Nonlinear Dynamical Systems
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Lecture - 31
Describing Functions:
- Sufficient conditions for Existence of Periodic Orbits
- Non Existence of Periodic Orbits

So, welcome everyone to this next lecture, today we will spend some time on finishing something small about describing function namely about sufficient conditions for existence of periodic orbits and sufficient conditions for nonexistence of periodic orbits. So, after this part we are going to do some problems on circle criteria, Popov criteria, the so called small gain theorem, these are problems we will do immediately after we finish this.

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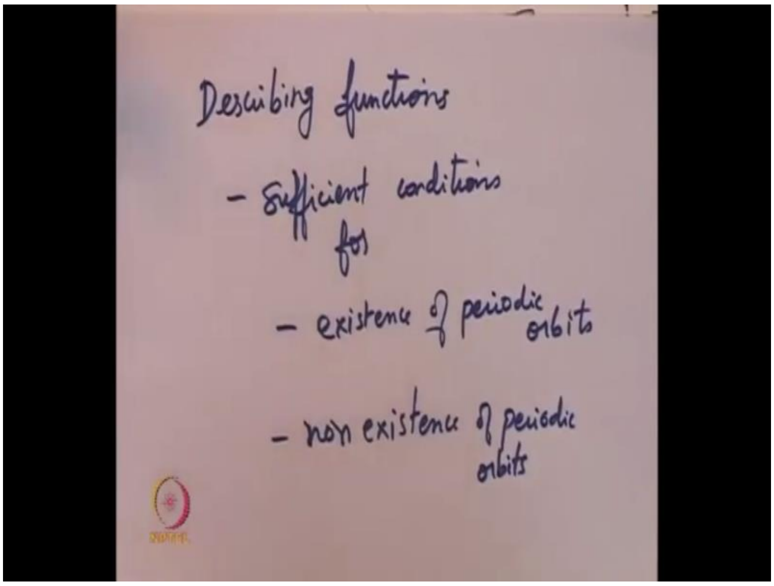

For existence for non existence one thing, one should note is in non linear dynamical systems in non linear control. It is very hard to have necessary and sufficient conditions, even the Lipchitz condition that we saw for existence of solutions with differential equation that was only a sufficient condition for existence. It is not necessary, there can exist solutions to differential equation even the even though the Lipchitz condition is not satisfied. ((Refer Time: 01:45)) Lipchitz condition is only a sufficient condition there. So, here also indeed existence of periodic orbits and non existence of periodic orbits seem like opposite, but we have sufficient conditions for existence and sufficient conditions for non existence.

So, in that sense we have only seen one we will have seen only one part of it why because necessary and sufficient conditions for existence of periodic orbits is indeed a very difficult thing to have. One requires to do to the best of our understanding, still requires research is required to be done to find necessary and sufficient conditions. Often, these are so complicated that at some point it becomes too artificial a little to artificial to pursue and have as a result.

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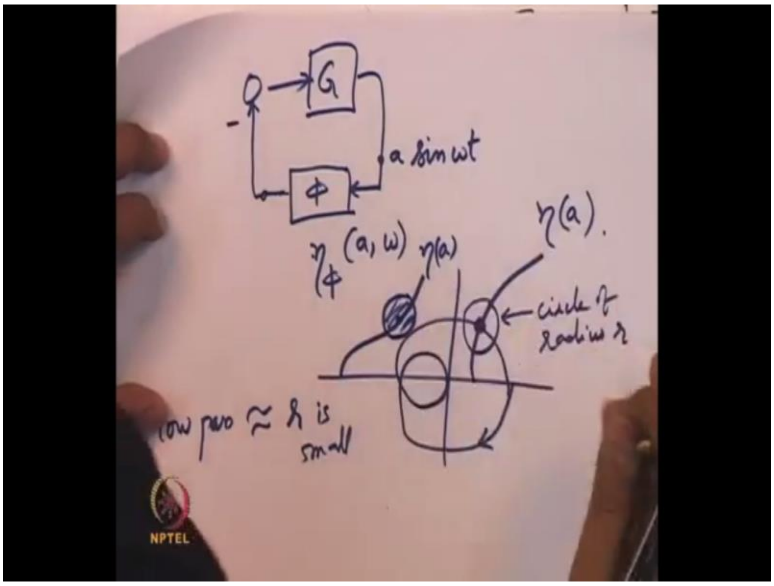

So, recall that we had recall that we had said that when you have G as a transfer function a linear system linear time infinite system and a nonlinearity phi. You consider that describing function of this nonlinearity phi function of a and omega, where a and omega are the amplitude and frequency of this sinusoidal input at the entrance at the entry of the nonlinearity. So, we have already assume we have already noted that the higher harmonics of phi when if phi is time invariant nonlinearity. Then, what comes and if phi is boundary input boundary output stable, then what comes out here is also a periodic signal in the same period, but it need not be a pure sinusoid, it might have higher harmonics.

Also, it might have co sinusoids also, but higher harmonics is being ignored when we define the describing function why because the describing function was defined using only the first harmonic of the Fourier series. The first Fourier series coefficient and this ignoring of higher harmonics was is justified if G is a low pass filter, so let suppose the Nyquist plot of G looks like this. Suppose, then describing function happened to be such that this is a plot eta for convenience, we will assume that this is a function only a. Otherwise, we do not have this convenience of plotting them separate and just looking at the point of intersection.

If eta dependent on a and omega, then this point of intersection had to be corresponding to the are frequency for the Nyquist plot and also for the dishaving function eta a comma omega. Now, notice that the higher harmonics that have been ignored that might, if they were not ignored. Then, the intersection is not here, but inside this ball inside this circle this is some circle of radius r where low pass means r is small, but one can quantify exactly how small r s and one has to look at such a ball.

Now, what is the sufficient condition for non existence a sufficient condition for nonexistence would be that this dishaving function this is eta of a this happen to be far such that such a ball even if you take such a circle. Even if you take such a ball and still there is no intersection with the Nyquist plot of G. For example, here you see that this ball is just touching, so how much big radius ball we should along the Nyquist along the dishaving function curve that depends on how much low pass G is why because the dishaving functions indicates only the first harmonic. Higher harmonics have to be ignored or not that depends on the extent to which G is low pass, so let me just recapitulate this again.

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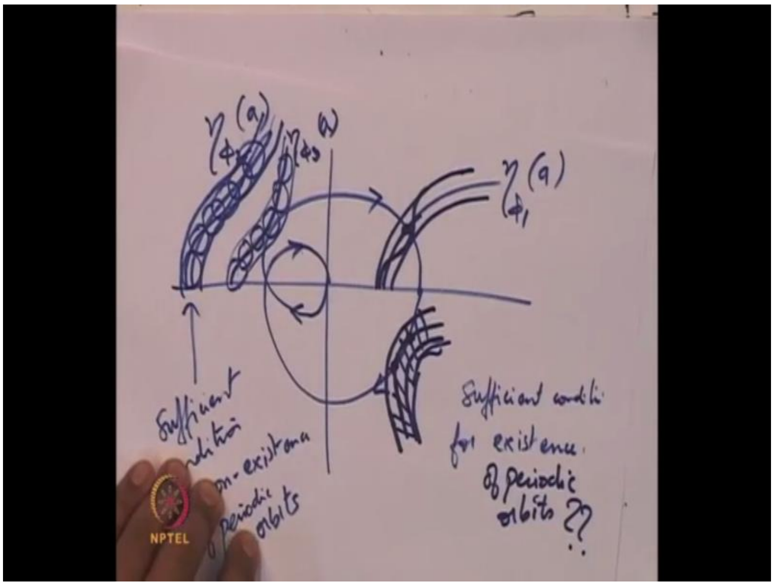

So, suppose this is the Nyquist plot of G the exact transfer function is not being discussed here, suppose the dishaving function was like this that is one eta 5, one a another one was like this and 5, 2 a here this Nyquist. This dis having function curve has a function of a and the Nyquist plot do not intersect while in this case it intersects. So, just it intersects cannot be feel that, maybe there exist periodic solution I said that this was only approximation method, but does that exist one close by at least yes or no question. We want to have for sure the value we might have some inaccuracy that is acceptable here.

On the other hand, we do not have intersection, notice that you may not have intersection even in this case, there is another let us say eta phi 3 a, but then here the ignoring of higher harmonics was playing a role. If you had not ignored maybe it did intersect, so why it is resemble to ask if a ball around this intersects or not here such a ball here such a band. So, this is instead of a ball, we can move this ball around along this particular curve and see if that band intersects. If the band itself does not intersects this, then one might be safe one might safely say that periodic solutions will not exist. One can go ahead and claim that periodic solutions do not exist if not jut the not just non intersection of this dishaving function curve.

Also, non intersection of this band in which the thickness of the band depends on how much low pass the transfer function G is the next harmonic how much the gain has decreased, how much the gain of G, how much in the Bode plot. The magnitude plot magnitude curve has decreased compared to the frequency corresponding to the first harmonic. So, this is one, so this gives us sufficient condition for non existence, now we will say that what will happen if the band intersects, even if the band intersects, even if the curve intersects, it is still possible that when you include that band that time you have non intersecting.

The actual curve the actual curve might be some curve inside this band that is why we have to check that the entre band is for intersecting and hence no curve inside this will also not intersect the Nyquist plot. So, the next question is we are going to ask for sufficient condition for existence if the opposite of this are sufficient condition for existence, what is the question? We have obtained a sufficient condition for non existence namely the band around the dishaving function curve as a function of a. You get the band and the band also does not the Nyquist plot that nonintersecting of the band has been proved in Vidyasagars book as a sufficient condition for nonexistence of periodic orbits.

What is the opposite of this statement that the band intersects, if the band intersects? For example, in this case it is not necessary that the curve intersect if the band intersects, it might still happen that the curve actual curve also does not intersect just like this curve that we have shown here. Actually, there were no periodic orbits, so intersection of the band with the Nyquist plot is clearly not a sufficient condition for existence. If periodic orbits the opposite of this condition that the band intersects with the Nyquist plot that band intersection with the Nyquist plot is clearly not a sufficient condition for existence of periodic orbits.

It turns out that it will be a necessary condition for existence of periodic orbits, but it is still not clear because look at this example here, the actual curve does not intersect, but the band intersects the band is like this. Just because of band intersects does not mean that the first harmonic curve intersects, it does not mean that the actual curve intersects either. So, look at this example where the actual curve intersects, this also is not a sufficient condition for existence id periodic orbits because we know we are not sure that this eta 5, 1 a is actual curve.

This is just a first harmonic actual curve might be something else, we need some condition that the actual curve intersection Nyquist plot and intersection of the actual curve which is all the harmonics considered together. Loosely speaking, that is a curve that we want to guarantee intersection, then we will have a sufficient condition for existence of periodic orbits. So, I will construct an example, where the first harmonic intersects at two points, but perhaps the actual curve is like this actual curve does not intersect and the actual curve is of course inside this band. So, inside this band, if it happens that the first harmonic curve intersects the Nyquist plot, but actual one does not.

That can happen if this band is sort of just touching or touching, then intersecting at two at two points. If it is touching, then clearly anything inside will intersect it at all, but if it is intersecting at two points that also is a situation where the actual curve perhaps may not intersect at all. The first harmonic curve might intersect at two points very close to each other because of which the actual curve actually can escape, it need not this is only a sufficient condition that we are aiming for...

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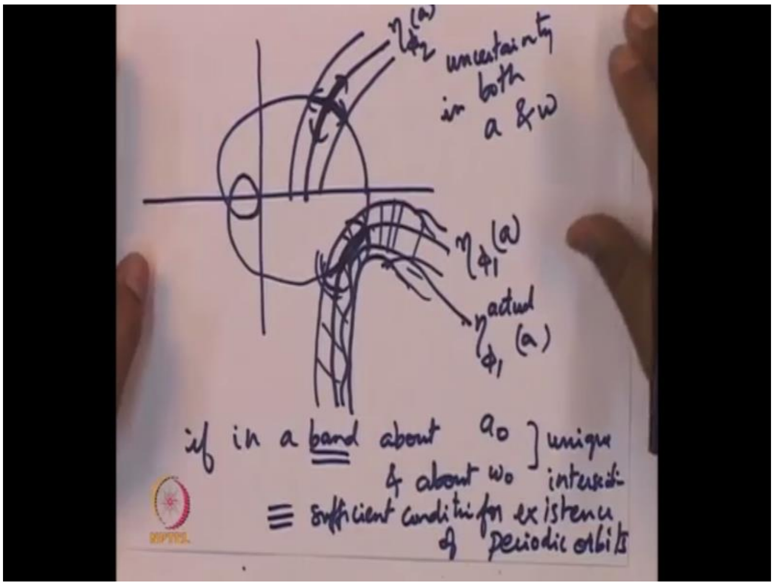

So, let us get a feeling for what is the sufficient condition for existence of a periodic orbit. So, here is an example this is eta phi one a, this is a actual curve that escapes eta phi 1 actual a, that is because there of course it has to be inside his bag. It is inside this band this band around, around this sorry this curve is should not be so far from this.

So, actual curve has escaped without intersecting the Nyquist plot, but we will like to say some there is some property of the first harmonic that is allowing is to happen namely inside that band inside any inside any small neighbourhood there are two intersections. Suppose, dishaving function was such that inside that band, so this band is actually both in the omega, so uncertainty in both a and omega, so you look at you look at this point of intersection of the dishaving function. You look at some band around the amplitude parameter, some uncertainty along the omega parameter, this the Nyquist plot is where omega varies and on the Nyquist plot along the dishaving function where that curve is where the amplitude varies and inside this.

If you consider just these two, there is only one point of intersection, in other words there is there is a certain function which is one to one that one to one is being noticed by the fact that inside the band of frequencies. Inside this band of amplitude, there is a unique intersection that is precisely what is not happening here along the omega, along the amplitude band. There were two intersections and this is the frequency band along the frequency band. Also, there are two intersections because of two intersections happening both within the same amplitude band and within the same omega band this situation that the actual curve escapes without intersection is possible.

So, if f in a band about a naught and about omega naught unique intersection a naught and omega naught of course are the point of intersection of the describing function as a function of a. The Nyquist plot as a function of omega, but then there is an uncertainty there is a band a naught that we should be considering because of ignoring the higher harmonics the same reasons also causes us to look for a band about omega naught and inside this band. Also, if this a unique point of intersection, then we will say that this is a sufficient condition for existence of what of periodic orbits. So, this has been proved in Vidyasagars book on non linear systems that it has been of course proved far more rigorously.

The purpose of discussing this was to give a feeling of what type of analysis arguments are used in proving such statement. So, this is also again only a sufficient condition just because this to say that this is not necessary means. I would like to point out that even if there is a situation where it is not unique intersection still these periodic orbits could exist.

In that sense, this condition is only a sufficient condition it is not a necessary condition for existence of periodic orbits is this has been described in detail in Vidyasagars book and we will not pursue this discussion further in this course. So, this completes the topic of dishaving function, we have also seen some problems where the di having function we have also seen some problems where dishaving function predicts the existence of certain periodic orbits, how to find the amplitude and the frequency?

We have also related that with how Popov criteria and circle criteria also give a very similar band for non existence of periodic orbits, why because they give a band for absolute stability and absolute stability automatically rules out periodic orbits. It rules out instability also, so we are going to now spend some time to solve some problems involving Popov criteria circle criteria and also pole gain theorem, which we only barely touched in our previous lectures.

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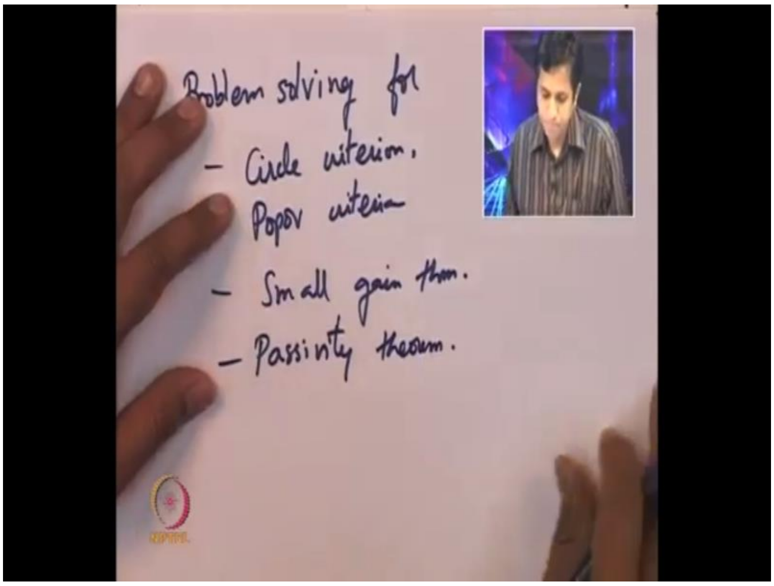

Problem solving for Circle criteria Popov criteria and the small gain theorem, so while solving these problems we will also very quickly recapitulate what this theorems are, what this very celebrated results are. So, these development of theory has happen over the last 50 to 60 years and is fundamental in the in the theory of non linear dynamical systems.

In fact, the listeners are recommended strongly recommended to have a look at the selected best twenty five papers that appeared in the last century on nonlinear dynamical system. This is in a book edited by Agrawal in which there are many people that speak about these results and also further development after this the Kalman, Kowich, Popov Lima. Popov criteria, the passivity theorem are undoubtedly the strongest best results that had happened in control theory in non linear dynamical systems.

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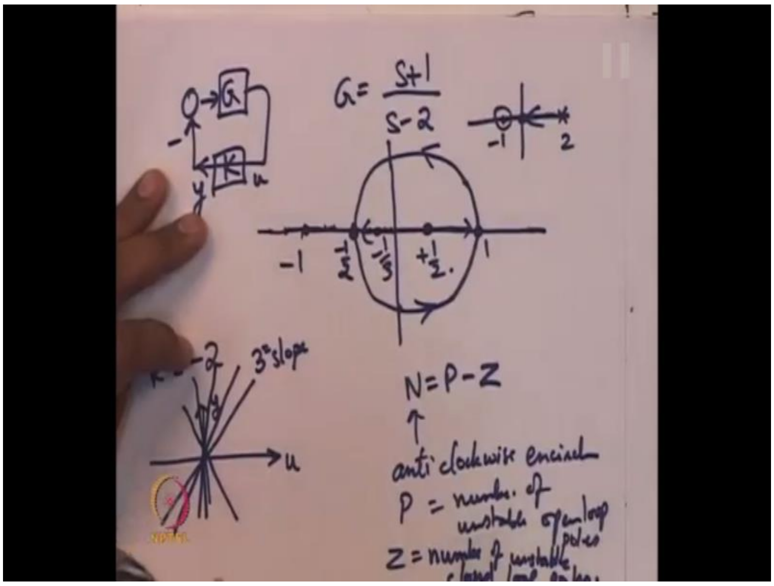

So, let us take an example of a transfer function s plus 1 over s minus 2, so this is Nyquist plot is this circle as I said for first order systems, the Nyquist plot is always a circle. It is a orientation that we have to spend more effort, why because the real axis intersections are also very easy in is for astride infinity and the other intersection is for s equal to 0, so minus 1 by 2. So, as I said one is to have a good understanding of linear system before one comes to non-linear systems. So, one should be familiar with bode plot Nyquist plot root locus for us to quickly benefit by using circle criteria this is where the pole is and this where the 0 is.

So, we know that for large gain the system close loop will become stable why because the this is how the root locus is after some value it will become stable. Hence, when we blow this Nyquist plot large enough then this particular, now we have unstable pole, so p is equal to 1 and it is not encircling the Nyquist system the point minus 1.

So, z will also be equal to 1, so indeed there is one unstable pole for the close loop for the close loop suppose this is G equal to this, so there is one unstable pole in the close loop also for unity gain, but when we take a gain that is larger than 1. That is nothing but making this circle larger and larger and by making this circle larger than by multiplying this by 2. We know that this point come here, so we know that for gain larger than 2 this will encircle the point minus 1 and then there is no other change that can happen for larger gain. So, for gain larger than 2, we know that this will have the close loop would have come into the left half complex plain.

So, we know that z will be equal to 0 for gain larger than 2 so n has to be one n has to be plus 1 for gain larger than 2. So, if it has to be plus one than this orientation has to already be anti clockwise that is a conclusion that we drew by using the Nyquist criteria where this is the number of anti clockwise encircleness p s number of unstable open loop poles. Open loop because we have nothing else in the feedback in the loop, we have only transfer function G. Hence, it is to call this open loop poles while z is the number of close loop poles number of unstable close loop poles the number of unstable close loop poles z is what we want to find out by using the Nyquist criteria for stability.

Now, we know that these all points are these all points here are the one that are encircled the correct number of points and so are these points also for this point. For example, corresponding to plus 1 by 2 that corresponds to gain of k equal to minus 2, so what the circle criteria says from the Nyquist criteria. We know that these all points in the interior the diameter are the ones that are encircled p number of times. If you want z to be equal to 0, then you should look for those points on the real axis which are encircled exactly p number of times if p is equal to 1 like in our example.

Then, look for all those points on the real axis, which correspond to which happen to get encircled t number of times anticlockwise and that gives us points and from those points, we can translate to gains. So, k is equal to minus 2 will correspond to an input output map u y where this is the k input is u output is y, so k equal to minus 2 is a line with negative slope like this.

Now, this suppose somebody says what about this suppose this is happens to be slope 3 slope 3 slope equal to 3. Then, that three corresponds to a point minus 1 by 3 is the point minus 1 by three inside the circle yes, is it encircled p number of times anticlockwise. What is p is equal to, so it is encircled one number of times anticlockwise, so there is also line that stabilise this, if you put k equal to 3 that will also result in close loop stability. So, the Nyquist criteria very easily gives us a sector correspond to lines that all cause close loop stability. Now, this is what we get by the Nyquist criteria, already the circle criteria says that you take you take correspond to that same diameter you construct a circle. If you have to construct a circle and that circle also should get encircled t number of times that circle might that diameter might have to be smaller, this is easily seen for various other examples.

In this case, you remain the circle as large as possible and it will come out to be same circle. So, for first order systems it will turn out that the sector of linearities is same as the sector of nonlinearities and this is an important conclusion worth noting down check that sector of linearities. So, the sector of linearities will turn out to be a open set and the all the slope values that cause close of stability. The slopes will be the sector of linearity that will be an open set, the boundaries will not get because the boundaries will cause imaginary axis poles of close loop is equal to sector.

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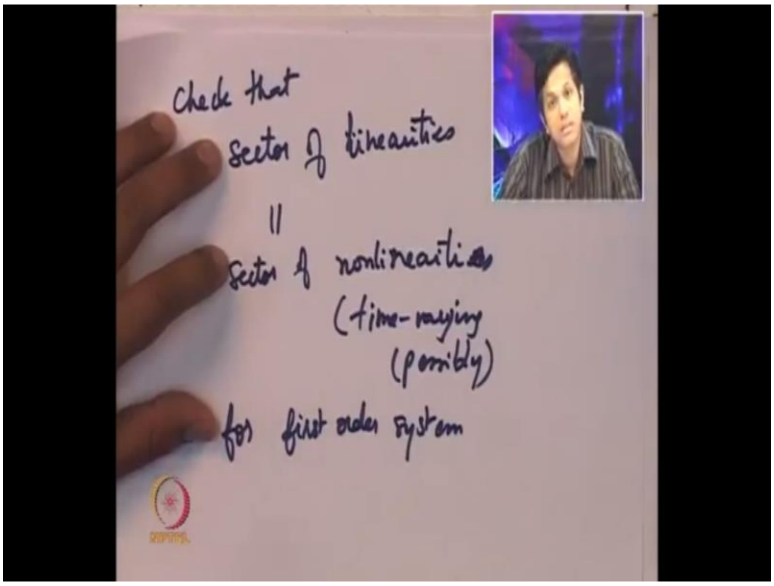

If we are using the Circle criteria, then this is possibly time varying, possibly time varying when will for what kind of systems are these going to be equal for first order, why would this happen because for first order system? The Nyquist plot itself is a circle and because it is a circle the circle criteria the largest circle will also happen to be the same circle. The largest circle will be either the largest sector the circle corresponds to largest sector will either be the largest circle inside this. It will be the smallest circle outside this when we were outside this because of that it can also happen that the open loop is already stable.

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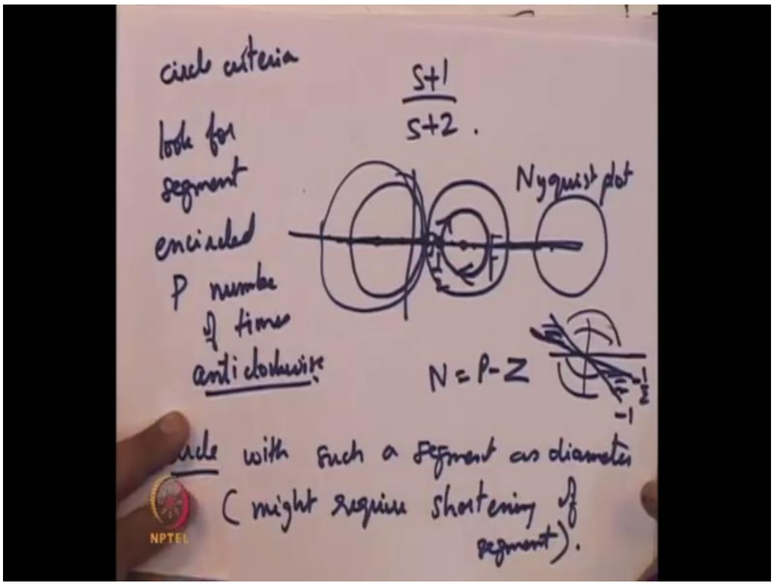

We will see another one such example take s plus 1 over s plus 2 here, we have 1 by 2, one a circle like this for s equal to 0, we have that it starts here. Then, this one is high pass filter first we encounter is 0 and then a pole, so the phase will be would be increasing and hence this is a orientation. So, here when we have n is going to p minus z, there are some points which correspond to negative gain that cause instability to this system. Those points are indeed getting encircled clockwise, why is it expected that it will get encircled clockwise, if for some point z should be equal to plus 1, we only have this minus sign here and p is equal to 0.

So, n would have to be minus 1 n would have to be negative, that is why this point would have to get encircled clockwise to begin with why is p 0 because open loop is already stable. Now, these are all points for which are encircled t number of times what is p is 0, so this all the points on the real axis which are outside this circle correspond to those which are encircled the correct number of times.

So, what a Circle criteria look for look for segment encircled p number of times anticlockwise, what is p about it p is a number of unstable open loop poles. So, if you encircle the a point any point on this segment p number of times that will be a linearity that causes close loop stability. So, that why because it ensures that z is equal to 0, but then of course a sector of nonlinearities will also have linearities. So, that is why this segment had better be encircled, now you not just this segment, it is not enough that this segment is encircle, but one should have take a circle with this segment has the diameter even the circle has to get encircled.

If you have to allow, not just linearities, but time varying possibly time varying nonlinearities, so circle with this segment circle with such a segment as diameter might have to make might require shortening of segment. So, what about in this case see these are this segment has everything outside except this diameter. So, one can in principle actually takes such a circle, so here if t is equal to 0, then all these points should not get encircled. So, one such circle is this segment is not encircled by the Nyquist plot and hence one can take a larger circle. Also, one can take this point left and left and the same thing is allowed here also and in fact one can take the two things together also.

Then, it will be that this Nyquist plot is encircled inside such a circle when would this happen if the open loop is already stable. That sector contains a vertical axis also in such a situation, this would get allowed, so here is an example where the largest sector the largest sector corresponds to everything except this. This is the line which slope minus one this is a line which slope minus 2, everything excluding this please note soon can take anything inside this sector and that will cost close loop stability even if these nonlinearity inside this sector are time varying. So, now we will see how the sector of linearities could be strictly smaller than the sector of nonlinearity if you allow time varying.

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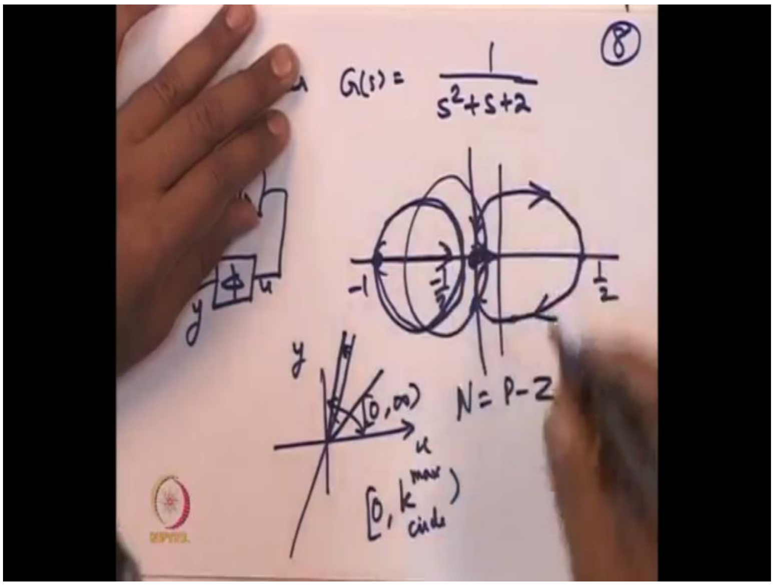

For example, consider I should note that this has to be a second order system, so p had considered this example in a similar context. Yesterday, that is 1 by 2, this is a point minus, so is this open loop is already stable, we always have this, so with phi equal to 1, with phi equal to the unity feedback with the negative sign is a close loop stable. It is stable because the one loop is already stable and the point minus is encircle 0 number of times and hence said this also equal to 0. So, in this equation n is equal to p minus z p is equal to 0 because of open loop is stable n is 0 because we are considering unity feedback and hence we are considering encirclement of the point minus 1.

So, it does not encircle point minus 1, hence n is 0 and that gives us z also equal to 0, so other point what about this point let us say minus 1 by 3 minus 1 by 3 point is also encircled 0 number of times. Hence, the close loop is stable, what about this sector what about this sector of linearities sector of linearities just requires us to check whether this segment is encircled 0 number of times, why 0 because p is equal to 0. The entire segment is not encircled, so n is equal to 0 for that segment also and that gives us that we have close loop stability for a sector of linearity. For any line inside this sector also, one can consider making this left and left until it goes in principle to infinity, so one can take this entire sector 0 to infinity this is u y.

For any line inside this sector, it is some line with positive slope that slope suppose it is k, then you look for the point minus 1 by k and that happens to be a point in this negative real axis and it is it encircled 0 number of times. So, does that mean that we can take a circle the circle cannot be taken for arbitrary, so you see this if the circle comes more and more to the left. Then, at some point it will intersect, this if you take a k that happens to be very large let us say k is equal to 1000 that will correspond to the point minus 1 over thousand, which is far inside this cave. Let us call this a cave and then when you construct a circle, it will intersect the Nyquist plot.

So, the entire circle is not being encircled t number of times the Nyquist plot should not be touching such a circle. So, clearly there is this, the largest sector cannot be too large if you are if you are concern with 0 to infinity. Then, it can go only up to a value k max k max come correspond to the circle criteria and it will have it will be open at some point it will intersect. So, this how to find this k max that that is nothing but you can take such a vertical line, if you take such a vertical line, then you can get from 0 up to this particular k max. That k max will be found as take this point find its real part and one over that will give us k max circle for circle criteria, if one wants to go for higher sector, if one wants to go for larger than 1 has to one cannot start from 0.

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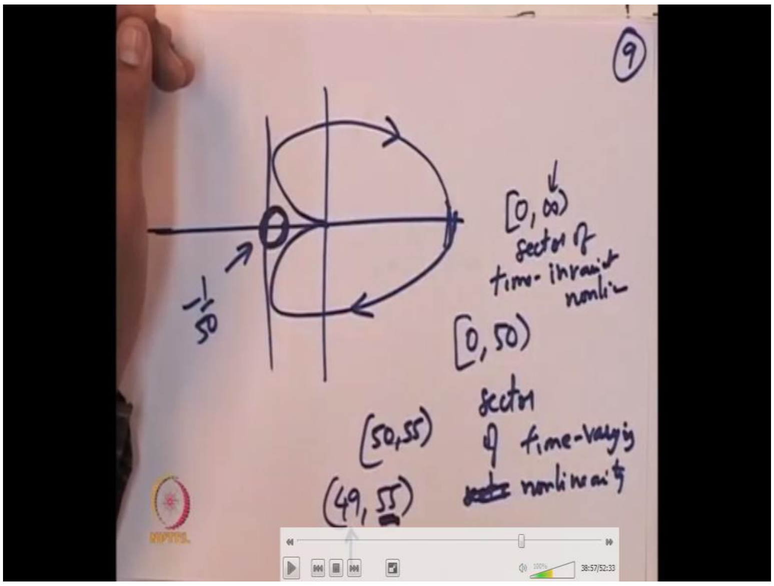

So, let us another figure, it explain this if you use the techniques that we have discussed yesterday for finding the left most point on the Nyquist plot, the part that causes real G of j omega to reach a minima, we plot that. Then, we will get that, and then we will get this and suppose this value is equal to minus 1 by 50. Then, we will get 0 to 50 as a sector of time invariant time varying sector, time varying nonlinearities of course time varying nonlinearities includes time invariant nonlinearities. Also, it includes lines, but time varying means possibly time varying and possibly non linear, suppose you want this, means that 50 to 55 sector is not allowed the minus 1 by 55 might be here.

Such a circle is also clearly encircled 0 number of times, so if you do not want to start from 0, what I mean it say is if you want the sector of nonlinearities to be varying only between 50 and 55. Then, you can that circle is also encircled correct number of times, so the lower value also plays a role in how much high you can go this 50 cannot be made larger because you are starting from 0. The nonlinearities could be time varying and could be varying from 0 up to 50, but if then if you say that the time varying nonlinearities will vary from only let us say 49, then one can go up to 55.

Then, there is depends on the Nyquist plot to what extent high you can and that depends on from where the nonlinearities from what value from what sector it starts from what slope it starts. So, this as an example where the sector of linearities is strictly different, strictly larger than the sector of nonlinearities for using the Circle criteria, there we saw that the lower value also lays the role as to how high you can make this slope. What about the sector of 0 to infinity, we already verified yesterday, sector of time invariant nonlinearities.

All these same example what the sector of nonlinearity, we can what the sector of time invariant nonlinearities, we can have up to 0 from 0 to infinity, the highest value go higher as infinity because the Popov plot. Using the Popov plot, we verified that a line with positive slope can be taken to intersect very close to 0 at minus 0.0001 also and still we can have that the line is to the left of the Popov plot, so we will now see an example of where the small gain theorem can be used.

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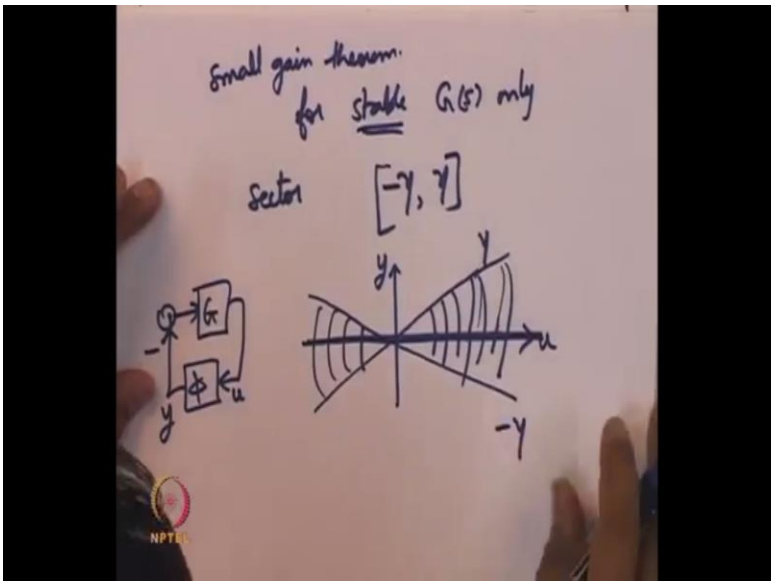

The small gain theorem or stable G s only a Circle criteria allows for unstable transfer functions. Also, the sector of nonlinearities can be found even when G is unstable because in that case when p is not equal to 0, when p is 1 or 2 or more that time such a circle has to just get encircle p number of times. This is about the small gain theorem does not allow because the small gain theorem considers sector of the type minus gamma to gamma. It considers such sectors in other words this are sectors which are symmetric about the horizontal axis, this is a line, which slope minus gamma this a line which slope gamma and any sector within this, any nonlinearity possibly time varying inside this only is allowed.

So, in particular the line which slopes 0 is also allowed, which means that open loop all already has to be stable. If you want closed loop stability for this sector, then you are allowing your insisting on close loop stability for line for the lines with 0 slopes, which is nothing but open loop because of the symmetry of this sector. This plus and minus sign does not play a role as far as a small gain theorem is concerned. So, if you allow a line with slope 0 also inside this sector, whenever a line slopes 0 around inside this sector that time that is like saying y equal to 0 even when u is non zero, which means that G has to already be stable only when you have open loop stability.

This you can have you can ask for appreciablility of the sector that also has 0, so that is why element for stable G because we are considering some such sectors which has 0.

Also, inside in fact here it is symmetric about the 0 about the line which slope 0, so what is the small gain theorem.

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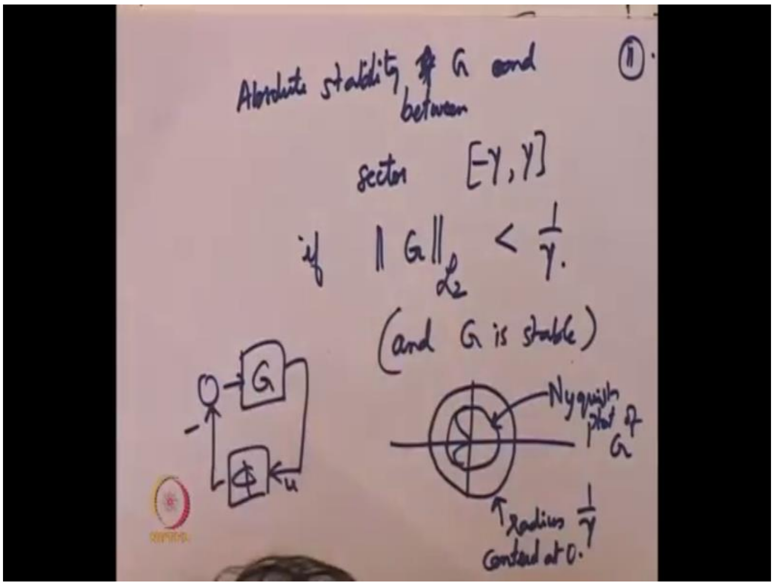

The small gain theorem says that Absolute stability or G between and sector bound time varying nonlinearities in the range minus gamma to gamma in this slope if G induce is strictly less than 1 by gamma. What does this mean, of course this already and G is stable writing this L 2 here induced already implies that G is stable; there are no poles on the imaginary axis. It has no poles in the right half plain, so what does this mean in terms of the Nyquist plot of G the Nyquist plot of G is at most gamma. In fact it is no point on the Nyquist plot is equal to gamma or more away from the origin. So, if this is a circle of radius 1 by gamma, then the Nyquist plot is contain inside this Nyquist plot naught of G the Nyquist plot of g need not be a circle because G could be of any order.

It could passed through the origin etcetera, but it should be contained inside a circle of radius 1 by gamma and centred at the origin centred at the origin. So, a statement that the of G which is nothing but the supreme overall omega in r of G of j omega and this is what we have already saw in a previous lecture that was about norms of signals and of operators.

There, we saw that the norms is nothing but the large farthest point on the of the Nyquist plot from the origin and to say that this is strictly lesson 1 by gamma means that the Nyquist plot is contained inside this. So, this small gain their and involves if you asked find the largest sector using the small gain theorem such that your stability of interconnection of b of G and sector bound nonlinearities of this form. That involves finding the smallest circle smallest circle that contains the Nyquist plot of G and of course G has to already be stable.

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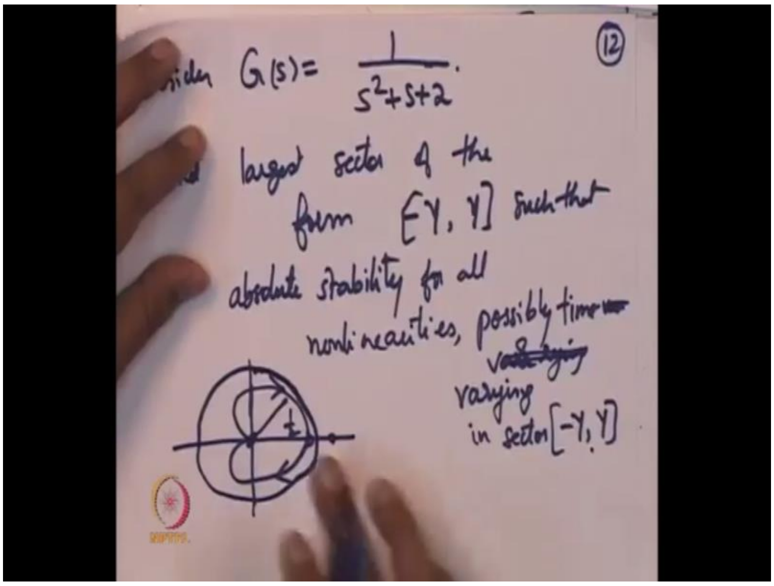

So, let us write this is a problem consider G of s equal to 1 s square plus s plus 2 find largest sector of the form minus gamma to gamma such that absolute stability, what is absolute about the stability. We are able to conclude, we are asked to conclude not for just nonlinearity inside this sector, but for a whole class of nonlinearities. That is what is absolute about it for not just one nonlinearity, but a whole class of nonlinearities absolute stabilities stability for all nonlinearities possibly time varying inside this sector in sector minus gamma to gamma.

So, if this is a question that we have been asked we notice that the sector that has been specified is of the 4 minus gamma to gamma. Hence, it is small gain theorem that we will use the Nyquist plot of this transfer function looks like this. This is 1 by 2, now we notice that any point here for example, on the real axis also corresponds to a point line inside such a sector. So, we are now going to look at the far at the point that is farthest from the origin perhaps the point that is farthest from the origin is here. So, then the smallest circle will be just tangential to this, let me draw another figure where this is better visible.

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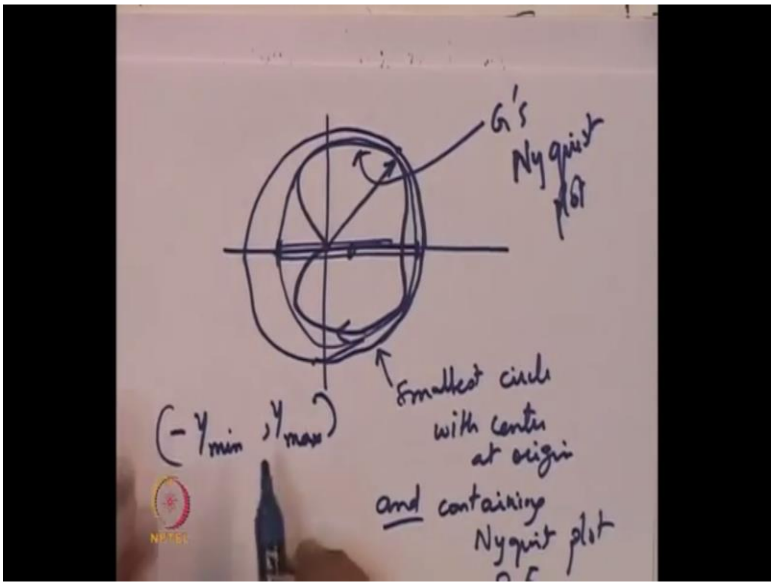

This is a smallest circle with centre at origin and encircling of course, if we did not have to encircle this, if one could make it even smaller and containing Nyquist plot of G, so right this of cours, this is the G, G is Nyquist plot. So, how will we find how will we find the radius of such a circle all one has to do is look at the farthest point in this case. It might be here look at the distance of the farthest point from the origin do 1 over that and that will give us the radius of the smaller circle centre at the origin that contains this, so clearly this forces want me to find this.

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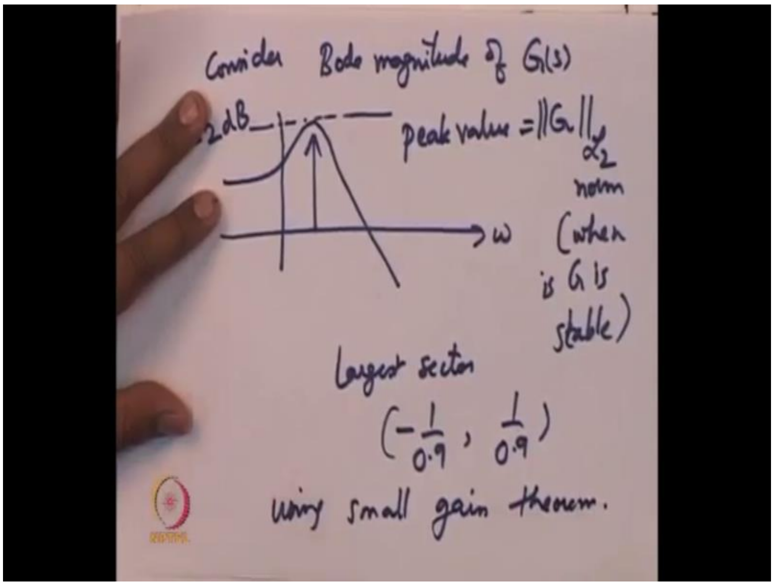

Consider Bode plot, Bode magnitude naught of G of s the peak value is a peak value peak value there are suppose this corresponds to some minus two d b for example, so this peak value is equal to G L 2 norm, of course when G is stable. This we had seen in detail in our various lecture on the norms of signal sign of operators. So, when G stable, so this of course we are not interested in b, we want the exact value this peak value is also the farthest point on the Nyquist plot farthest from the origin, the distance of the farthest point from the origin on the Nyquist plot. So, this will decide, suppose this equal to this is an absolute value this is equal to 0.9, then I am taking values here, I could be wrong here.

So, one will say that largest sector minus 1 over 0.9 to 1 over 0.9 an open brackets, one cannot take equal to 1 over 0.9 because that will cause circle to touch the Nyquist plot, it will intersect. That is not allowed, so that is why we are have saying only open bracket largest sector using small gain theorem. So, coming back to this example, what would the circle criteria say in this case circle criteria is not forced to have this circle that smallest circle centred at the origin the circle criteria also allows the smallest circle to contain the Nyquist plot.

Why is it allowed to contain, because G is already stable, since G is stable the circle does not have to be encircled, but the circle can in fact encircle the Nyquist plot of G, but the smallest circle is not forced to have it centre at the origin. Hence, the smallest circle could have this point this has a diameter, so one can think of the smallest circle like this, so if one removes the constraint that the centre of the smallest circle should be at the origin, it could be somewhere to the to the right of the origin. For this example, then that becomes polar, so the in such a case of course the sector that you get the largest sector will not be symmetric about the horizontal axis, see this is an example where the circle criteria is closely link to that small gain theorem.

The criteria of the largest sector we get from the small gain theorem only that the small gain theorem forces that the sector is of the type minus gamma to gamma. In other words it is systematic about the horizontal axis while the circle criteria allows from minus gamma min to gamma max. It allows of this type also where it is symmetric about the horizontal axis this value and this value need not be equal. So, we will see some more examples if possible before we start out last topic, which is namely tangent spaces and maybe Foles. That is what we will see in the following lecture, this complete several problems on small gain theorem, on circle criteria and Popov criteria.

Thank you.