Nonlinear Dynamical Systems
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Lecture – 1
Introduction

Welcome everyone. This is a course taught on non-linear dynamical systems by Madhu. N. Belur, that is me, and my colleague Harish. K. Pillai, we both are in the control computing group in department of electrical engineering IIT Bombay.

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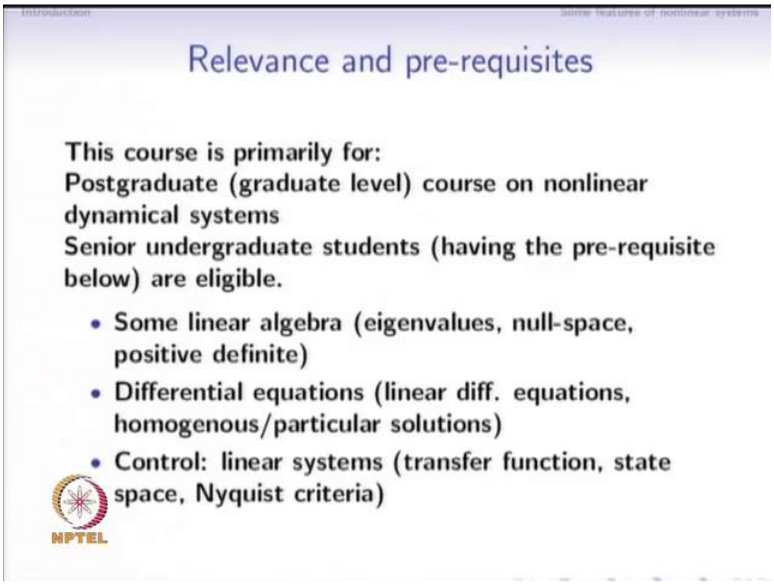

So, this course is relevant for primarily postgraduate students who are interested in non-linear dynamical systems also senior undergraduate students are eligible for this course. The prerequisites for this course, which is more important is some amount of linear algebra essentially Eigen values, null space - the concept of null space, and positive definite matrices.

We will also require some basics about differential equations in particular linear differential equation, homogeneous and particular solutions of linear differential equations. Some information about control in particular linear systems that we will require are about transfer functions, some state space concepts, and the Nyquist criteria for stability.

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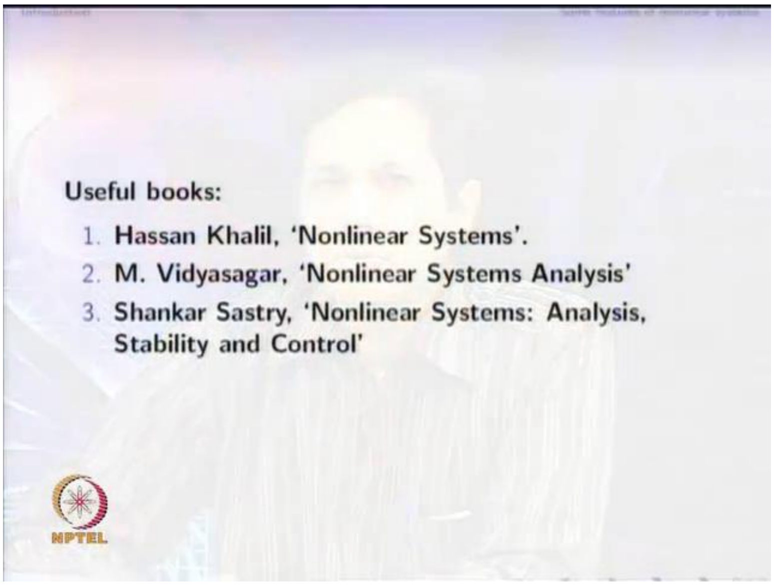

Some useful books that we will need in this course is the book by Hassan Khalil on non-linear systems, the book by M. Vidyasagar on Nonlinear systems analysis and the book by Shankar Sastry called non-linear systems analysis stability and control. These books will be very useful, so the outline of this course will be as follows. We will first begin with some properties of linear systems input output systems and autonomous systems. Then we will see some features that is present only in non-linear systems. Then we will move into existence and uniqueness of solutions to non-linear differential equations. We will also see the notions of stability, linearization, we will also see the Lyapunov theorem for stability, we will see the La Salle invariance principle.

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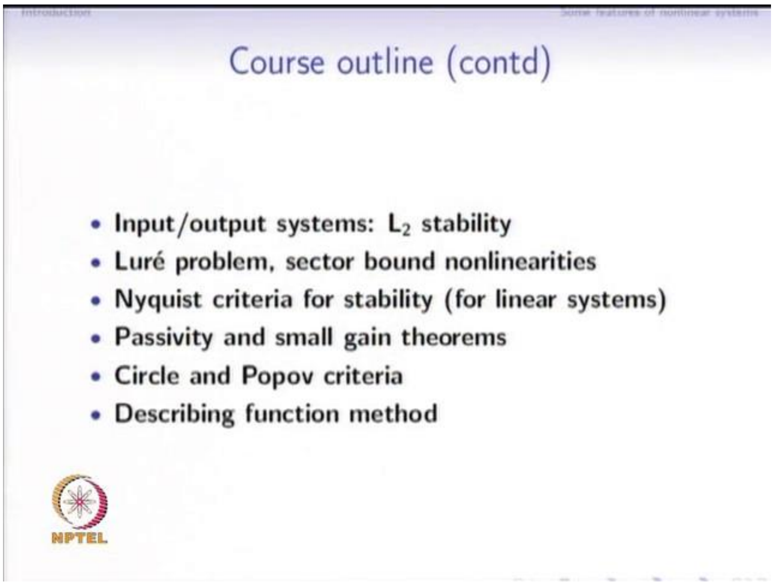

Then we will see input output systems in particular l 2 stability. We will also see sector bounded linearities in particular the lure problem. The Nyquist criteria for stability even though that is applicable only for linear systems, we will review that particular part because that will play extremely important role even for non-linear systems. We will begin will passivity and small gain theorem as the main results for sector bound nonlinearities and then we will see more generally circle and the povov criteria. We will also see the describing function method in this course.

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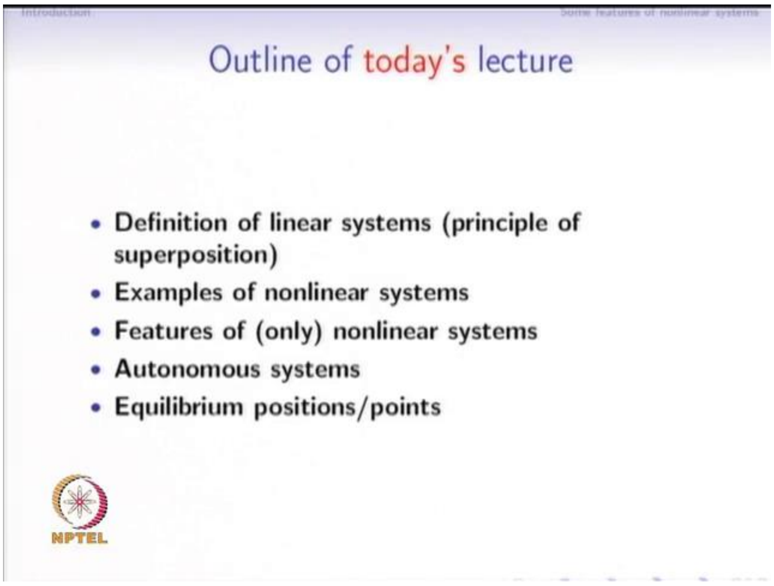

The outline of today's lecture will be the definition of linear systems. We will review the principle of superposition. Then we will see some examples of non-linear systems. Then we will see some features that characterize only non-linear systems, we will see autonomous systems what is their definition. Then we will see what the notion of equilibrium point or equilibrium position.

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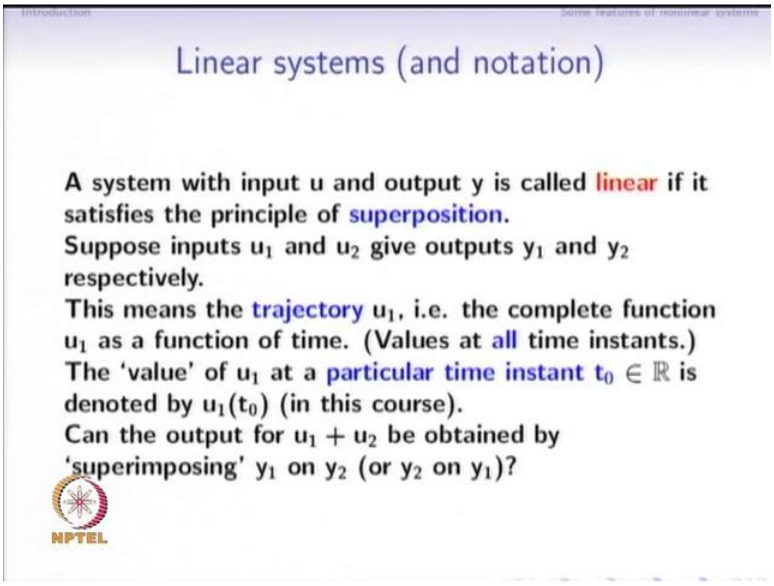

So, we will begin with the definition of a linear system, so when do we call a system as linear a system with input u and output y is called linear, if it satisfies the principle of superposition. What is the principle of superposition? Suppose inputs u 1 and u 2 give outputs y 1 and y 2 respectively. Then we can ask what does the input u 1 plus u 2 give as the output. So, in this context when we mean that the input u 1, we mean the trajectory u 1.

So, I would like to spend a few minutes on the notation we will use in this course. So, this particular lecture 1, should also keep looking regularly in the midst of the course because it contains important notational aspects also. So, when we mean input u 1, we mean the entire trajectory complete function u 1 as a function of time, which means we mean the values of u 1 at all-time instants. On the other hand when we are interested in the value of u 1 at a particular time instant t naught, t naught is some real number some time value.

Then, we denote that as u 1 at t naught, this will be the notation in this complete course. So, coming back to the principle of superposition, we can ask the question. Can the output of the system for the input u 1 plus u 2? Can that output be obtained by superimposing y 1 on y 2 or in other words superimpose y 2 on y 1?

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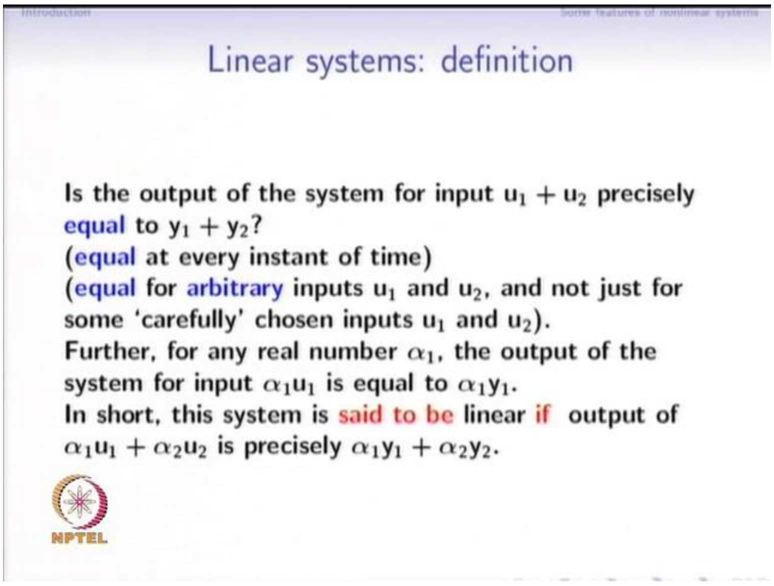

So, we will use this ability to superimpose as the definition of linear systems is the output of the system for input u 1 plus u 2 precisely equal to y 1 plus y 2. So, this equality should be understood in the sense that it is equal at every time instant, why? Because u 1 and u 2 are complete trajectories. So, at every time instant we want the output to be equal to y 1 plus y 2 the trajectory y 1 plus y 2. The corresponding outputs for inputs u 1 plus u 2, also we will like that the output is equal to y 1 plus y 2 for arbitrary inputs u 1 and u 2 and not just for some carefully chosen inputs u 1 and u 2. So, it is important that this ability to superimpose works for arbitrary inputs u 1 and u 2.

Moreover we will also like that if we scale the input by a real number alpha 1. Then the output is the same output scaled by the same amount alpha 1 in other words for any real number alpha 1. The output of the system for input alpha 1 u 1 is precisely equal to alpha 1 y 1. These two properties, the sum of the outputs and the scaling of the output these both can be captured by just 1 sentence. In short a system is said to be linear, if output of the system for the particular input alpha 1 u 1 plus alpha 2 u 2 is precisely equal to alpha 1 y 1 plus alpha 2 y 2.

So, again coming back to the notation we will say is said to be linear. If, so and so property holds in my definition, it is a if and only if statement. So, we can rewrite the same definition in a few other equivalent statements.

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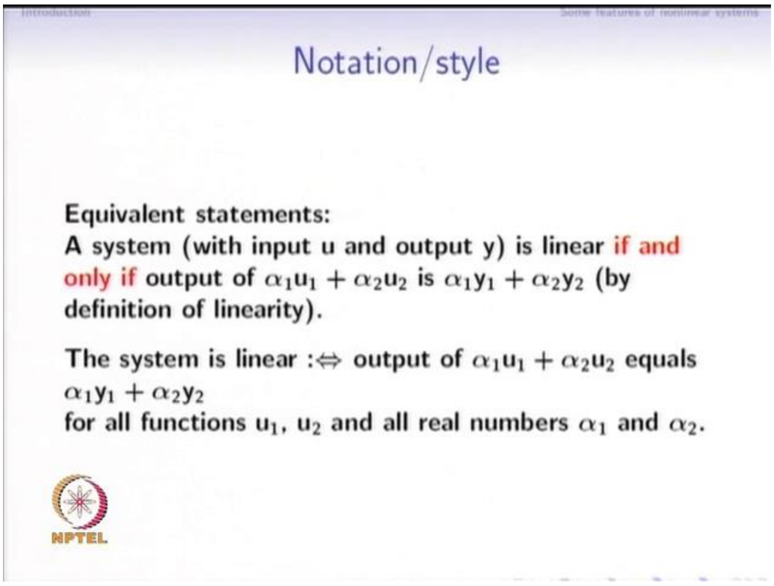

So, a system with input u and output y is linear if and only if, output of alpha 1 u 1 plus alpha 2 u 2 is equal to alpha 1 y 1 plus alpha 2 y 2 by definition of linearity. We can also state, this as the system is linear if and only if output of alpha 1 u 1 plus alpha 2 u 2 equals alpha 1 y 1 plus alpha 2 y 2. So, please note that we have this implication if and only if in particular, we put this colon on the left side which means the left side of the statement is being defined by the right hand of the statement of the if and only if sign. And as I said this should be true for all functions u 1 and u 2 and for all real numbers alpha 1 and alpha 2.

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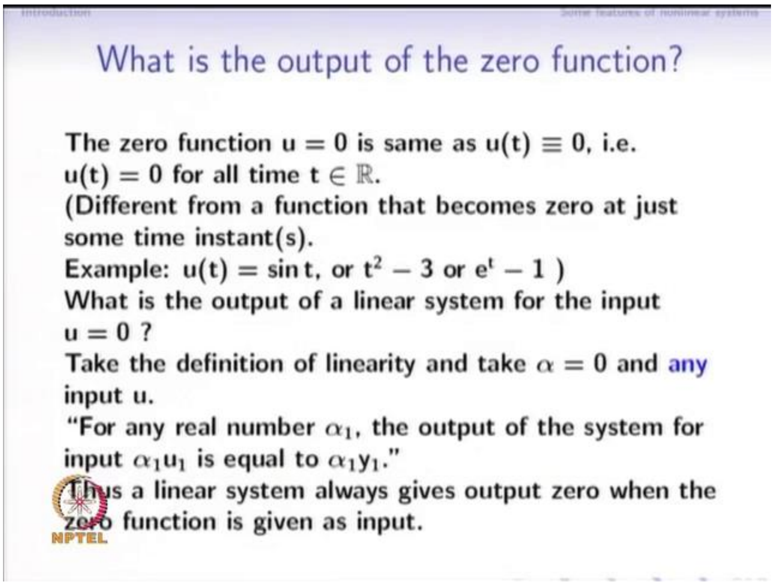

For a linear system we can ask, what happens when we give the input 0? So, the 0 function when that is given as the input, this function is same as u of t equal to 0 for all time t. That is u of t equivalently equal to 0. This is different from a function that is equal to 0 only at a few time instants. For example, the function u of t equal to sin of t or u of t equal to t square minus 3. These are functions that become equal to 0only at specific time instants, but not at all time instants, unlike the 0 function which is equal to 0 for all time t.

So, what happens to the system when we give the 0 input the output, we will like to say that the output is equal to 0. So, how do we obtain this as the consequence to the definition? We already saw, so take the definition of linearity and take alpha equal to 0 and take any input u. So, recall that we had this sentence for any real number alpha 1 the output of the system for input alpha 1 u 1 is equal to alpha 1 y 1. Here, if we substitute alpha 1 equal to 0. Then will obtain that a linear system always gives output equal to 0. When the 0 function is given as input.

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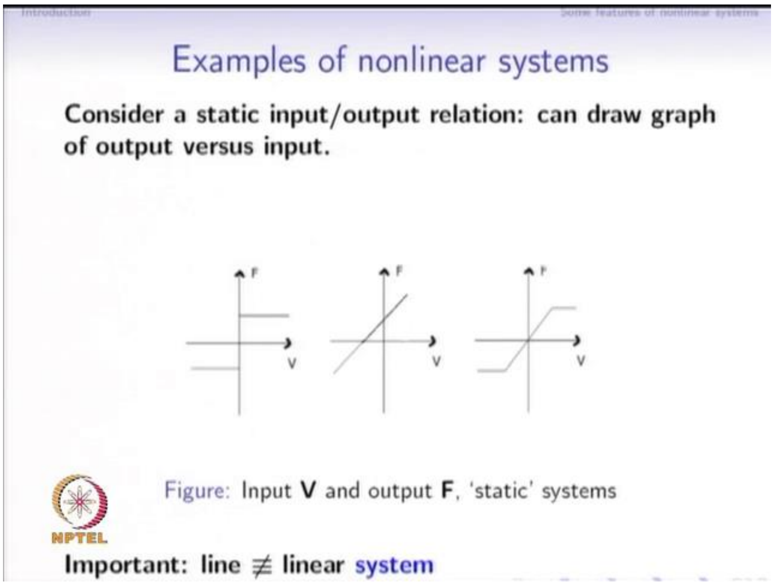

Let us see some examples of linear and non-linear systems so consider static input output system. So, if it is a static system which means that the output depends only on the value of the input and not on its derivative or integral. So, in such a situation we can draw a graph of the output versus input. So, the first situation these are 3 examples, where the force is plotted against the input v f is plotted against v, v is the input f is the output. So, the first one is clearly non-linear the last one which corresponds to saturation nonlinearity is also non-linear. The middle one also is non-linear that requires a little more careful look. So, please note that just because the graph is a line, it does not mean that the system is a linear system. That is written here. So, a line is not equivalent to a linear system.

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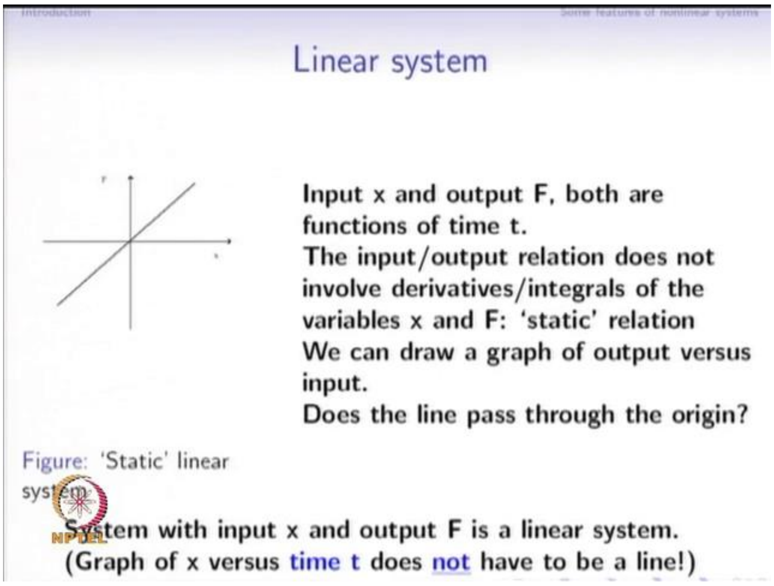

But consider this system input x and output F, both are functions of time x and f are functions of time. This is another example of a static linear system, the input output relation does not involve derivatives integrals of the variables x and f, hence we can plot the output variable as a function of the input variable at any time instant. So, does this line pass through the origin? When we ask this question then we see that the input 0, the 0 function gives output 0 and that important property is to be satisfied for a line also and only then we can call that this system is a linear system.

So, the system with input x and output F is a linear system. This is not related to the graph of x of the variable x as a function of time t. It is not related to that graph being a line. So, please note here that F and x are variables of the system 1 is the input 1 is the output and it is this graph which is incidentally a line. If, this line passes through the origin then this system is linear, but more generally for systems which do not involve a static relation between the inputs and outputs such a graph, we do not draw in that situation. We have to go back to the principle of superposition and check that for checking whether the system is linear.

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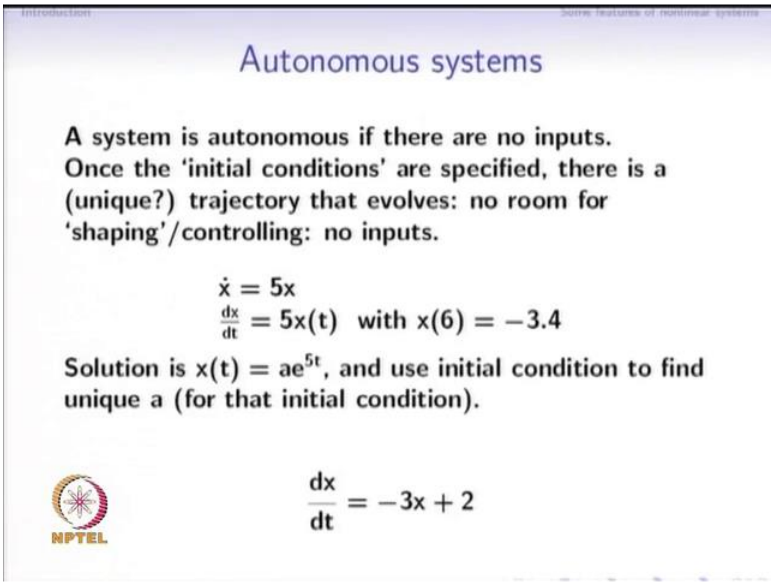

We now go into an autonomous system, when do we call an autonomous system linear for that particular question, we will quickly see the definition of an autonomous system, a system is autonomous if there are no inputs. In other words one of the initial conditions are specified there is a trajectory that evolves a unique trajectory, that evolves. There is no room for shaping or controlling, that is because there are no inputs to the system to what extent the trajectory is unique? These are some important questions. We will analyze in detail, so consider the differential equation x dot is equal to f of x once the value of x at time t equal to 6 is specified.

Suppose, it is specified as minus 3.4, then we are able to see that for the solution x of t is equal to a times e to the power 5 times t. This is how the solution to this differential equation looks and if we use this initial condition then we are able to get a unique value of a and this value of a corresponds to that initial condition. So, we see that this is an autonomous system. Another important example is that d by d t of x is equal to minus 3 x plus 2 this is another autonomous system. These are systems for which once the initial condition is specified. There are no inputs and hence the trajectory is fully determined.

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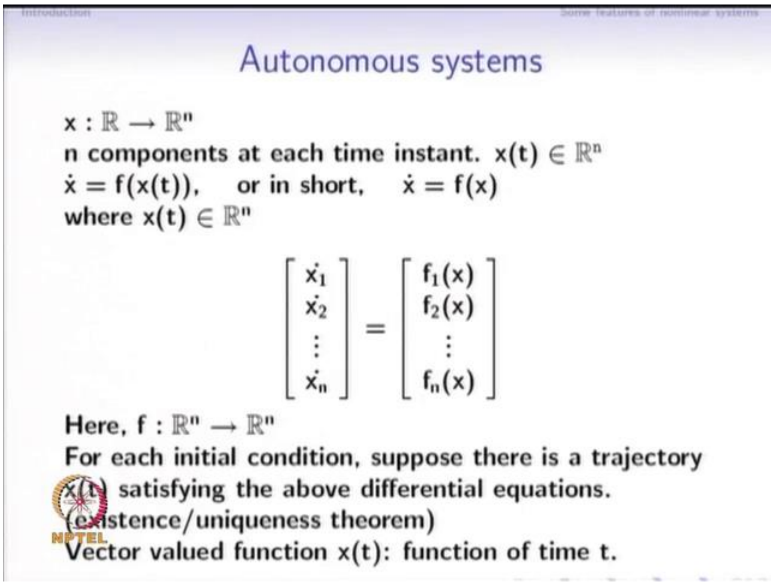

More generally if x is a map from R to R n in which the input space R, we interpret as time and the output space R n is a vector space, which has n components. So, at each n component at each time instants this we denote as x of t is an element of R n this is also suppose the differential is x dot is equal to f of x at time t or in short we will suppress the variable t and write x dot is equal to f of x. So, the dot means derivative with respect to time. So, here at every time instant x of t is an element of R n. So, this short hand notation x dot is equal to f of x has actually n equations within.

So, the first equation is x dot is equal to f 1 of x second 1 is x 2 dot equal to f 2 of x etcetera. So, f also is a map from R n to R n. So, f takes a value of x which is an element of R n and gives out another vector which is again in R n. So, for every each initial condition, suppose there is a trajectory x of t satisfying the above differential equations, so to what extent we can say that for every initial condition, there exists a solution and to what extent is that unique. These are some important questions, we will address for the time being please assume that for each initial condition, suppose there is a trajectory that evolves from that initial condition. So, this trajectory itself is a vector valued function x of t is a function of time.

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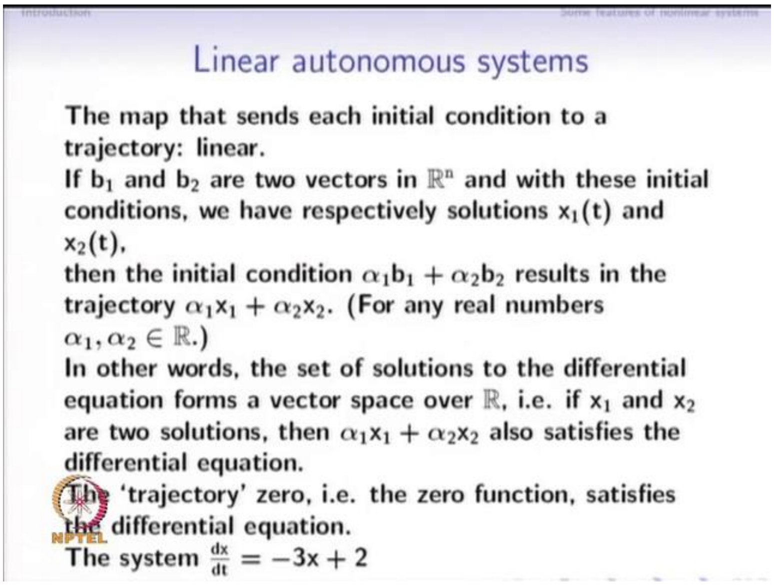

Then we can ask is this map linear, which map? The map that sends each initial condition to a trajectory is that map linear. In other words, if b 1 and b 2 are 2 vectors in R n and with these initial conditions with the initial condition b 1 and b 2, we have respectively solutions x 1 and x 2 as a function of time. Then the initial condition alpha 1 b 1 plus alpha 2 b 2 results in a trajectory alpha 1 x 1 plus alpha 2 x 2, suppose this property is true. Then we will say that map is linear and in such a situation we will also like to say that the autonomous system is a linear autonomous system.

So, again as I said these are required to be true for any real numbers alpha 1 alpha 2 and for any 2 vectors b 1 and b 2 in R n. This also equivalent t saying, the set of solutions to the differential equations forms a vector space over R, that is if x 1 and x 2 are two solutions. Then alpha 1 x 1 plus alpha 2 x 2 also satisfies the differential equation. This alpha 1 x 1 plus alpha 2 x 2 is also a solution to that differential equation. We can ask is the trajectory 0 a solution to the differential equation.

The trajectory 0 now again means the 0 function. So, we can now see that the system d by dt equal to minus 3 x plus 2. Here, we can substitute the trajectory x of t equivalently equal to 0 and check whether the 0 function satisfies the differential equation. And we will obtain that 0 is not a solution to this differential equation and hence this autonomous system is not a linear autonomous system.

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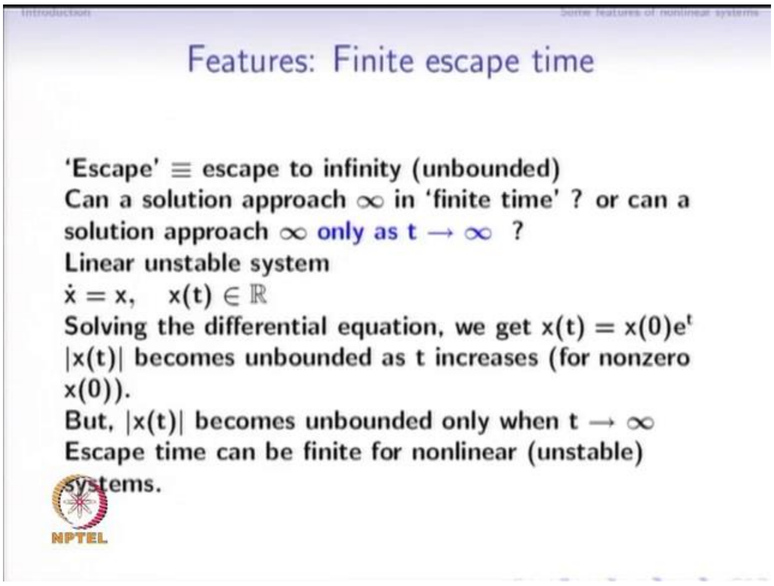

So, we come to some features that is present only in non-linear system. This makes the study of non-linear systems extremely interesting and challenging also. So, what are some features? That is present only in non-linear systems? The first important feature is what we will like to say finite escape time so let me explain these terms 1 by 1 escape in this situation means escape to infinity.

So, escape to infinity means does the solution x of t become unbounded in R n. So, can a solution approach infinity, can it become unbounded in finite time instant? That is the question escape. Here, means escape to infinity means, here it becomes unbounded and can. This happen when t itself is finite that is the question we are asking here or is it that the solution x of t can become unbounded only as t tends to infinity? So, this knows the important question that we are going to address for a linear unstable system, exactly the definition of unstable we will see later. But for now we see that the differential equation x dot equal to f of x in which at any time instant x has only 1 component.

So, x of t is an element of R, solving this differential equation, we get x of t is equal to x of 0 times e to the power t. So, we see that x of t becomes unbounded as t increases for a non-zero initial condition x of 0, but we also see that x of t becomes unbounded only when t tends to infinity. It does not become bounded for a finite time t. If, anybody gives us a finite value of time t, we can evaluate x of 0 times e to the power t and see that it is again a finite number, but for non-linear systems the escape time can be finite for non-linear systems, which is not possible for linear systems.

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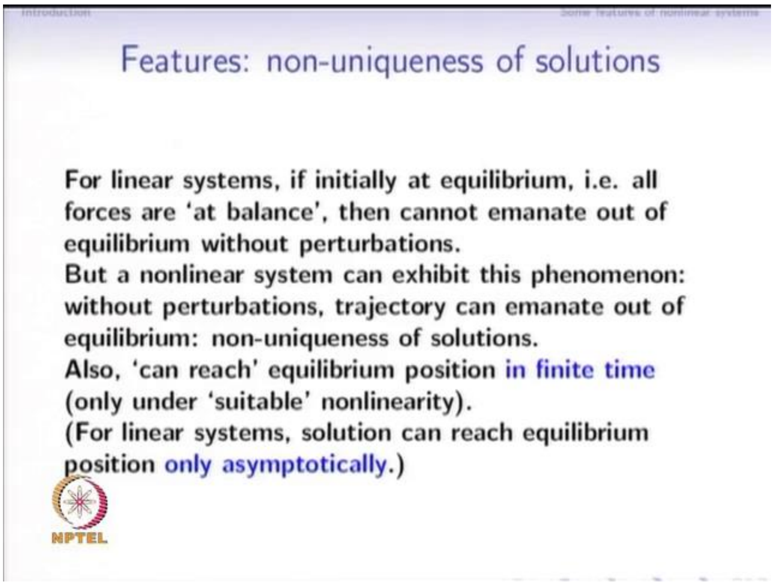

Another important feature is for a linear system we can ask, if initially the system is at equilibrium, what is equilibrium? All the forces acting on the trajectory are at balance in such a situation we can ask, is the system going to remain in equilibrium? If, we are at an initial condition, which is such that the system is at equilibrium, does it mean that the system will remain at equilibrium for all future time? This is the question of uniqueness and non-uniqueness of trajectories and we are going to address this situation for linear systems. So, for linear systems it turns out that if initially the system is at equilibrium then it cannot emanate out of equilibrium without perturbation.

So, but a non-linear system can exhibit this phenomena without perturbations also the trajectory can emanate out of equilibrium in the absence of perturbations also and in that sense we see that we also have non-uniqueness of solutions possibly in non-linear differential equations. The analog of this particular situation is that the question can we reach an equilibrium point in finite time under some suitable nonlinearity. It turns out that we can reach the equilibrium point in finite time and this aspect is also not there in linear systems. So, for linear systems what is possible? For linear systems the solution can reach the equilibrium only asymptotically not in finite time, but only as t tends to infinity.

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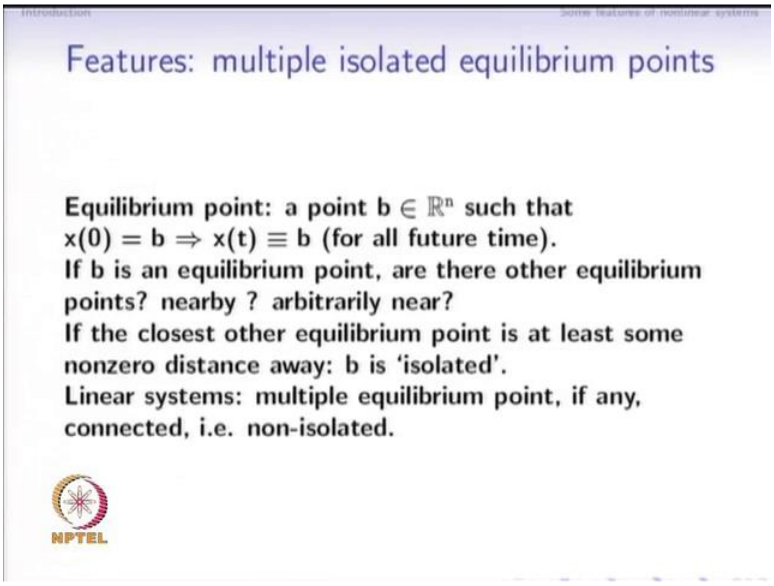

Another important feature for non-linear systems is the notion of equilibrium point, while equilibrium point is also present in linear systems. We will see that the equilibrium points for linear systems are all connected unlike non-linear systems, where we could have multiple equilibrium points which are not connected. In which case we will call them isolated? So, we will see this in detail now, so an equilibrium point is a point b such that if initially the trajectory is at b then it remains at b for all future time this definition, we will see more carefully very soon. The equilibrium point we understand as an initial condition, if we start of there we remain there for all future time? So, if b is an equilibrium point are there other equilibrium points. This is the question we can ask.

The next question, we can ask, is if there are other equilibrium points are these other equilibrium points close by, and if they are close by can they be really close in other words can they be connected. So, if the closest other equilibrium point is at least some non-zero distance away. In other words there is some small distance within which there is no other equilibrium point, other than the point b in such a situation, we will call b is isolated. When we will call b isolated? If, in the situation that there are other equilibrium point every other equilibrium point is at least some non-zero distance away from the point b. In such a situation we will say that there are multiple equilibrium points, but b is an isolated equilibrium point.

So, for linear systems in case, there are multiple equilibrium points they are all non-isolated in other words they are all connected to each other. We take any equilibrium point for a linear system and for any small enough distance, we will see there is another equilibrium point in the vicinity.

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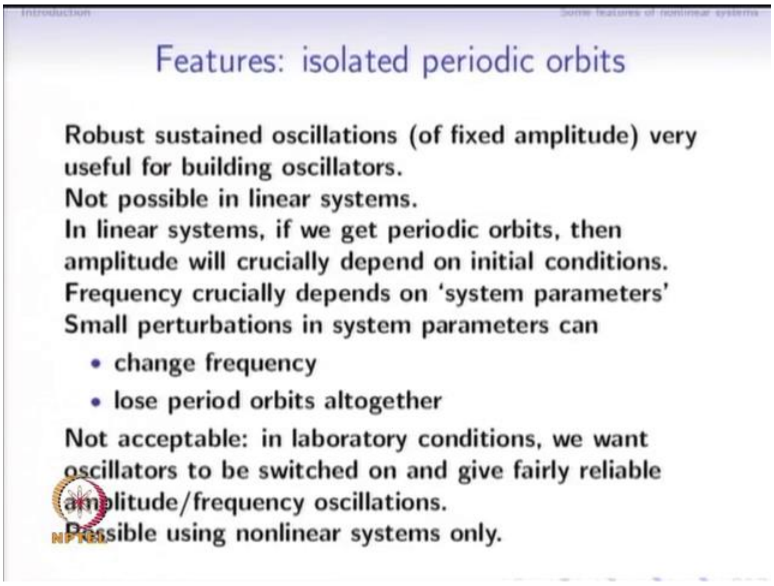

Another important feature of non-linear systems is that we can have periodic orbits which are isolated, even the periodic orbits can be isolated just like the equilibrium points can be isolated. So, this is relevant in the context of robust sustained oscillations, we will see these terms carefully now. So, why are robust sustained oscillations important if an amplitude is fixed, and if the frequency is fixed. Then they are very relevant for building oscillators in a laboratory, so such a situation it turns out is not possible for linear systems.

So, in linear systems if we get periodic orbits for a certain linear system, then the amplitude will very crucially depend on the initial conditions. If, the initial conditions are different then the amplitude will no longer be the same. It is very unlikely that by changing the initial condition we will get the same amplitude. Also the frequency of the periodic orbit also depends crucially on system parameters, small perturbations in the system parameters can change the frequency. In fact it could also lose the property of periodic orbits altogether.

So, why is this not acceptable this is not acceptable in laboratory condition? We will like that oscillators are just switched on and they give fairly reliable amplitude and frequency oscillations. So, that we are able to build an oscillator using this particular differential equation. So, this is possible only using non-linear systems.

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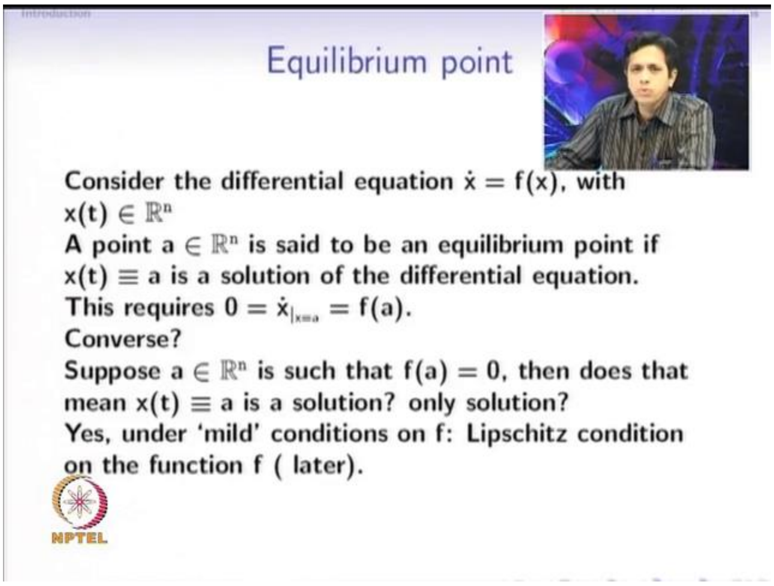

Now, we will see what a non-equilibrium point. So, consider the differential equation x dot is equal to f of x in which x at any time t x of t is an element of R n. There are n components in the vector x, a point is said to be an equilibrium point if x of t is always equal to a. If, this is a solution of the differential equation, so take the differential equation x of t equivalently equal to a for this particular trajectory. If, we see that this is also a solution to the differential equation, then the point a is said to be an equilibrium point. So, what does this require from f, what we see is if it should always remain at a the rate of change of x with respect to time should be equal to 0, when evaluated at the point x equal to a.

So, at the point x equal to a x dot is nothing but f of x in other words when f is evaluated at a then we get 0 the 0 vector. Now, we can ask is the converse true, what is the converse of the statement. Suppose, a is a vector in R n such that f evaluated at a is equal to 0 then does that mean that x of t equivalently equal to 0 is the solution to that differential equation. So, this really suggests that this converse should also be true and we will see that under some fairly mild assumptions. This is indeed true and it is also the only solution. So, what are these mild conditions? We will see that there is an important condition called the Lipchitz condition and the Lipchitz condition, we will like to say is mild because this is how most functions f would really look like, but this is the topic that we will do in detail later.

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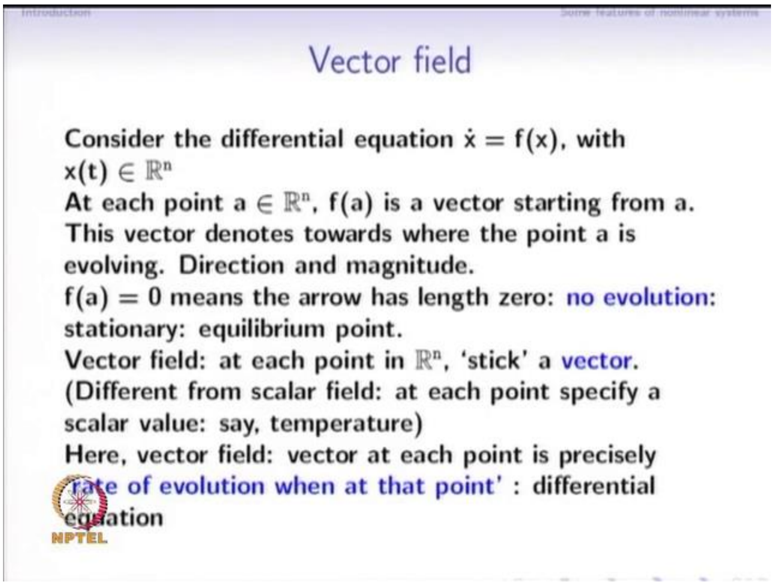

Now, we will quickly see another interpretation of a differential equation. So, consider the differential equation x dot is equal to f of x with x of t an element of R n. So, at each point a in R n f of a is a vector starting from a, so at each point a we will evaluate f at a this also is an element of R n. This vector we will like to place as starting from a what does this vector denote it, denotes where the point a towards where the point a is evolving both towards direction and magnitude.

So, f of a equal to 0 means the arrow there has length 0 in other words there is no evolution from the point. In other words the rate of change at that point is equal to 0. This is what we will like to also like to say is stationary if at that place we will start then everything is stationary and the system does not evolve this also what we call an equilibrium point. So, what is vector field about it?

At each point in R n we are sticking, we are attaching a vector there; this is unlike a scalar field where at each point we could also specify a scalar value. For example, the temperature at every point in the room, this would be a scalar field, but in our situation at every point a in R n we have a vector with equal number of components. Hence, we will say this is a vector field and moreover this vector at every point is precisely. The rate of evolution when we are at that point, this is what is a differential equation a first order differential equation is exactly this notion? Where at every point a we will stick a vector there, and this vector denotes the rate of change of that particular point under the action of that differential equation.

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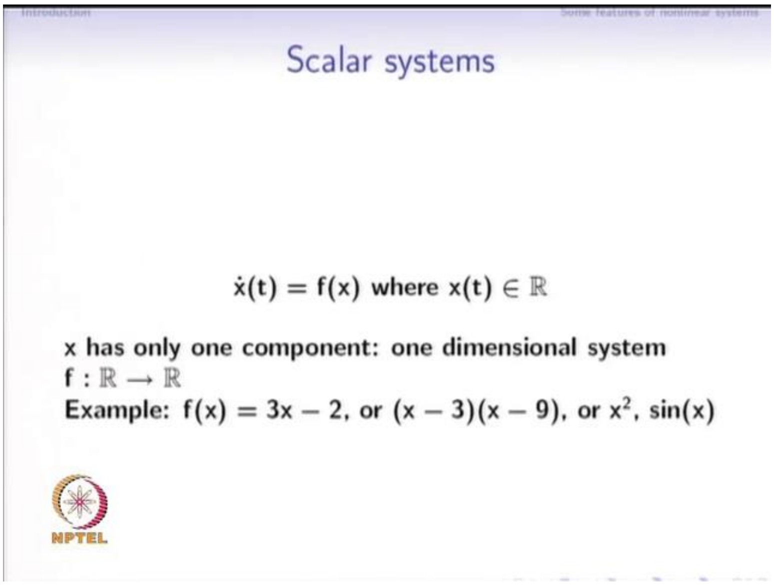

So, we will end today's lecture beginning with the topic of scalar systems. So, consider the scalar differential equation what is scalar about it x dot t is equal to f of x where x of t has only one component. It is a real number, so this is also called a one dimensional system in such a situation f is a map from R to R. For example f of x is equal to 3x minus 2or f of x equal to x minus 3 times x minus 9 or x square or sin x. These are examples of f that we will see today. So, this particular situation is best seen using a figure.

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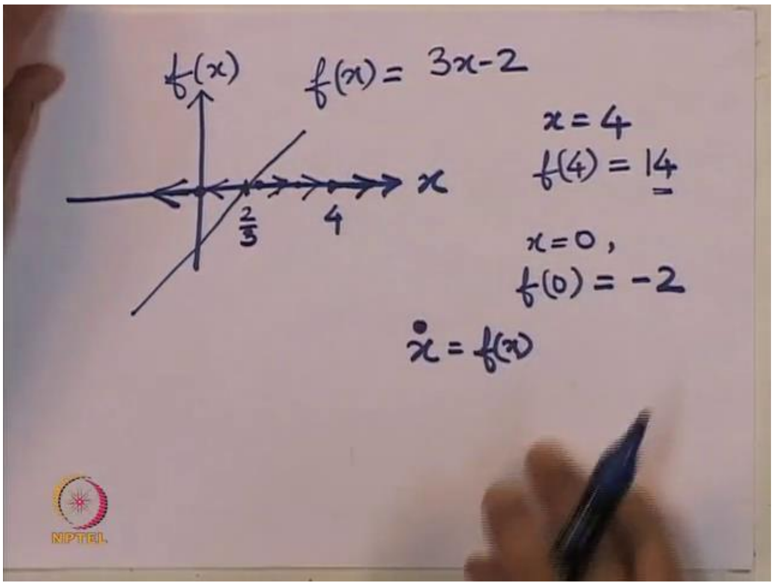

So we are going to attach at each point a vector. So, consider f of x is equal to 3 x minus 2 and suppose we take the point 4 y x equal to 4 for this point x equal to 4 we will evaluate f at 4 and we obtain 14. So, what it means at this particular point. There is an arrow which is starting from the point 4 and it is to the right. Why it is to the right? Because this number 14 that we have obtained is positive and moreover in addition to being to the right towards the direction of increasing x. It is a vector of length 14 at another point for example, x equal to 0 we can check, what is f evaluated at x equal to 0 and for that we get minus 2.

So, it means that at 0 we draw a vector which is towards the negative direction of x and it has length equal to 2. So, for a scalar differential equation x dot is equal to f of x it means that at each point, we can draw a vector to the right or left depending on whether f at that point is positive or negative. So, in this particular situation we see that f of x is equal to 0 precisely at x equal to 2 by 3. So, suppose 2 by 3 is a point here and f of x is a line.

So, now we are going to plot a graph of f versus x even though x itself was a function of time, we are plotting f as a function of x and we see that we get this line, which passes through the point x equal to plus 2 by 3, at this point f becomes equal to 0. So, at this point this vector has length 0 everywhere to the right we see that this vector is pointed to the right of this point, why is it to the right because we see that f to the right of the particular point is all positive. So, this particular point let me draw a slightly bigger figure.

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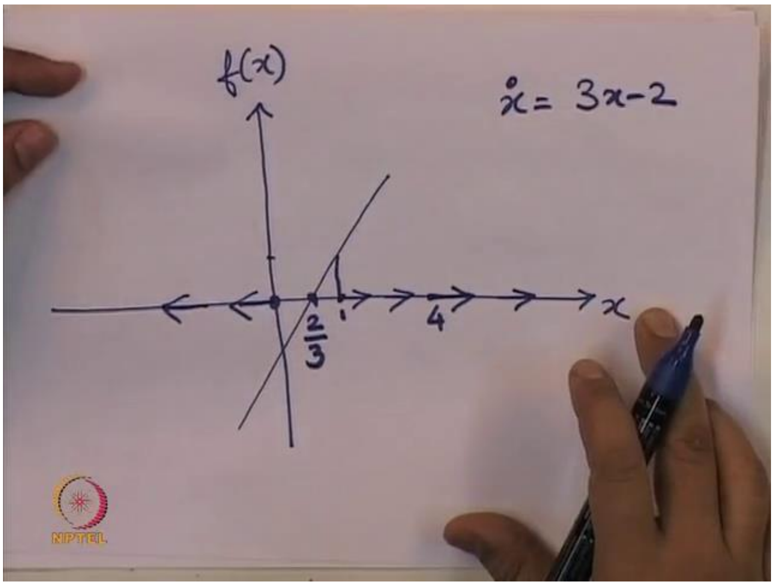

We are considering the differential equation x dot is equal to 3 x minus 2 and we are going to plot f versus x even though itself was a function of time. We will also later plot x as a function of time, but for now we are interested in drawing the vector field we took a point 4 there is a point 2 by 3 here and there is a point 0 at the point 4. We already saw the vector is directed to the right at the point 2 by 3. The vector has length 0 and at the point 0 the vector is directed to the left towards decreasing direction of x.

So, what does this mean that when we draw when we plot f of x versus x see that to the right of the point 2 by 3. The arrow is marked to the right why is it to the right because at this particular point say 1, we see that f is positive at x equal to 1 because f is positive, it means x dot is positive in other words x is increasing. So, more generally we can see that if we are given with a function f and if f is scalar we can draw a graph and decide at which points x is increasing, which points x is decreasing by just seeing whether f is positive or negative, at that particular value of x.

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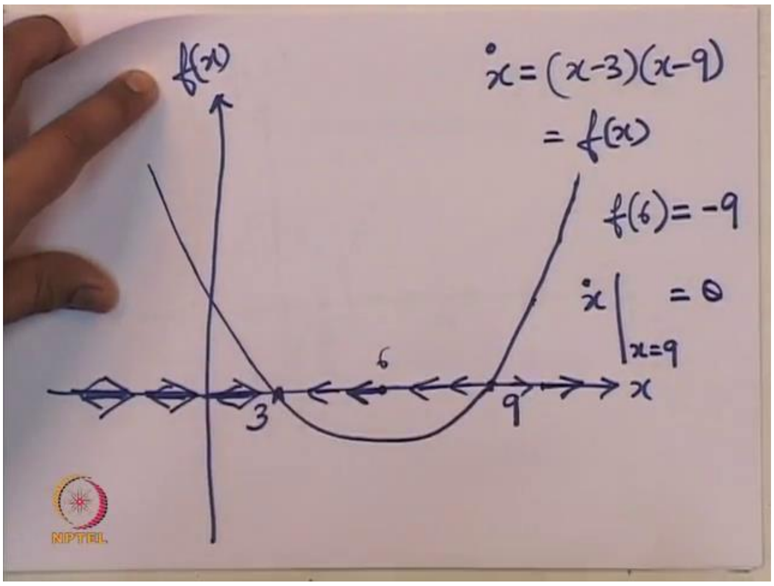

This is away we will analyze the other example that we saw. So, consider the differential equation x dot is equal to x minus 3 times x minus 9 which we want to call as f of x. So, this is equal to 3 this is equal to 9 and the graph of this function looks roughly like this. This function has roots at x equal to 3 and 9 and hence it is passing through 0 precisely at x equal to 3 and 9 and if we take a point 3. Then we see that the vector at the point 3 has length 0 and hence we plot it neither to the right nor to the left.

On the other hand consider the point 6 at x equal to 6, we expect that f will be negative, we can check that f at 6 is equal to 3 times minus 3 which is minus 9. So, since it is negative we can also see that from the graph. Here, this is a vector pointing to the left towards decreasing direction of x and that we can also see because x dot is equal to f of x. So, at x equal to 6 x dot being negative x would eventually start decreasing. It would decrease and that is precisely what this arrow shows the arrow shows the direction in which x will evolve. On the other hand suppose we take x equal to 11 at x equal to 11 we can easily draw the graph the arrow to the right why because at x equal to 11 the function f takes a positive value.

So, we are able to draw all the arrows for this particular example, all we have to do is we have to see where the equilibrium points are and to the left and the right of the equilibrium points, we can draw the arrows towards increasing direction of x or decreasing direction of x depending on whether f takes positive values there or negative values. Another important point we can note is that if we start at the equilibrium point x equal to 9. We, will of course, remain at n1 because x dot is equal to 0 x dot evaluated at x equal to 9 is equal to 0 and hence x does not change at all it will remain in the point 9. Similarly, x equal to 3 is also an equilibrium point.

So, as we can see we have made a mistake here all these arrows for x less than 3 because f is positive it cannot be towards decreasing direction of x. On the other hand we should be seeing that all the direction, all these directions have to be reversed. They are all towards increasing direction of x, so this particular figure I will quickly draw again.

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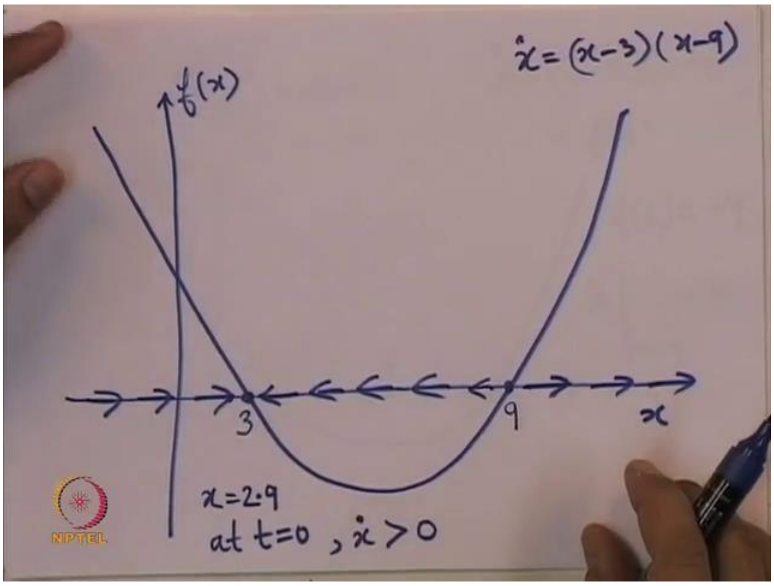

So, this is the differential equation x dot is equal to x minus 3 time x minus 9. Another important feature we can see here is that if we start slightly to the right of equilibrium point 9 equilibrium point 9 if we start to the right then x is going to increase and it will go farther from 9. So, for a very small perturbation 9 to the positive side of n1 takes that initial condition, further away from 9 even though we noted and at the point 9 the trajectory remains at 9 for all future times, but slightly to the right for a very small perturbation the trajectory goes away from 9.

Also slightly to the left of point 9 we see that the trajectories are again directed away from 9, why? Slightly to the left of 9 the function f is negative. So, the x will become further away from 9 it will further decrease. So, we will like to say that this particular point is an equilibrium point, but it is also an unstable equilibrium point. For very small perturbations both to the right and left we see that trajectories are going to move away from this equilibrium point.

On the other hand please note that 3 is also an equilibrium point, but for very small perturbations to the right all the arrows are pointed towards the point 3. We expect that for small perturbation towards the positive direction of 3 the trajectories are moving back towards 3. On the other hand if we move slightly to the left of 3 meaning if you start from an initial condition. For example, x equal to 2.9 at t equal to 0 we see that x dot is greater than 0 that is why the arrow is marked to the right and hence it will increase and approach 3 again.

So, this equilibrium point we will like to call is a stable equilibrium point. In the context Lyapunov's stability, we will see more precise definitions of stable unstable asymptotically stable equilibrium points, for now for a scalar system looking at the graph of f versus x, we are able to decide which are the equilibrium points. We are also able to decide whether these equilibrium points are stable or unstable. So, now we will see another example from the list of examples we saw.

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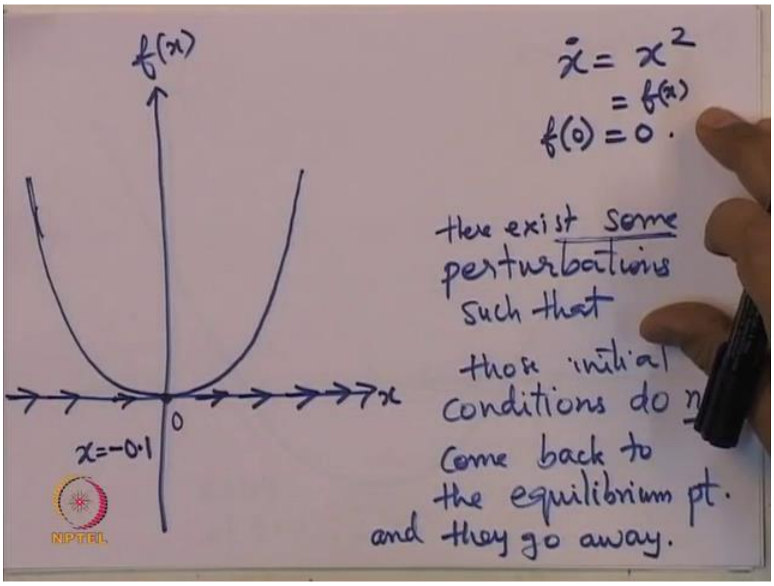

Consider this graph, we see that f. So, this is equal to f of x this is f at 0 is equal to 0 why because 0 is a root of this. So, we see that the point 0 itself is an equilibrium point if we start there, we are going to remain there slightly to the right we see that f is positive and hence the arrows are directed towards the right. On the other hand slightly to the left of the point, we see that f is positive and hence again arrows are directed towards the right. In other words the value of x is going to go on increasing whether it is to the right or to the left of the point 0 and only at the point 0, the value of f being 0 x does not change x dot is equal to 0.

So, now we will like to ask is this equilibrium point stable or unstable the property of stable or unstable, we like to give only to equilibrium points this x equal to 0 is an equilibrium point. Now, we see that slightly to the left we see that when x is slightly negative. For example, x equal to minus 0.1 the value of x dot is positive and hence x is going to increase and approach 0. So, can we call the 0 a stable equilibrium point? We can answer after we analyze to the right of the point 0 to the right x dot is again and hence x is going to further increase and become away from 0.

So, we see that for certain perturbations it comes back to 0 and for certain other perturbations it goes away from 0. So, we can say that there exist some perturbations such that such that initial conditions do not come back to equilibrium point. So, in such a situation we are going to say that this equilibrium point is unstable when we will call it unstable there are just some bad perturbations there exists. Some perturbations such that those initial conditions they do not come back to the equilibrium point and they go away, in such a situation that equilibrium point is unstable.

We are not going to be satisfied with some perturbations which come back to the equilibrium point. We are unhappy that there are some perturbations that are going to go away from that equilibrium point, and hence that equilibrium point has been classified as unstable. So, we will see more about stability, unstability, asymptotic, stability in the following lectures, but we will end this lecture by seeing a similar graph which we will like that is d1 as homework for which examples?

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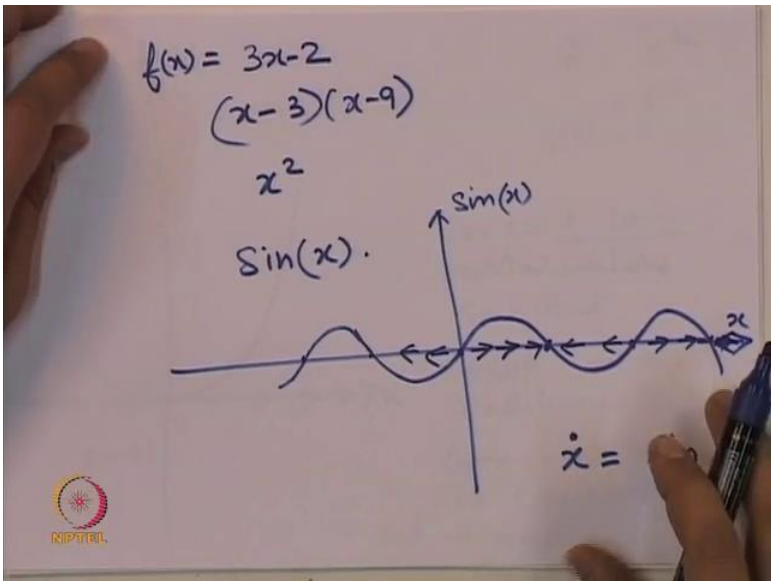

We have already seen 3 x minus 2, we have also seen x square. Now, we will quickly decide what are the equilibrium points for this particular function sin of x and whether they are stable or unstable? So, there are several equilibrium points there are several equilibrium points for the differential equation x dot is equal to sin of x. Please note and x itself is not a sinusoidal trajectory; it is a differential equation in which sin comes in. Suppose, this is an example so here we see that all the 0 crossings are equilibrium points and depending on whether before and after that equilibrium point, whether this sin of x is positive or negative based on that we are able to classify these equilibrium points as stable or unstable.

So, here we can draw these arrows to the right and here to the left, similarly here again we can draw. So, we see that this equilibrium point is unstable this one is stable, this is unstable, this is stable. So, for this particular differential equation we are able to see that there are several equilibrium points and alternatively they are, alternately they are stable or unstable. So, this is something that we expect the viewer to carefully verify, so with this we end today’s lecture. We will continue with these aspects from the next lecture in more detail.

Thank you.