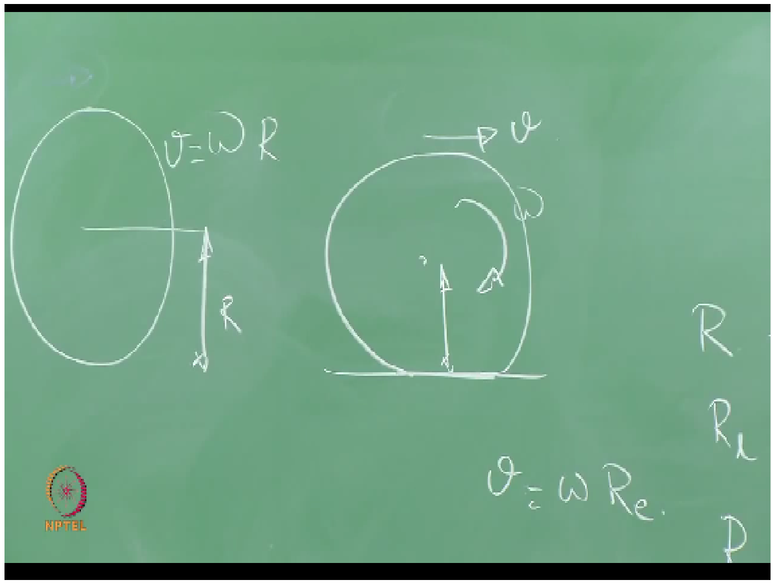

The first, very first thing that we notice in a tire, pneumatic tire, is that the tire which is like this, let us say that it is an inflated condition, okay. It gets deformed or is deformed when I apply load and put it on the road, okay and when it rolls, this deformation again takes place obviously. So in other words, there are different radiuses that are present, okay, in the tire. One is this free radius, okay.

Now because it gets deformed here, the free radius is not maintained and hence you have at this point, we will call this as loaded radius, okay.

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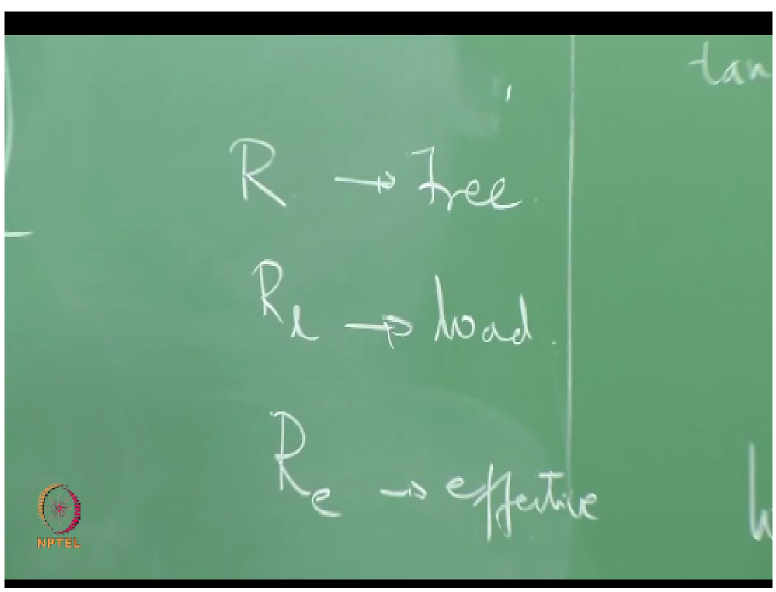

So I have a free radius R. Let us say that that is the loaded radius Rl and in between the 2 what you do not see here is what is called as a effective radius Re. What is effective radius, very simple, if this guy is not to be deformed, is not deformed, and we just rotates with an angular velocity omega, then the tangential velocity v is given by omega*R, right. So in other words, as the wheel rotates with an angular velocity omega and the wheel travels with the longitudinal velocity v, then v is given by omega*Re.

So Re is defined with the same formula that you are all familiar with from fundamental dynamics, right. Before we go further, let us understand this contact patch, okay. From a plan view, let us say that the contact patch looks like this and lot of niceties in this, we will come to that a bit later. Let us say that that is the contract patch. First things first. When the tire is stationary, like that, omega is not there.

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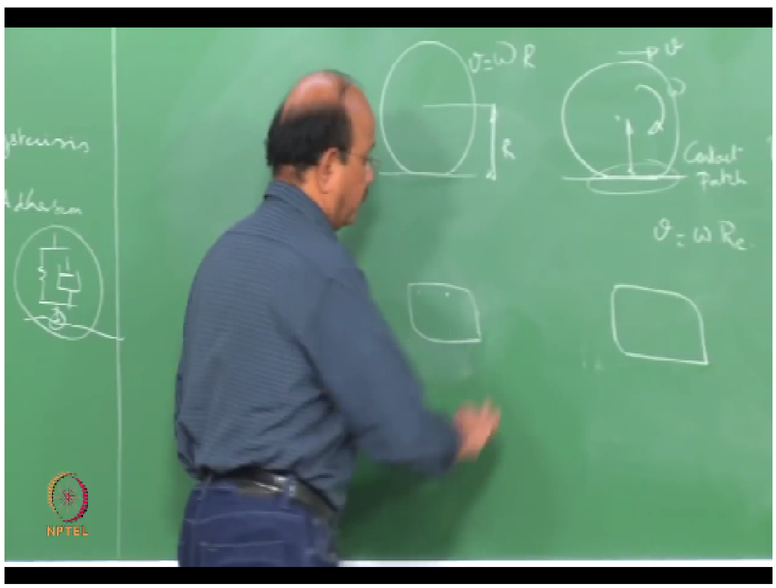

I am saying that this is stationary, then you have a contact patch, of course, region where the tire is in contact with the road, okay. Then at different points on the tire which sit at different point on the road, so all these guys sit in different points of the road, right. It is simple to understand, nothing very difficult. On the other hand, let us see what happens to the contact patch as the tire rolls, okay.

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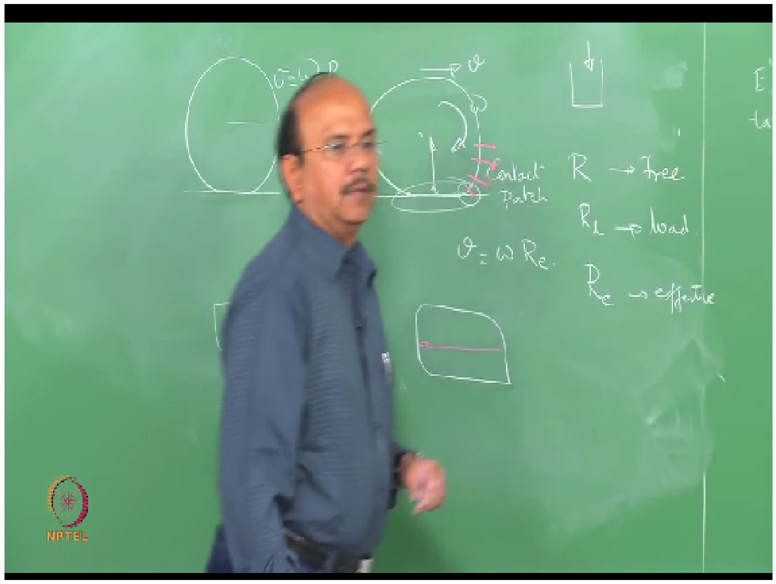

In order to understand that, let us do a small thought experiment, okay and the thought experiment is necessary to dispel some of the ideas which you may have which I have over the year's students asked, okay. So we are going to do this thought experiment. Let us say that these are the treads or the bristles that are sticking out, right. So let us say that you are going to sit there, okay.

Do not worry, it is only a thought experiment, you are not going to be hurt, just sit there. Ask your friend to go and sit at this place, okay and another friend who sits here. Now the tire rolls, right. So as the tire rolls what happens, you would go and hit the ground, right, you would go and hit the ground. So the mu situation would be like this, so you would have hit the ground here in this region, okay, let me extend it, so it is not that you are going to become 0.

So you will get compressed, right. So you will be at this position, you are just going to hit the ground. The first thing is that you are not going to travel in space like this, you are not going to travel in space like this. Then what happens? So you hit here, let us say that you are not slipping, your forces are such that, adhesion is such that you are going to stick there. Then what would happen? Your friend who is sitting here, that guy would hit the ground before you okay.

Then your other friend, he will hit the ground before that guy. So in other words, you can imagine that the contact patch is actually travelling over you. It is not that you travel over the contact patch but the contact patch travels over you because as new guys keep hitting, relatively you would be moving back in the contact patch. So in other words what happens is as I hit here, I will stand here, there is another guy who goes there, someone else and so on.

So with respect to the ground, with respect to the contact pressure, actually I will be going behind, okay. So ultimately I will come to a situation where because of this rolling, I will come to a situation where I am at the end of the contact patch. I will rise up and go out, okay. Now let us see what happens when you stick. You are subjected to a pressure, okay. So you are subjected to a pressure.

So let us draw that situation. Let us just take that out, okay, you are there. You are subjected to a normal pressure and let us say some force is applied and you are also subjected to a tangential force which becomes a sheer stress, okay and so on, right, or traction. Under these conditions, what actually happens, I will go stick to the ground. As long as possible, I will be at the place, okay.

I will be pulled so I will be stretched and so on, okay, depending upon there is a breaking or there is acceleration. I will be shifted in the 2 directions, different directions. But I will stick to my ground because I have already established my relationship with the ground and the nice guy is holding, right, and at one point of time, I will not be able to stand there because of all of these forces, okay. I have to slip a bit. I will just move. In other words, there is a micro-slip.

From here, I will just move like that, it is a micro slip, right. Then again I will be subjected to a contact force which will not be the same as what it was before. We will see that. So again I may stand here, right. So in other words, if at all I move in the contact patch, I move over very short distances, micro-distance, micro slip, very short distances, right. I do not slide or go from here to here, okay.

The contact patch as I said goes above me and at the maximum I am going to just move like that. So the first thing that is important to understand is the motion of this contact patch. You are not moving in the contact patch, contact patch is moving over you, okay. That is the first lesson. Let us get back to the tire again, right and let us see how actually the change in radius has an effect on the velocity, okay.

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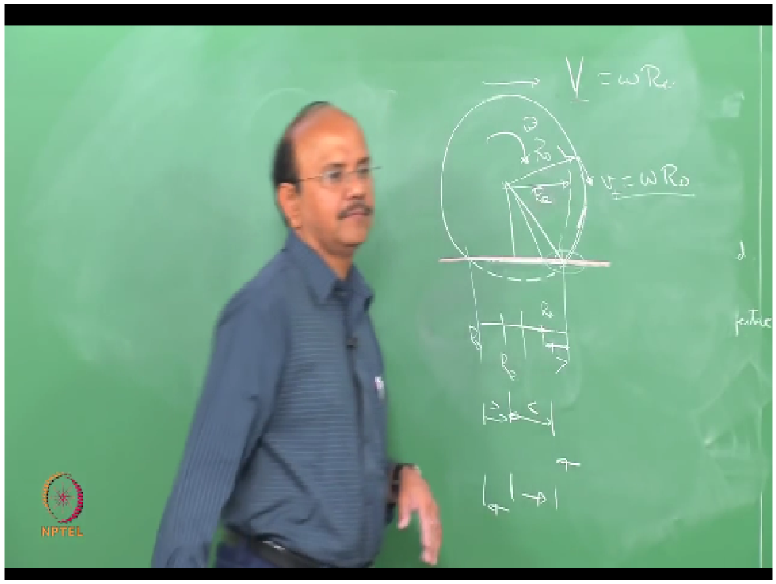

So that is my tire and since I am going to hit the ground, that is my ground. I cannot penetrate the ground obviously. So I will be like this. That is the free radius. What it will be called as R0, what is that, I think we should maintain, let us call that as R0, right what it would be. Let me call that as original radius. Let me call that as R0. If I change the symbol, please let me know so that I do not confuse you with various symbols.

Let us say that the original or the undeformed radius is R0. Let us say that the Re factor, what is Re, this is the equal like we used Re, yes equivalent radius, okay. That is smaller than this R0. So let us say that that is Re and the loaded radius Rl, right, okay. Let us now look at what happens with the contact patch. We look at the physics and then we will look at the actual values. Now again the thought experiment, go and sit here.

My tangential velocity v omega R0 where omega is the angular velocity with which I am rotating, right. So there are 3 conditions that exist in the contact patch, right. What are these conditions? 3 conditions are there. There are situations where the radius is more than, so that would be the radius, would be more than Re or = R0, more than Re, but < or not and lastly it will be less than Re or in other words, this is the actual case.

In other words, R0 changes to a loaded radius, Rl and Re sits between the 2, right. So there is a region that, let us just take that line, this line. When I just hit it, I am R0. Somewhere at this point, my radius is equal to Re, that is the point and somewhere at this point or the centre I am Rl, again as I go back, I have Re, go out I am at R0, just go out. So there is a continuous variation of radius from R0 back to R0, clear, okay.

Now let us now look at the velocities. The tangential velocity here is omega R0, right. V tangent, let me be very clear so that you do not get confused. Let us say that the velocity with which the vehicle is moving. Either you can look at the velocity of travel v and the stationary ground or you can say that the wheel is stationary, the ground is moving, it does not matter. It is a relative velocity. The ground is moving in the opposite direction, does not matter, okay.

However, you want to imagine, you can do that and it is not going to make a difference. Now obviously this velocity, tangential velocity will be more than the velocity of travel which is equal to omega Re. So in other words, here is the region where the tangential velocities are more than the velocity of travel V, we will call that as more. Here exactly the same. From here to here, it is less. Here to here, it is more. Get up and go, you are back to R0. Oh, that is a complex situation.

Imagine that this is happening as the tire rolls. The guy is really confused. Initially he is happily coming, hitting the ground. Suddenly you know he is slipping in one direction then he meets Re then again goes in the other direction then again comes back to a situation where he has to go to the opposite direction. Wow, wow. So mechanics is very complex and that is what makes tire mechanics very interesting, right.

So if you look at the direction, okay, you will see that they are in one direction up to this. From here to here, they will be in the other direction. From here to here, they will be in this direction and it goes of. What are they? They are likely slip. An interesting point is that we just now saw that without that slip, there is no friction. So your fiction concept has to now change because friction is now contributed due to hysteresis, okay.

It is that energy loss as well as due to adhesion, so without slip, it is an oxymoron, but it is actually like that. Without slip there is no friction here. So tread is subjected to a very complex phenomena at this position, okay. When it breaks, the situation is very different. Added to this are some subtleties. What are the subtleties? We will just mention the subtleties and stop here. So imagine that you are so tired, standing there, getting hit all the time on the ground.

Let us say that you are going to lie down there happily in a bed like this, okay. Now lying in that bed, you are going to approach the ground. There is already a signal that ground is approaching. How? Because of this radius change, when you are here, okay, the circumference gets reduced or is reduced and you are going to feel that. As you go near, you are going to shrink actually, your bed is going to be compressed. So that is going to be compressed.

So as you go in, you are going to get actually compressed and as you go out, you are going to get elongated and go out. So in a circumferential sense, the radius change results in a circumferential change. So there is a compression, okay, more compression, tension, in the sense release of compression back, right. So what is that affect and we will see that in the next class.