Nonlinear Dynamical Systems
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Lecture - 11
Bendixson and Poincare Bendixson Criteria
Van-Der-Pol Oscillator

Hello everyone, I am Sriram C Jugade and I welcome you all to the lecture number 11 of non-linear dynamical systems.

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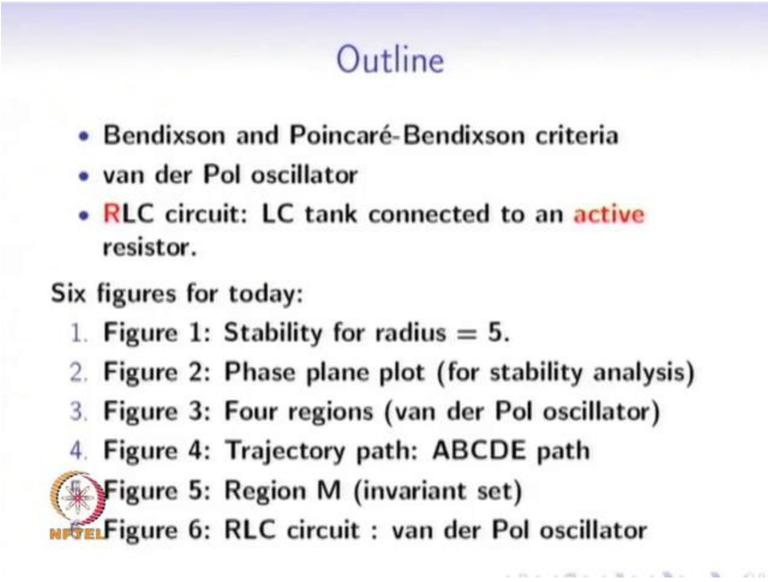

In today's lecture we will look into the Bendixson and Poincare Bendixson criteria, the application of it we will consider one, two examples ((Refet Time: 00:31)). Next we will consider Van-der-pol oscillator, we will study Van-der-pol oscillator which is a non-linear oscillator. Then we will take example of Van-der-pol oscillator which is RLC circuit, LC tank connected to a active circuit. During the lecture we will refer to the following figures six figures.

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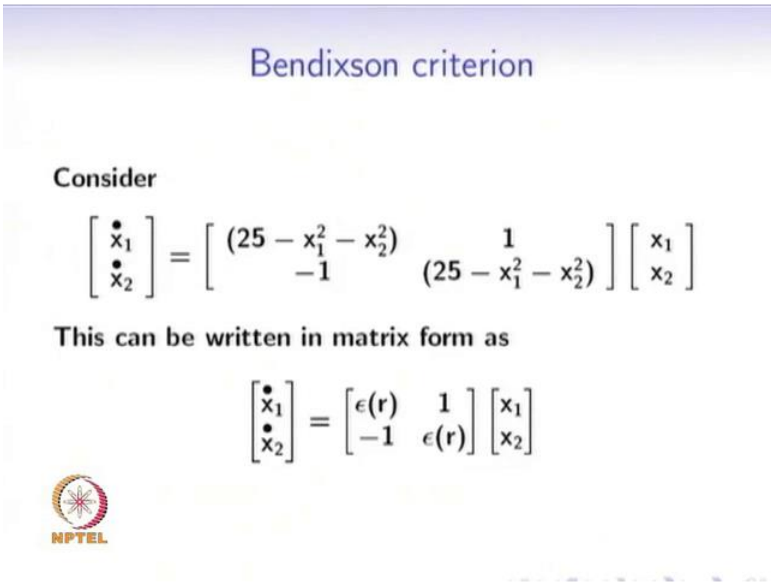

First we will consider the Bendixson criteria, here the state equations of an examples, here we can see in the matrix the function 25 minus x 1 square minus x 2 square. This function is dependent on both x 1 and x 2, let us say the radius is r then we can say that x 1 square plus x 2 square is equal to r square. So, the function 25 minus x 1 square minus x 2 square is can be written as 25 minus r square, so the function is dependent on r. Let us represent that function as epsilon r a function dependent on r. So, the state equations reduce to the following form, which are shown in this slide.

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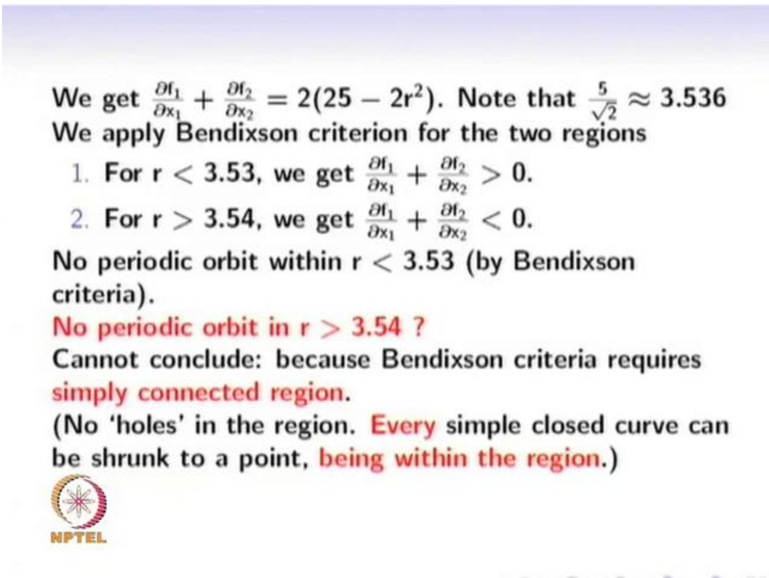

Next, let us consider the expression dou f 1 upon dou x 1 plus dou f 2 upon dou x 2, this results out to be 2 into 25 minus 2 r square the root of this equation is r equal to 5 by root 2, which is approximately equal to 3.536. So, at r equal to 3.536 we will have this expression value to be 0. Now, let us try to apply Bendixson criteria to the following example, for this we will consider two regions; the first region we will consider bounded by all r which is bounded by 3.53. So, r is strictly less than 3.53, now for this region we will get the expression dou f 1 upon dou x 1 plus dou f 2 upon dou x 2 as strictly greater than 0.

Now, the second region we will consider for r strictly greater than 3.54 for this region we have the value of the expression dou f 1 upon dou x 1 plus dou f 2 upon dou x 2, strictly less than 0. So, we can see in both the regions the sign of the expression dous not change in the first region the sign of the expression remains positive, and in the second it remains negative. So, let us try to apply a Bendixson criteria let us consider the first region where r is strictly less than 3.53, so there is no sign change we can say that by Bendixson criteria no periodic orbit exist in the region.

Now, consider a region where r is strictly greater than 3.53 now the question arises whether we can apply Bendixson criteria to this? The answer is no we cannot conclude in this case because Bendixson criteria requires simply connected region. Now, what is simply connected region? A simply connected region is a region which has no holes or we can define it in other sense, if we take a that region and if we take a simply closed curve in that region and shrunk it to the point then it should also remain within the region. During the shrinking every curvature and up to the point it should be remain in the region. So, that is how we define simply connected region.

So, for r greater than 3.54 we cannot conclude whether there are periodic orbits or not or we cannot apply Bendixson criteria. Since, this region is not a simply connected region. So, we conclude here that Bendixson criteria is not applicable for a region r greater than 3.54.

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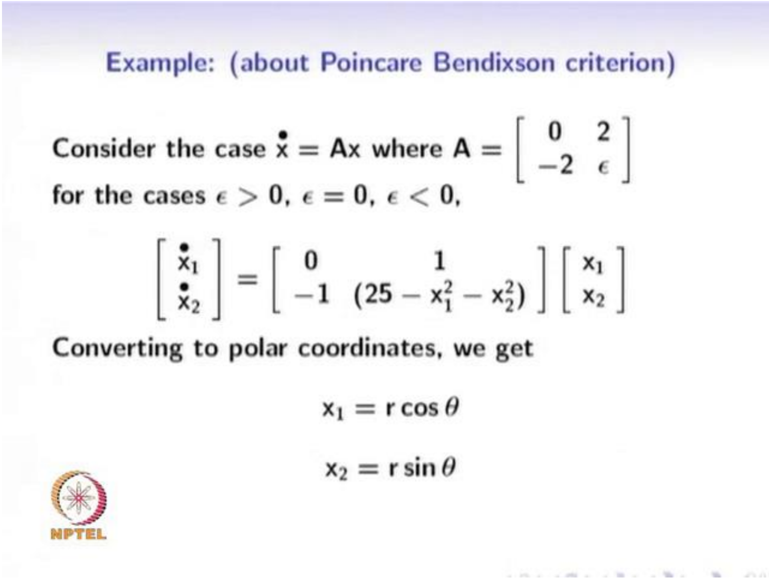

Next, we will consider an example of Poincare Bendixson criteria, here we take a system of the form x dot equal to A x where A is equal to 0 2 minus 2 epsilon the diagonal elements are 0 and epsilon. Now, in this case as epsilon will vary, the behavior of the system will change. So, we will consider three cases in the first case we will consider epsilon as greater than 0, in the second case we will consider epsilon equal to 0, and in the third case we will consider epsilon less than 0. Now, let us take that example pervious one in the previous one we had both the diagonal elements as 25 minus x 1 square minus x 2 square.

Now, in this example we have 1 diagonal element as 0 and the second as 25 minus x 1 square minus x 2 square. So, we will try to analyze the behavior of the system and for that we will convert the coordinates into polar coordinates. So, we will have a clearer picture so we can convert a polar coordinate into the following form. So, x 1 will become r cos theta and x 2 will become r sine theta, where r is the radius and theta is the angle.

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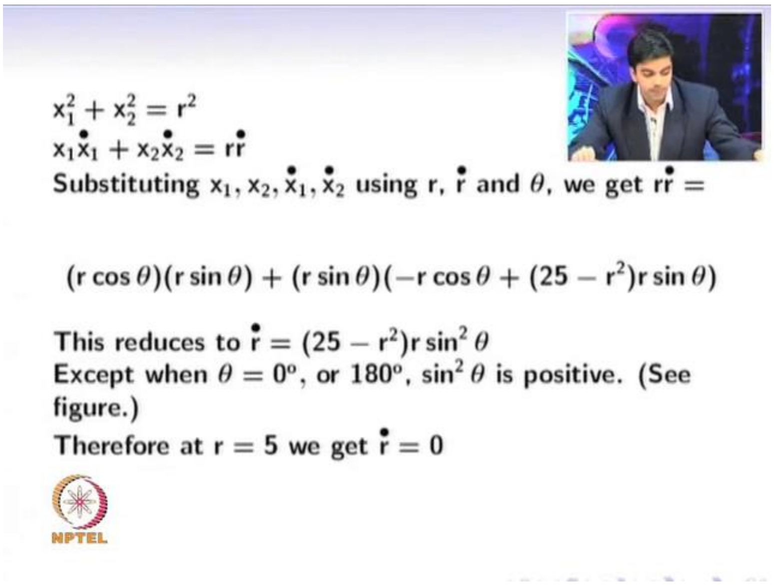

So, x 1 square plus x 2 square we get as equal to r square differentiating it with respect to time, we will get x 1 into x 1 dot plus x 2 into x 2 dot equal to r into r dot. Now, we have the expressions for x 1 and x 2 and x 1 dot and x 2 dot from the state equations. So, we can substitute in this expression and we will get the following expression at r dot equal to expression shown on the slide. So, we will cancel out the common factors and rearrange it and the equation will reduce to the final form r dot equal to 25 minus r square into r into sine square theta. So, except theta equal to 0 or theta equal to 180 degrees theta is sine square theta is always positive. So, let us consider one figure.

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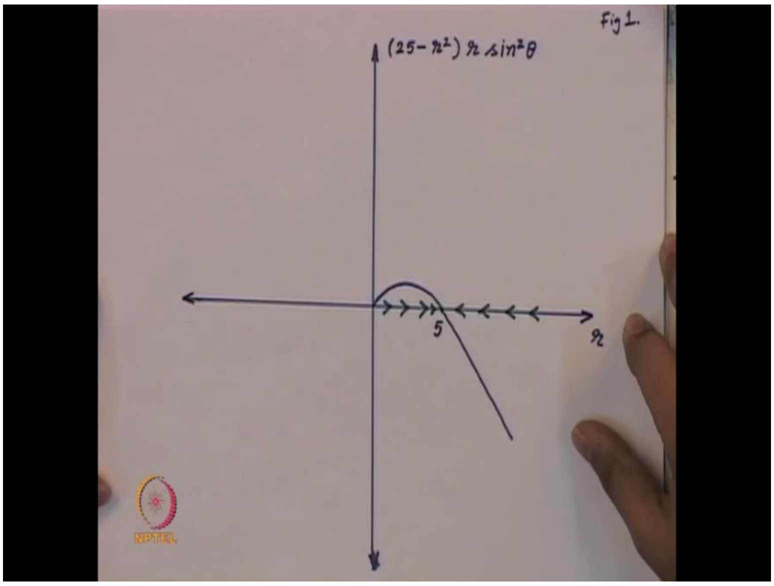

In this figure we have plotted 25 minus r square into r into sine square theta versus r. In the first case we have consider r greater than 0, so we can find it for r equal to 5 the expression reduces to 0, so we can see that for r equal to 5 r dot is equal to 0. So, when r dot is equal to 0 we can say that the circle with radius 5 is a periodic orbit. Since, the radius is not changing.

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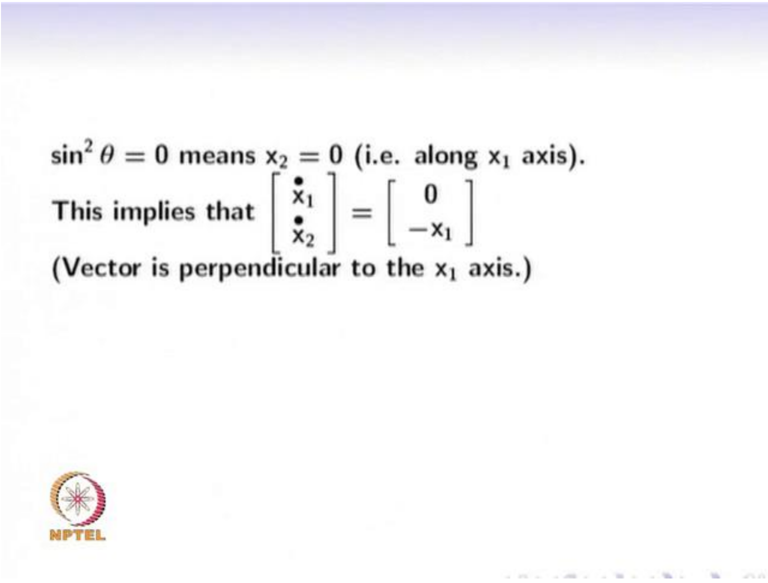

Now, we will consider the second case where sine Square theta equal to 0, means theta is equal to either what 0 or 180 degrees, so in that case we will get x 2 equal to 0. So, the x 2 equal to 0 is a is along the x 1 axis. So, the if we substitute these values in the state equation then our state equation reduce to the following form where x 1 dot equal to 0 and x 2 dot equal to minus x.

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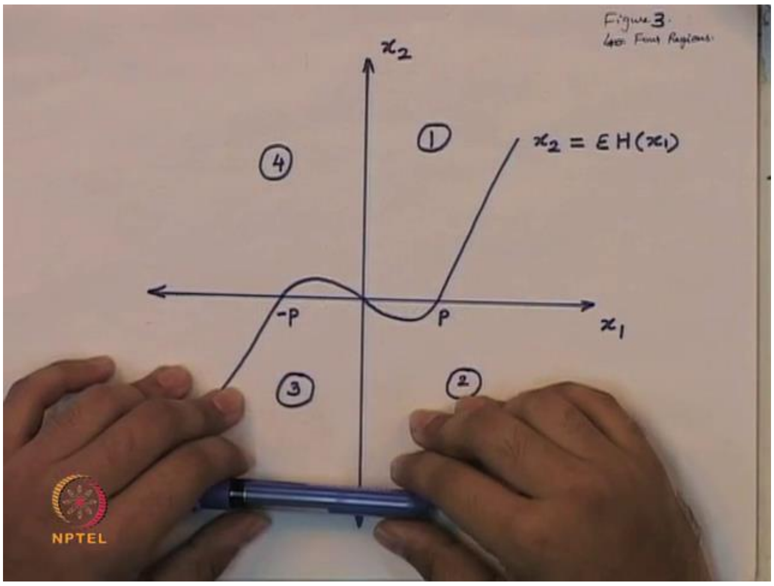

In this case we have a point on the x 1 axis, which is the initial condition then according to the state equations we will have x 1 dot equal to 0 and x 2 dot equal to minus x 1. So, for x 1 to be when x 1 is positive we will have a vector pointing in downward direction, and it will perpendicular to the x 1 axis. Similarly, when x 1 is negative direction vector of the vector will be pointing outwards and it will be perpendicular to the x 1 axis. So, the magnitude of this vector will depend on the value of x 1. So, we can see that for a x 1 not equal to 0 the vector is always non-zero.

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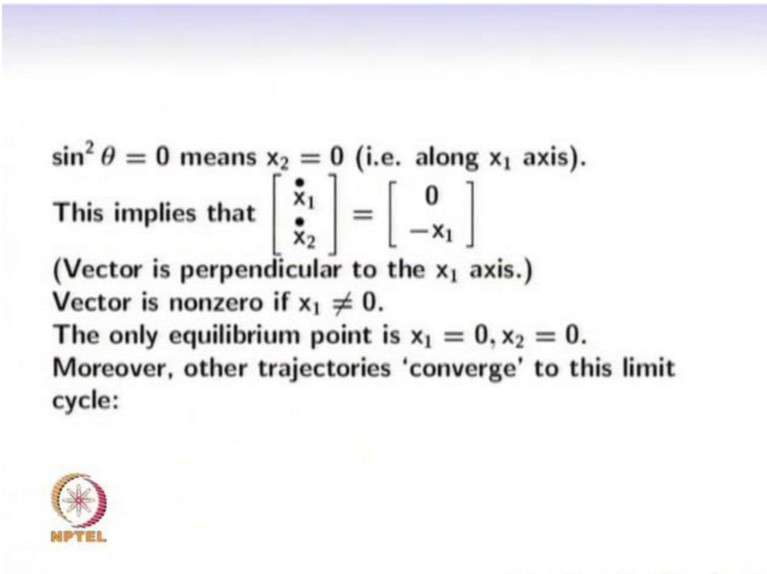

So, from the state equation we can see that for the equilibrium point we need x 1 dot to be 0 and x 2 dot to be 0. So, in this case the only equilibrium point we can see is when x 1 dot is equal to 0. So, the only equilibrium point is x 1 equal to 0 and x 2 equal to 0 so we will go back to the previous figure, where we have drawn plotted the r dot versus r in this case we can see that for r equal to 5, if there is a disturbance or perturbation then the trajectories are approaching towards r equal to 5, when r is greater than 5 or r is less than 5.

So, the point at r equal to 5 on the periodic orbit we will conclude that it is stable, we can also say that the limit cycle is a isolated. The meaning of isolated is if we consider a small region around r equal to 5 then we will have no periodic orbits, so in that region r equal 5 is the only periodic orbit existing. We already proved that how it is stable since for disturbance all the trajectories are pointing towards r equal to 5, so it is a stable limit cycle.

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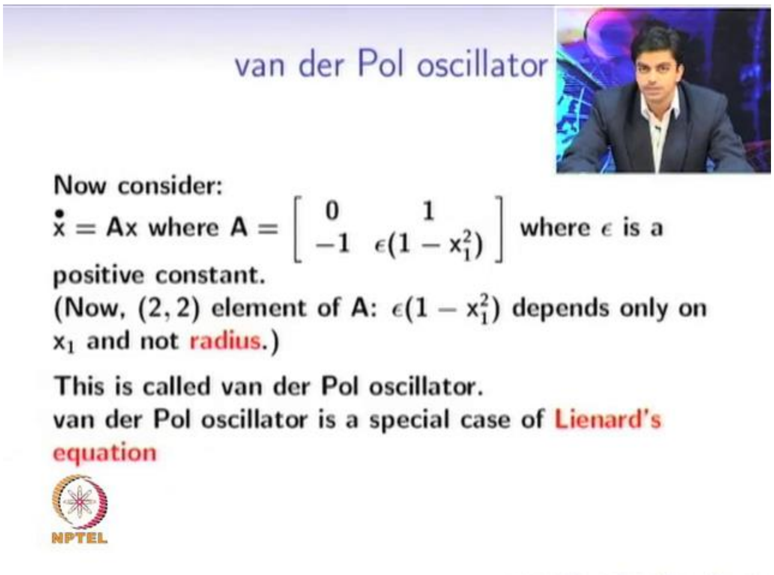

Next, we will consider van-der-pol oscillator van-der-pol oscillator is the non-linear oscillator, if we consider the same for system form as x dot equal to x where A is equal to 0 1 minus 1 and epsilon into 1 minus x 1 square. So, epsilon here is a positive constant here we can see that 1 diagonal element is dependent on x 1 square. So, it is not the diagonal element is dependent only on x 1 square it is not dependent on x 2 square. So, we cannot conclude that it depends on radius it dous not depend on radius, it depends only on x 1 square. So, this system we will call it as a van-der-pol oscillator and van-der-pol oscillator is a special case of Lienard's equation.

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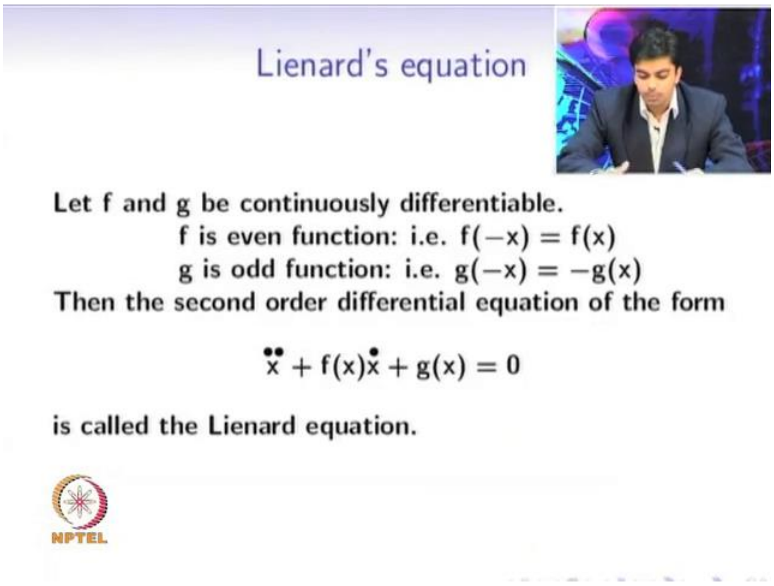

Next, we will consider Lienard's equation as we previously mentioned that van-der-pol oscillator is a special case Lienard's equation defines the generalized case for the non-linear oscillators, we let us consider 2 functions f and g which are continuously differentiable. Let us consider that f is a even function that is f of minus x is equal to f of x and g is a odd function so that g of minus x is equal to minus g x. So, the second order differential equation of the form x double dot plus f of x into x dot plus g of x is equal to 0, this equation is called Lienard's equation this is the generalized form of the equation for non-linear oscillators. In general non-linear oscillators are considered for the modeling of the physical oscillation.

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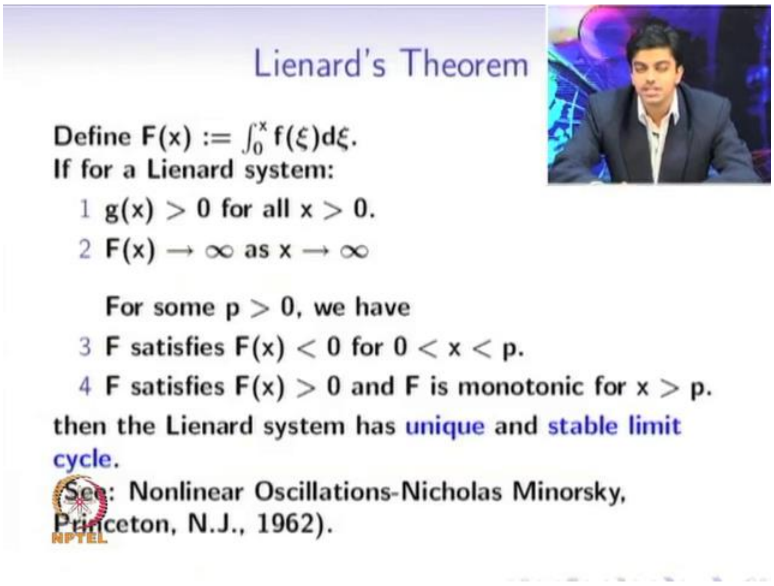

We have defined the generalized Lienard’s equation for the non-linear oscillators. We will have to next look into the stability of the oscillator for the non-linear oscillator for that first we will define a function capital F of x which is equal to integral of small f of x. Then for a Lienared’s system if we consider the following conditions like g of x is greater than 0 for all x greater than 0 and capital F of x tends to infinity as x tends to infinity.

And for some p capital F of x satisfies that it is negative for the range, when x is between 0 to p and f of x also satisfies that it is positive and monotonic, for x greater than p. That is when x is greater than p the f of x is monotonically increasing, if these conditions are satisfied then we can say that the Lienard’s system is having a unique and a stable limit cycle and this is what is called Lienard’s theorem. Lienard’s theorem gives the conditions for the stability of oscillations for non-linear oscillators. For the reference you can see the book titled non-linear oscillations by Nicholas Minorsky with the respective edition.

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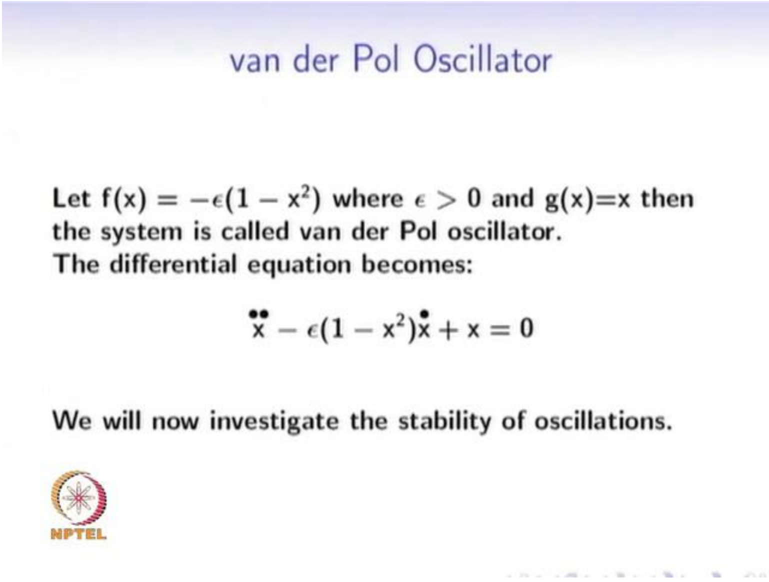

Next, let us consider f of x is equal to minus epsilon into 1 minus x square where epsilon is a scalar and it is strictly greater than 0 and g of x equal to x then the system is called van-der-pol oscillator. Previously we consider a Lienard's equation where it was a generalized case, now we are defining van-der-pol oscillator in that equation where f of x and g of x are defined as I said before. So, the differential equation the Lienard's equation gets transformed to the form x double dot minus epsilon into 1 minus x square into x dot plus x equal to 0.

Let us now investigate the stability of oscillation we had Lienard's theorem which gives us the condition for the stability of oscillation. For van-der-pol oscillator we can specifically investigate for it is stability like if epsilon is much greater than 0 then our then our oscillations for the van-der-pol oscillator are very stable. Now, as epsilon goes on decreasing their relative stability of the oscillations goes on decreasing, when epsilon is equal to 0 we can see that the equation is transformed to x double dot plus x equal to 0. So, the oscillator no longer remains a non-linear it turns into a linear oscillator.

And for the third case where epsilon is less than 0 we will have unstable oscillations. So, we can see that the van-der-pol oscillator will have stable oscillation only is epsilon is greater than 0. So, we can also conclude that for epsilon greater than 0 f of x and g of x will satisfy the Lienard's condition given for the stability.

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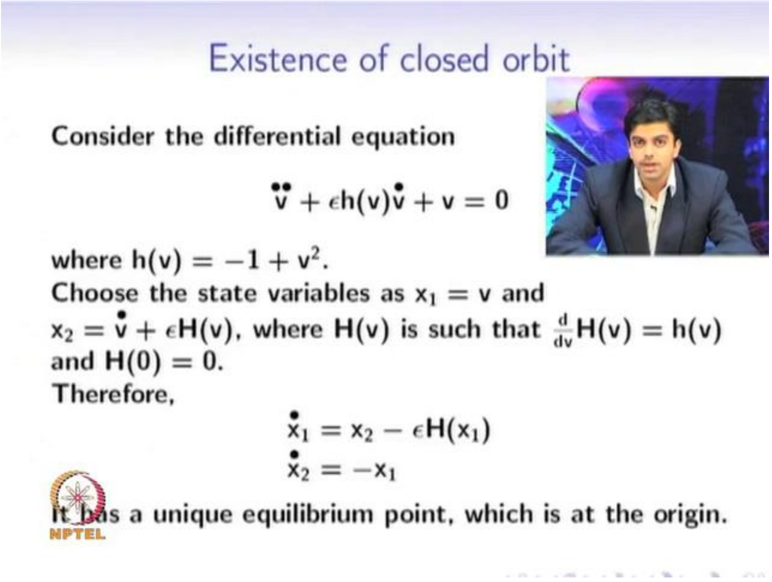

Next, let us look into the existence of a closed orbit for the van-der-pol oscillator. So, we consider the same equation where v double dot plus epsilon h v, h of v into v dot plus v is equal to 0, where v can be a voltage across the element in the given circuit. Now, h of v here is equal to minus 1 plus v square, now for analyzing the behavior let us choose state variables as x 1 equal to v and x 2 equal to v dot plus epsilon into capital H of v. Now, here we will define capital H of v as a d by d v of capital H of v is equal small v and capital H of v at v equal to 0 is equal to 0.

Therefore, if we differentiate the equations of x 1 and x 2, we will get the following state equations where x 1 dot is equal to x 2 minus epsilon x of h 1 and x 2 dot equal to minus x 1. So, we can see here if we put a x 1 equal to 0 and x 2 equal to 0 there’s a unique equilibrium point. Since h capital H is equal to 0 only at v equal to 0, it is the only equilibrium point. So, origin is the only equilibrium point for this van-der-pol oscillator.

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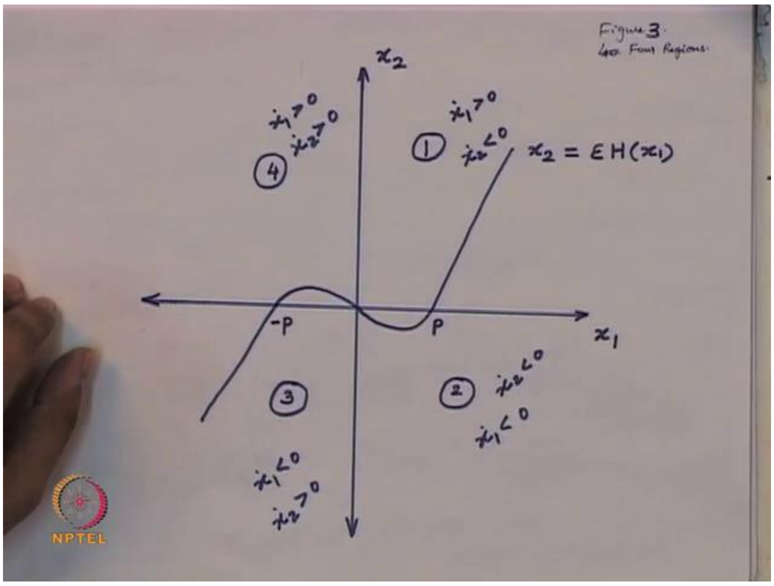

Next we will consider one figure let us look at the state plane where x 1 and x 2 are the axis. So, we will divide this plane into four regions with the help of the curves given as follows x 1 dot is equal to x 2 minus epsilon h of capital H of x 1. So, it is this curve where x 2 is equal to epsilon into capital H of x 1 and the second curve is x 2 dot equal to minus x 1 equal to 0, which is the x 2 axis so we will next look into how this curve divides the plane into four regions, we can say that each curve divides or separates x 1 x i dot greater than 0 from x i dot less than 0.

Like for example, let us consider the first curve where x 2 is equal to epsilon capital H of x 1 so above this curve in this region, we can have x 1 dot greater than 0. Since x 2 is greater than epsilon capital H of x 1, so x 1 dot is greater than 0 in this whole region and below this region, we have x 1 dot which is less than 0. Now, let us consider the second curve which is the x 2 axis now to the right side of the x 2 axis, we have x 2 dot less than 0. Since x 2 dot is equal to minus x 1, so to the right side of x 2 axis x 1 is positive, so x 2 dot will be negative. So, x 2 dot is less than 0 to the right half of the plane and to the left half of the plane x 2 dot is positive.

Now, we will consider the four regions, now in the first region we have x 1 dot greater than 0 and x 2 dot less than 0. In the second region we have x 1 dot less than 0 and x 2 dot less than 0. In the third region we have x 1 dot less than 0 and x 2 dot greater than 0. And in the fourth region is x 1 dot greater than 0 and x 2 dot greater than 0. So, as we are seeing here the 2 curves are dividing the state plane in the four regions. Now, we will see how these four regions will be helpful to us in finding the existence of the periodic orbit.

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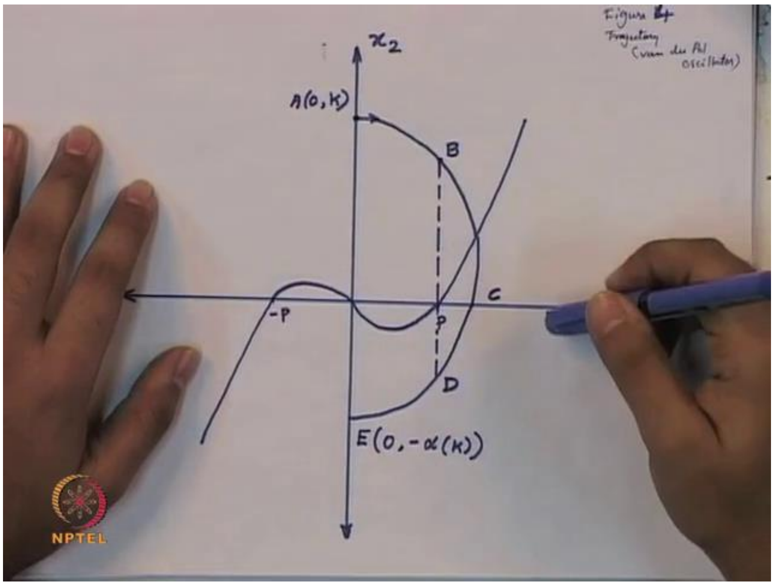

So, we will consider another figure so let us take the initial condition on the x 2 axis so that x 1 is equal to 0 and x 2 is equal to minus k, these are the initial conditions we have taken. And here k is greater than 0 let us name the point as A if we draw the trajectory according to the directions given, so here is the trajectory which will be intersecting x 1 axis to at point c and x 2 axis again at point E. Let us say that the coordinates for the E point is 0 and minus alpha of k where alpha is positive. So, alpha is greater than 0 the reason I have taken alpha as a function of k because alpha depends on k.

Now, if we change the initial condition or if we change k the value of k then will then we will get a different alpha. So, the value of alpha is actually dependent on k so the intersection at point E or at the x 2 axis again, so it is dependent on the value of k. So, we can say that alpha is a function of k, now if we take k as large enough then we can prove that alpha k is less than k, so that it is same as saying if we start out with the initial condition and around the orbit if we consider 180 degrees curvature. So, trajectory will come closer to the periodic orbit since the k is greater than alpha of k.

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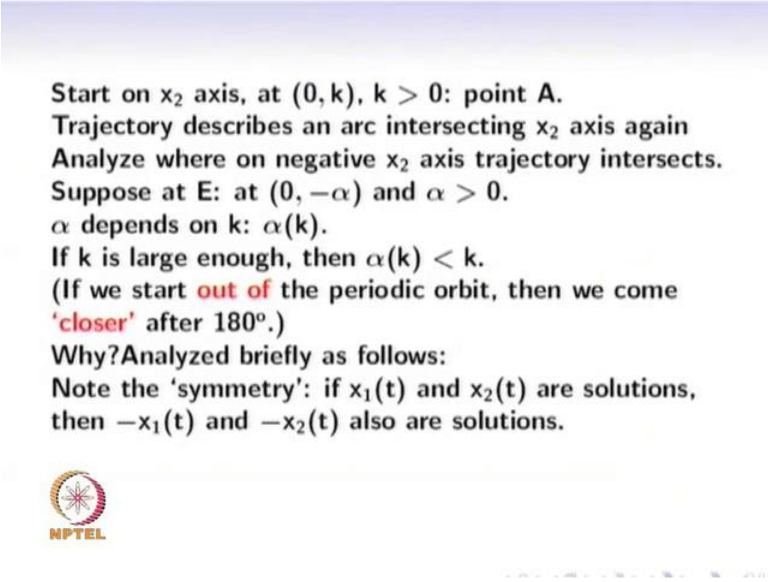

Now, let us look why it is like that in the first slide we will we consider that x 1 dot and x 2 dot the state equations in that we can see that they are the function of capital H of x 1 and x. So, both capital H of x 1 and x are odd functions so we can say that if x 1 and x 2 are the solutions to the van-der-pol oscillation, then minus of x 1 and minus of x 1 x 2 t are also the solution. Now, as stated before the reason for this is h is an odd function. Now, let us consider that if the trajectory completes 360 degrees then if alpha k is less than k the trajectory will become more closer to the periodic orbit, let us consider the function v of x equal to x 1 square plus x 2 square upon 2.

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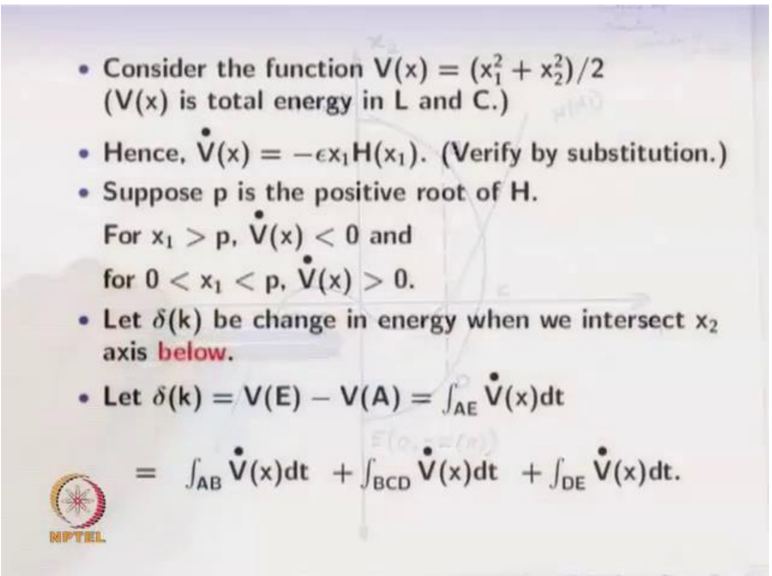

Now, V of x is equivalent to the total energy in a L C circuit in a LC type circuit. So, if we differentiate V dot V of x with respect to time then we will get is equal to minus epsilon into x 1 into capital H of x 1. We can verify this by substituting the values of x 1 dot and x 2 dot from the state equations. Now, suppose we consider the curvature of H so this is the curvature of h of x 1. So, it has a positive root p when x is greater than 0 and a negative root x 1 is less than 0. So, when x 1 is greater than p, v dot x is less than 0.

Since, for x 1 greater than p h of x 1 is positive and x 1 is also positive so from the expression we can see that v dot x is negative, if we consider the region for x 1 to be between 0 and p then we will get that v dot x is 0 greater than 0. So, we can conclude that the v dot x is changing along the x 1 axis. Now, let us take delta k as a change in energy when we intersect the curvature to the x 2 axis. So, delta k is the change of energy from point a to point e, so we can define delta k as v e minus v a, where v e is the energy at e point e and v a is the energy at point a. So, it is equal to the integral along the curvature a e of v dot x with respect to time.

We can divide the following curve into three curves a, b where b is the point just above the x 1 equal to p so at point b along the curvature x 1 is equal to p, points c is the intersection of the trajectory with x 1 axis, point d is another point where x 1 is equal to p on the curvature and d e is the remaining curvature. So, the whole curvature from a to e is divided into three curves a b b c d and d e so delta k can be represent in the as a change of energy from a to b b to d and d to e.

Let us represent it in the form of delta 1 k delta 2 k and delta 3 k, delta 1 k is the change of energy from a to b, delta 2 k is the change of energy from b to d and delta 3 k is the change of energy from d to e along the curve. Now, let us take the case where delta 1 k is greater than 0, we can say that delta 1 k is greater than 0 because x 1 here is greater than 0 and h of x 1 as we can see is negative. So, delta 1 k is the greater than 0 so we can say that the change of energy from a to b is positive.

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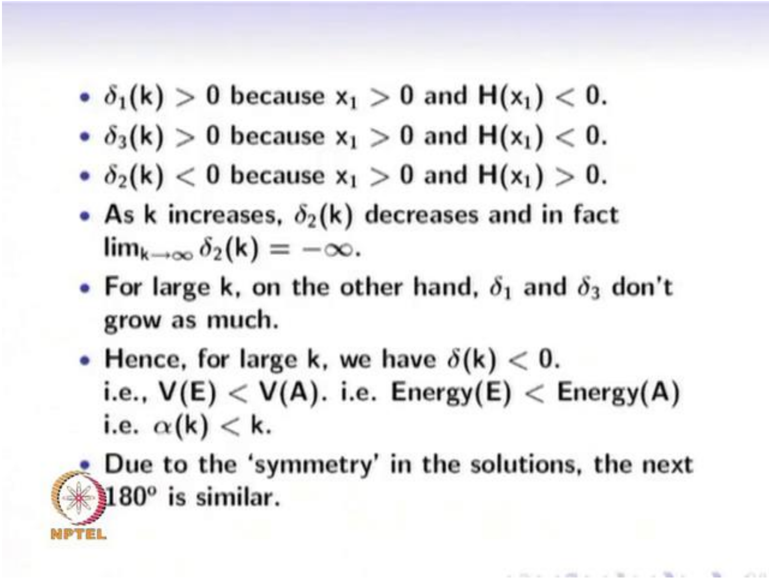

In the second case we can say that delta three k which is the change of energy along the curve from d to e is positive, it is similar to delta 1 k. In this case also x 1 is greater than 0 and h of x 1 is less than 0 h of x 1 is less than 0 because x 1 is restricted to the point p. As we can see b and d are the points along the x x 1 equal to p x, so we will have delta 1 k greater than 0 and delta 3 k always greater than 0. Now, let us consider the change of energy along the curve b c d which is denoted by delta 2 k, here we can say the change of energy along the b c d is less than 0. And we can give the reason because x 1 is greater than 0 in along the curve and h of x 1 as we can see x 1 is greater than p so capital H of x 1 will be always greater than 0. So, delta 2 k along the curve b c d will be less than 0.

Now, as we see as I increase the initial conditions as I increase the k the curvature will expand and delta 2 k that is the change of energy along b c d will go on decreasing. And we can also say that as limit x tends to minus infinity the change of energy along b c d that is delta 2 k will go to minus infinity. In the other context of we look at the delta 1 k and delta 2 k expressions that is the change of energy along a b and d e, then for large k they will not grow as much as delta 2 k, so as k will go on increasing the delta 2 k will grow much faster than delta 1 k and delta 3 k.

So, since delta 2 k is negative so the net summation of delta 1 k delta 2 k and delta 3 k will be negative. Hence for a large value of k we will have delta k less than 0 since delta k is less than 0, we can say that along the curvature from a to e the energy at point e is less than energy at point a. That is the energy along the curve is decreasing since the energy is decreasing along the curve, we can say that alpha k is less than k so the trajectory will move closer to the periodic orbit or it will approach the periodic orbit. Now, due to the symmetry in the solutions as we stated before if x 1 t is the solution and x 2 t is the solution then minus x 1 t and minus x 2 t are also solutions. So, the next 180 degrees will be similar to that.

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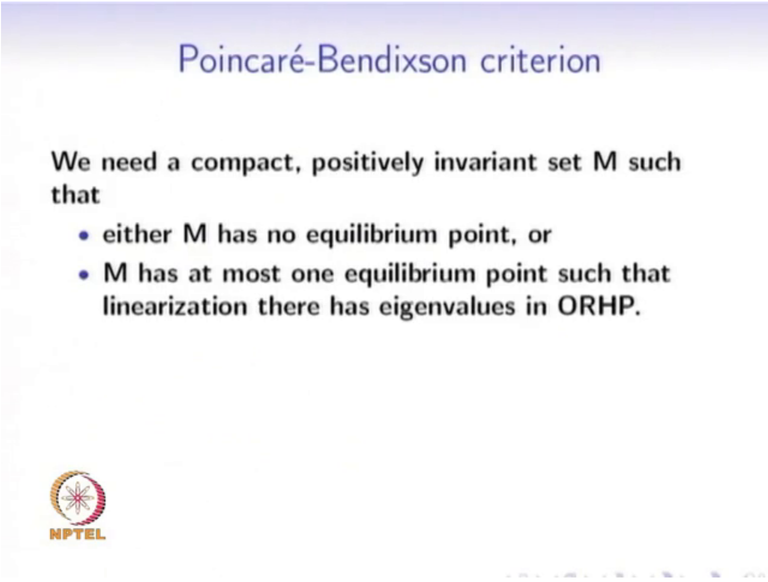

Let us consider Poincare Bendixson criteria again for applying Poincare Bendixson criteria, we need compact positively invariant set m such that either m has a no equilibrium point or it can have at most 1 equilibrium point such that after linearization, if we consider the Eigen values then will be in a open right half plane. So, the equilibrium point if it is there in the m region it will be unstable. So, here we can choose a region we consider the curve a, b, c, d, e before now let us consider another curve, where the initial condition is 0 minus k so the curvature will be f g h i and back to a.

So, if we consider this whole region if we connect close this region, then we can say that this region is a positively invariant set. Now, this curve is also contained in the van-der-pol oscillation or it is also solution with due to symmetry as stated before that if a portion x 1 t and x 2 t is in the solution, then a portion minus x 1 t and minus x t will also in the solution. So, if we consider the whole region it will be a positively invariant region, that is for an initial condition in this region the trajectory will approach a main in the region and it will approach periodic.

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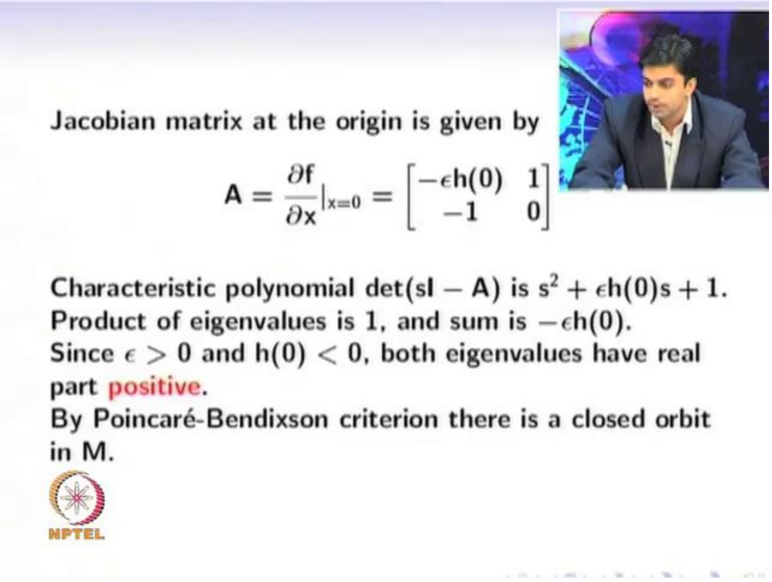

So, next we will consider a Jacobean matrix which will be formed after linearizing the system, so that A is the Jacobean matrix which is defined a dou f by dou x at x equal to 0. Since, we have the equilibrium point at x equal to 0 so we will get the matrix as shown in the slide, which is a minus epsilon into small h of 0 1 minus, minus 1 and 0. So, the characteristic polynomial of A will be given as s square plus epsilon into h of 0 s plus 1. Now, from this equation we can say that the product of Eigen values is 1 and sum is equal to minus epsilon h of 0 because the roots of this characteristic equation are then Eigen values of the system.

Now, since we define before epsilon is greater than 0 and h of 0 as negative then both of the Eigen values we will have as having positive real parts. So, we can say since the Eigen values are having positive real parts the equilibrium point is unstable. Now, the conditions we concluded for this system or van-der-pol oscillator are that we have a invariant set m, then we have a a equilibrium point inside that which unstable. So, we can apply the Poincare Bendixson criteria here and by Poincare Bendixson criteria we can say that there is a closed orbit in m.

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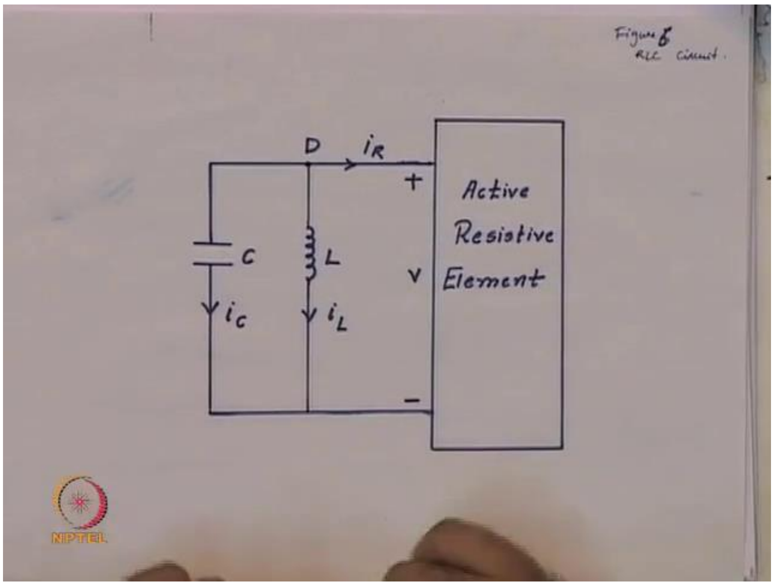

Now, for the example of the van-der-pol oscillation we will consider a RLC circuit, where R is the active resistive element. Now, let us consider this figure in this figure this is an RLC parallel circuit this is a capacitance c which is greater than 0 L inductance which is greater than 0 and we have connected in parallel active resistance element, this we is having v i characteristic as i equal to capital H of v, where we had defined capital H of v before.

Now, so we can say that the capital H of v satisfies the following conditions capital H of 0 is equal to 0, then capital H dash that is derivative of capital H with respect to v satisfies capital H dash of 0 is less than 0. And capital H of v tends to infinity as v tends to infinity now we have a point saying that capital H of v is similar to capital H of f in Lienard's equation. We can see that the conditions that are satisfied by capital H of v capital F of x are same. So, we can say that H and F are both odd functions.

Now, let us go back to the circuit again here we consider a point d and we will apply k cl here i c is the current to the capacitor i l is the current to the inductor, i r is the current for the active resistance element. So, at d we will apply k c l, so that we will get the summation of all the three currents is equal to 0.

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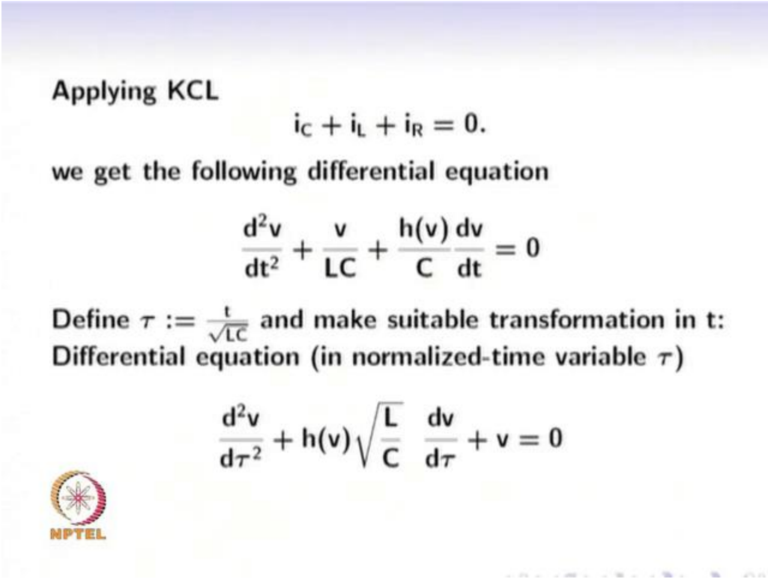

Then in the differential form, we will get the following expression where we have h of v coming into the picture. Now, we will use the transformation we will define an element toe equal to t upon root L C and substitute in the other following equation. So, we will get a normalized time variable equation, it is the second order differential equation and we can see that it is in the form of van-der-pol oscillator, where we have epsilon equal to root of L C, here we can see that root of l c is epsilon is greater than 0.

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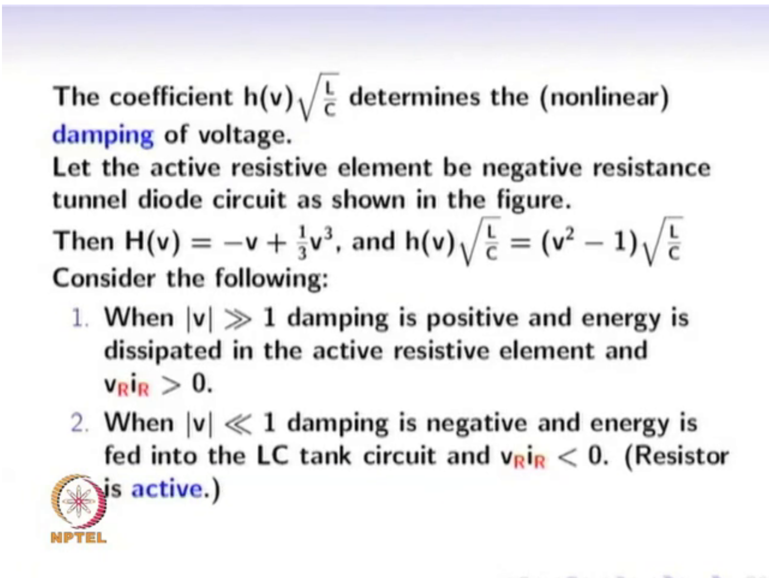

Now, next we will consider the coefficient of v dot which determines the damping of the system. So, the coefficient of v dot is h of v into root L C and this determines the damping of the voltage the damping here is the non-linear damping. We can practically implement active resistance element in the form of tunnel diodes, so it will act as a negative resistance for some value of v and a positive resistance for other values of p. Now, we will define capital H of v as minus v plus v cube by 3 and small h of v into root l c is equal to v square minus 1 into root L C.

So, let us analyze this system consider v much greater than 1, so we have much greater than 1 the damping coefficient is positive so we can say damping is positive. And since the damping is positive energies dissipated in the active resistance element the transaction of energy is from L C circuit to active resistive element, and it is getting dissipated in the resistor element. The resistive element in this case will be positive the value of the resistor will be positive and we can say that since the energy's getting dissipated v r into i r is strictly greater than 0.

Now, in the second case we will consider v value of v or voltage is much less than 1, so in that case the damping will be negative as a damping constant coefficient v is negative and the energy will be fed into the L C tank circuit. So, the transaction energy is from active resistance element to the L C circuit, so we can say that the for the active resistance element we have v r into i r less than 0 and here that since the damping is negative and the resistance is actually an active element, this is acting as an active element or we can also say the resistance is negative in this case.

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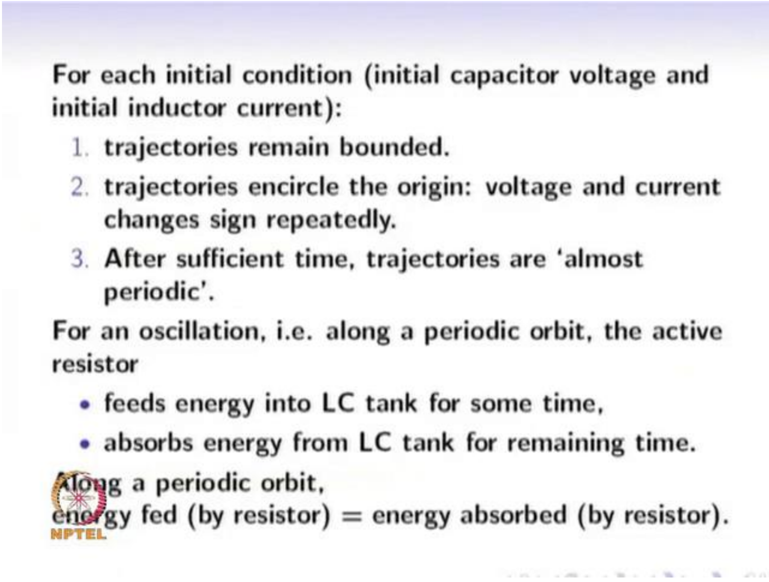

Let us look at the behavior of the system, for given an initial condition now that initial condition may be a voltage across a capacitor, a initial voltage or it may be a current of the inductor. Now, in this case the trajectories will remain bounded, now have can we say that as we look before for v greater than 1 and v less than 1, we have damping positive and negative. So, we can say that trajectories are remaining bounded when damping is positive, the trajectories are approaching towards the orbit and further when damping is negative the trajectories in that case also the trajectories are approaching the orbit. So, we can say that a trajectories are remaining bounded.

Now, the second point we can say the trajectories encircle the origin, now for this case we will consider when for a initial given condition as said before the initial voltage the damping is positive and negative depending on v is remaining v greater than 1 region or v less than 1 region. So, the value of v and i repeatedly changing the sign so in that case we can say that since v and i repeatedly changing the sign. So, the trajectories are actually encircling the origin.

Now, we have concluded the 2 points that the trajectories remain bounded and that they are also encircling the origin. Next we will consider after sufficient time that is for a given initial condition, if we after a sufficient time we can say that the trajectories are almost periodic. Since for a given initial condition the trajectories are approaching the periodic orbit so after much sufficient time we can say that they are almost periodic, they cannot be periodic the 2 trajectories cannot intersect. So, they will be almost periodic.

Next, we will consider oscillation along the periodic orbit when we have a oscillation along a periodic orbit, the active resistor feeds the energy into L C circuit for some time that is when active resistor is negative, the damping is negative it feeds the energy into L C circuit. And it also absorbs the energy in the L C from the L C circuit when it is positive in that case damping is positive. So, we can say that during the periodic orbit active resistance element is feeding energy and also absorbing energy for some time.

Now, let us see how can we say that the periodic orbit is stable or not. Now, suppose along the periodic orbit energy fed by the resistance is equal to the energy absorbed by the resistance, then we can say that periodic orbit is stable. We can day this because when the energy is equal fed is equal to the energy absorbed net energy expend is equal to 0. So, the periodic orbit in that case will be a stable one. So, we will have a stable oscillation for van-der-pol oscillator.

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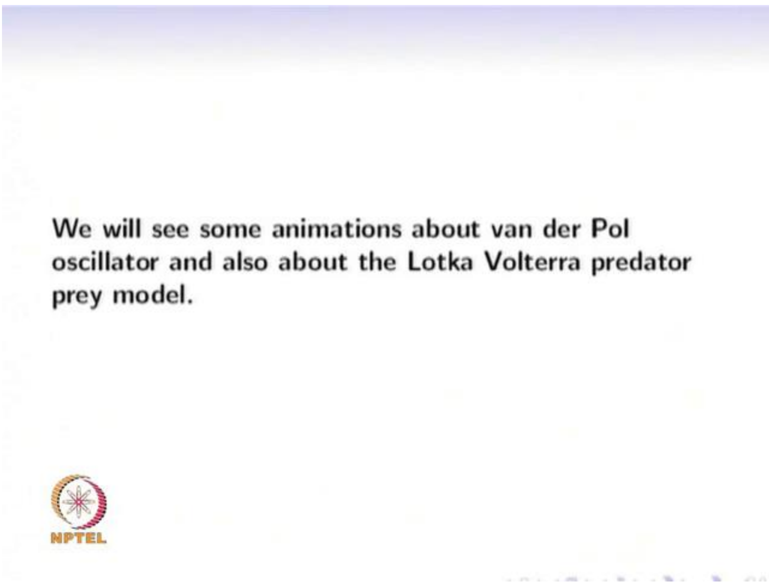

Further we will see some animations about van-der-pol oscillator and also about the Lotka Volterra predator prey model.

Thank you.