Digital Signal Processing & Its Applications
Professor Vikram M. Gadre
Department of Electrical Engineering,
Indian Institute of Technology, Bombay
Lecture No. 04 d
Advantages of Phasors in Discrete Systems

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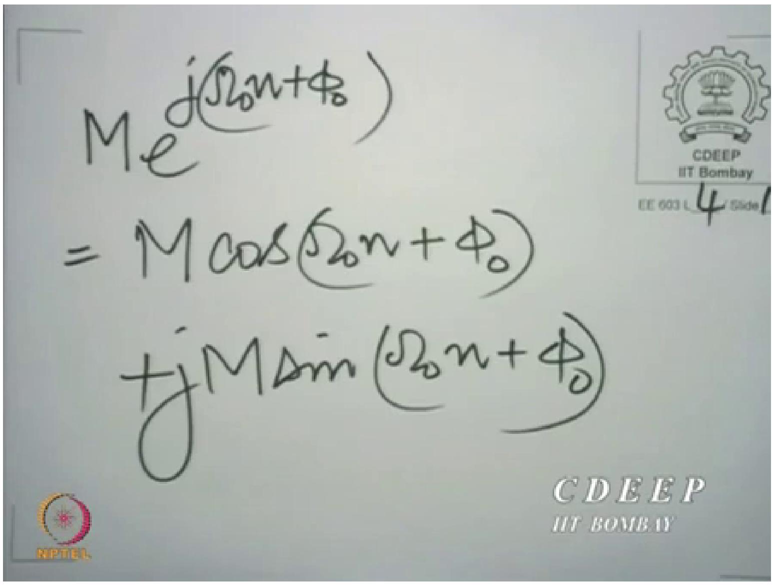

I also see the advantage of dealing with phases. You see, what I am going to do is I am going to think of a sine wave as the real or the imaginary part of a phasor. And instead of dealing with a sine wave, I am going to think that it was the phasor which was applied to the discrete system. And this would result in a phasor coming out with the same frequency, but a possible change of amplitude and phase.

And the beauty of dealing with a phasor rather than the sine wave is, this process of changing the amplitude and the phase, amounts to multiplying that phasor by a complex constant. So, I can represent the action of the system as multiplication by a complex number at that frequency, I want to be able to describe a system.

Now, this requires some thought. Now, this is another way of looking at, the other way is, I can think of a sign, of course, I can think of a sine wave as the real or the imaginary part, but how do I get real or imaginary parts? I must be able to get them with some kind of an operation. So, another way to think of it is when I add two phasors with the same frequency in opposite directions, what I mean by that is.

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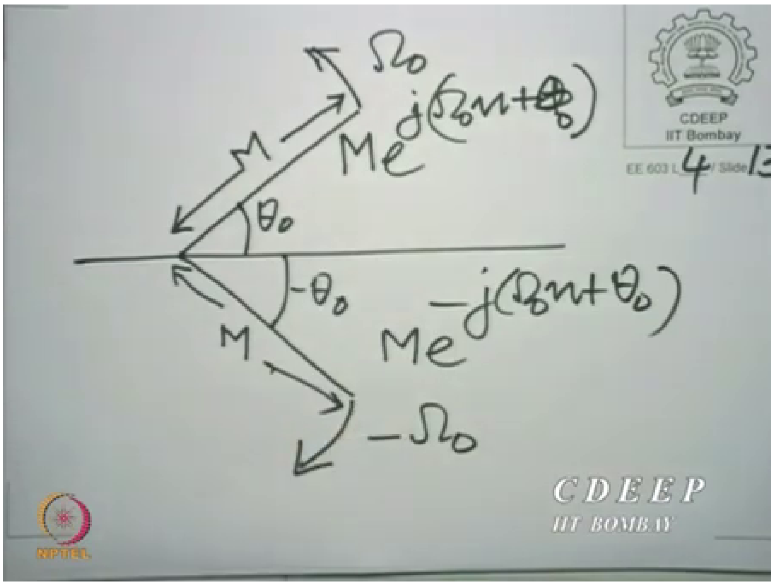

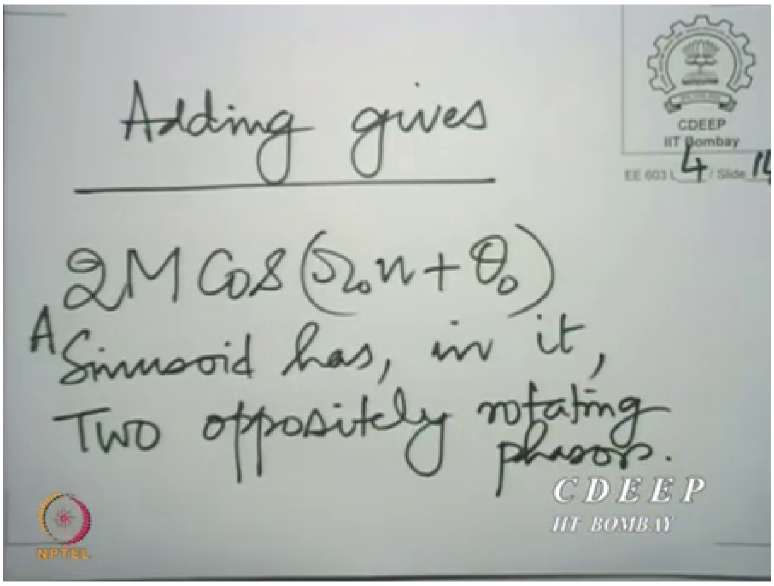

If I were to take two phases, both of magnitude M with opposite initial starting angle, so it is like mirror images, the phasors are mirror images of one another, they rotate with a mirror image angular velocity. So, you have omega not for this one and minus omega not for the other one. How would I describe this one? This is, of course, M, and of course, both of them are sampled.

So, I have $Me^{j(\Omega_0 n+\theta_0)}$ if you would like. Here, I have $Me^{-j(\Omega_0 n+\theta_0)}$, there are complex conjugates, these phasors are complex conjugates if they begin from the opposite starting angle. If they rotate with opposite velocity of the same magnitude or they have the same magnitude n then they are complex conjugates they always remain as complex conjugates.

And I am adding them to $2Mcos(\Omega_0 n + \theta_0)$. So, in other words, a sinusoid really hides or has in it, two oppositely rotating phasors, that is another way to look at it. Now, in many systems that I deal with, in discrete-time processing, it is sufficient for me to see what the system does to one of these two, they have the same frequency notionally, but with opposite signs.

So, one of them has the frequency omega not the other one has a frequency minus omega not. So, in most of the systems in many I will not say all systems, we will deal with some exceptions, but in many of the systems that we will deal with at least when we implement systems in this first course, what would happen is?

What that system does to one of these phasors is a mirror image of what it does to the other. So, if I take the phasor rotating the anti-clockwise So, here I have this, you know you go back to those two phasors, think of this phasor being applied to a system here.

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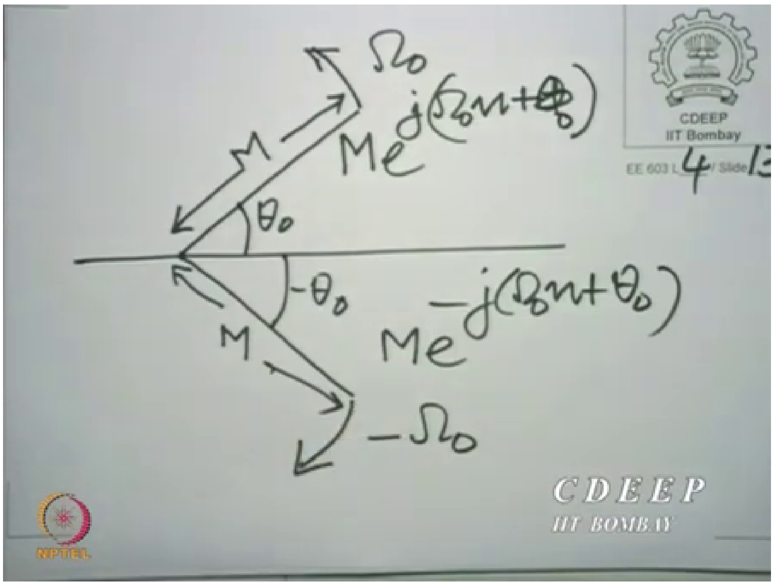

And then think of this phasor being applied to the system. When this phasor is applied to the system, what is going happen is, outcomes are phasor with the same frequency with a change amplitude and phase, apply this phasor same thing outcomes, a phasor with the same frequency, same frequency means minus omega not here and a change of amplitude in phase.

Now what is going to happen in most of the systems that we are going to deal with at least implement is that the change of amplitude for both of these will be the same. So, they will both of them will be multiplied by the same and 1/M. But the change of phase here, that means of the phase starting phase goes from θ_0 to θ_1 here, the starting phase here would go from $-\theta_0$ to $-\theta_0$. So, a change of phase would be mirrored. That is what the situation would be, in most cases.

That means if I know what happens to one of the phasors, I know what happens, I do not need to analyze what a system does to both individually. And therefore, I can replace my analysis of the sinusoidal by analysis of what happens when I put one of those phasors into the system. Do I know what it will do to the other phasor? And once I know what it does to the other phasor, I know what it does to a sine wave.

So, what I am saying is, instead of dealing with sine waves, I prefer to deal with phasors and what is the reason for that dealing with phasors means putting a multiplicative constant multiplying that phase by a constant. So, what is my requirement. Now, let me put down my requirement of the system, and then I am going to start discussing the properties that will give me that requirement.

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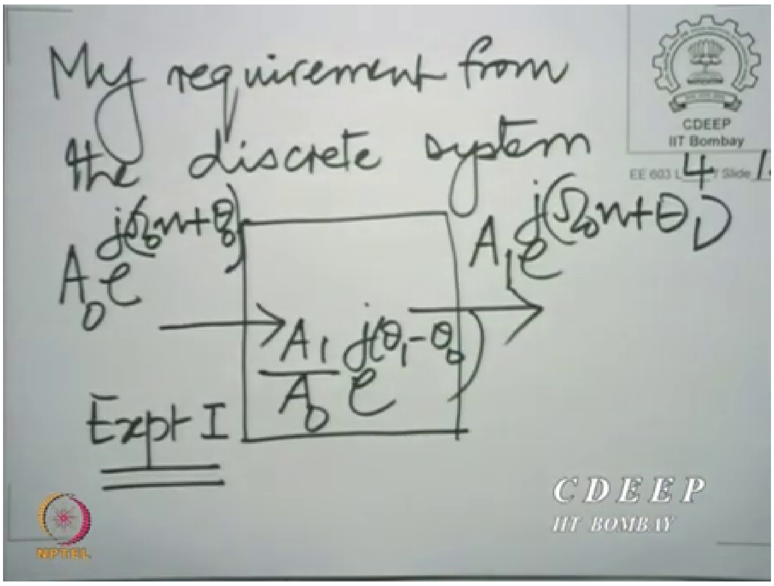

My requirements from the systems that I design, my requirement is when I put in. In fact, let me do three experiments. Experiment one, I put in a phasor with frequency omega not and initial phase theta not outcomes a phasor $A_1 e^{j(\Omega_0 n + \theta_1)}$ and therefore, the system can be described by just this ratio $A_1/A_0\, e^{j(\theta_1 - \theta_0)}$. I can describe the system just by this operation, just by this multiplication at that frequency.

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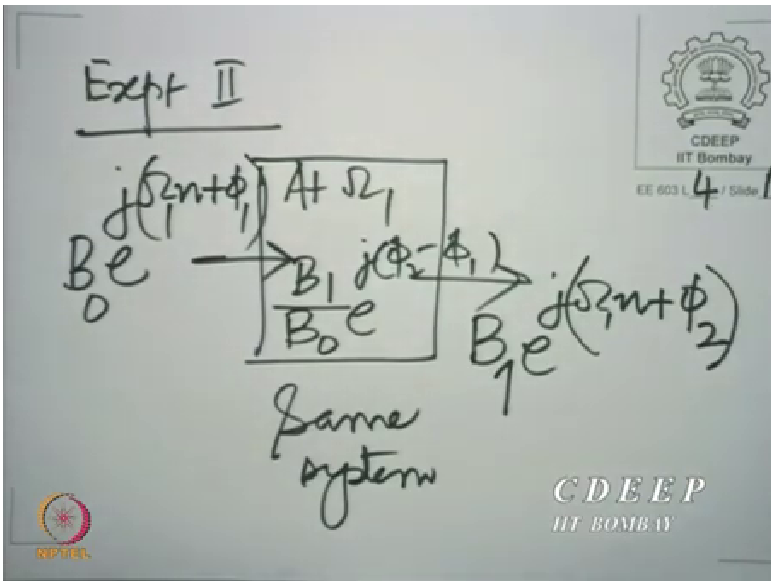

Experiment two, same system but different input, $B_0 e^{j(\Omega_1 n + \phi_1)}$ with different phase ϕ_1 outcomes $B_1 e^{j(\Omega_1 n + \phi_2)}$ same frequency, but of course, phase and amplitude can change. And therefore, at Ω_1 at the different frequencies is to be described by the ratio $B_1/B_0 e^{j(\phi_2 - \phi_1)}$, experiment number two, experiment number three. So, the same thing but a frequency. Experiment number three,

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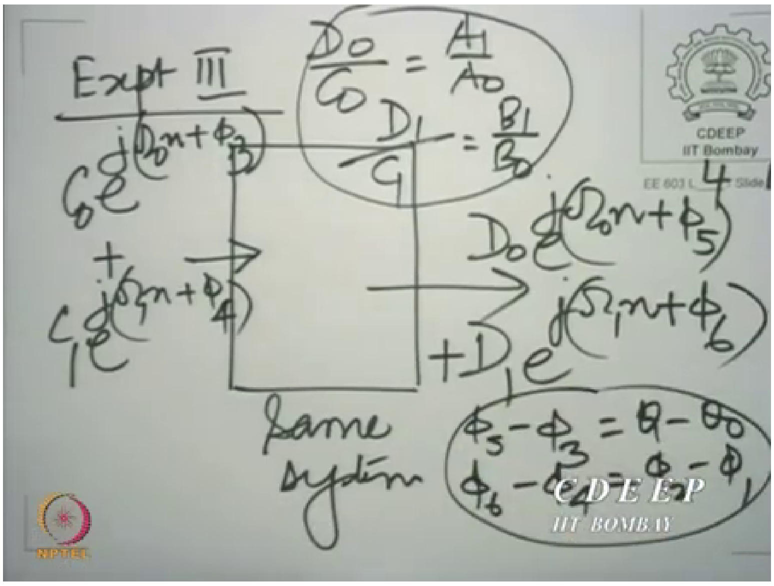

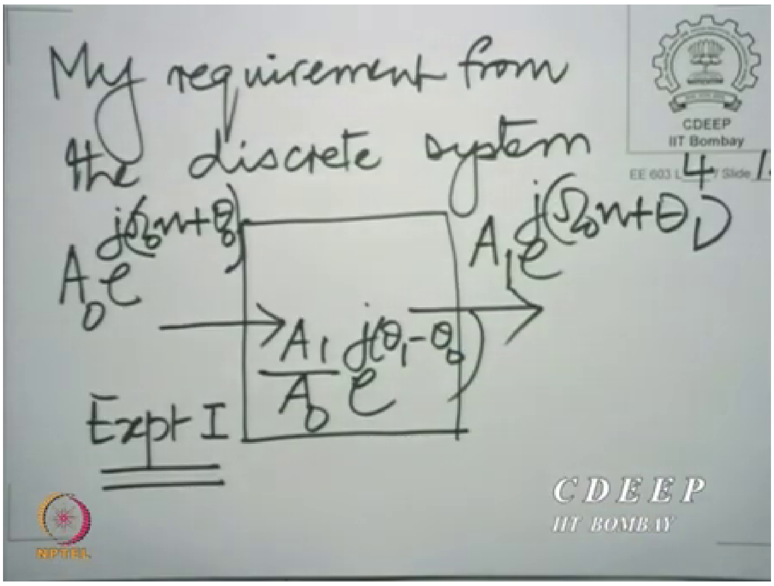

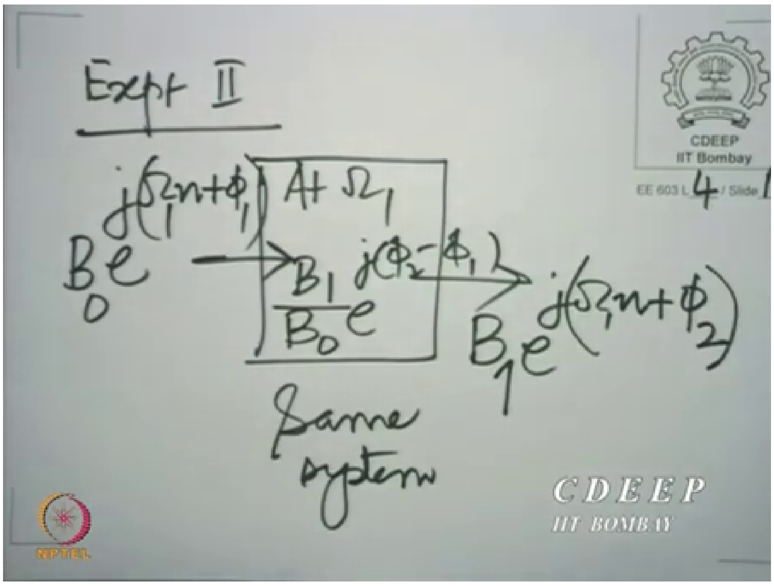

Same system, now, put in $C_0 e^{j(\Omega_0 n + \phi_3)} + C_1 e^{j(\Omega_1 n + \phi_4)}$ plus outcomes $D_0 e^{j(\Omega_0 n + \phi_5)} + D_1 e^{j(\Omega_1 n + \phi_6)}$ and that is not all. $D_0/C_0 = A_1/A_0$, $D_1/C_1 = B_1/B_0$. First, secondly, $\phi_5 - \phi_3 = \theta_1 - \theta_0$.

And $\phi_6 - \phi_4 = \phi_2 - \phi_1$. All this must be true. Just to remind you what these quantities are, A1, A0, where the amplitudes in the first experiment, θ_1, θ_0 are the amplitudes. In the first experiment, in experiment two, the amplitudes are ϕ_2, ϕ_1, and sorry, the amplitudes are B1 and B1, and the phasors are ϕ_2 and ϕ_1.

What we are saying? Is that the way the phase changes and the way the amplitudes change remains the same when I perform the third experiment. So, what are we saying? we are saying something very profound. And this is true for any omega0, omega1, any A0, A1, B0, B1, and any θ_0, etcetera. It is a very serious demand I am making of my system.

In other words, what is my demand? When I put in two complex phasors, with different frequencies, what I must get is a sum of two complex phasors of the same two frequencies, the way the amplitudes and the phasors change should be the way they change when I give those as individual inputs.

So, if I look at what happens to the first phasor, and if I look at what happens to a second phasor, in their own right, without the other being present, I must be able to tell what happens when I put them in combination. Now, this is a very serious demand of a system. And to be able to satisfy this demand, I will need the system to obey many properties. I shall list the properties today and we shall tomorrow take those properties up one by one.

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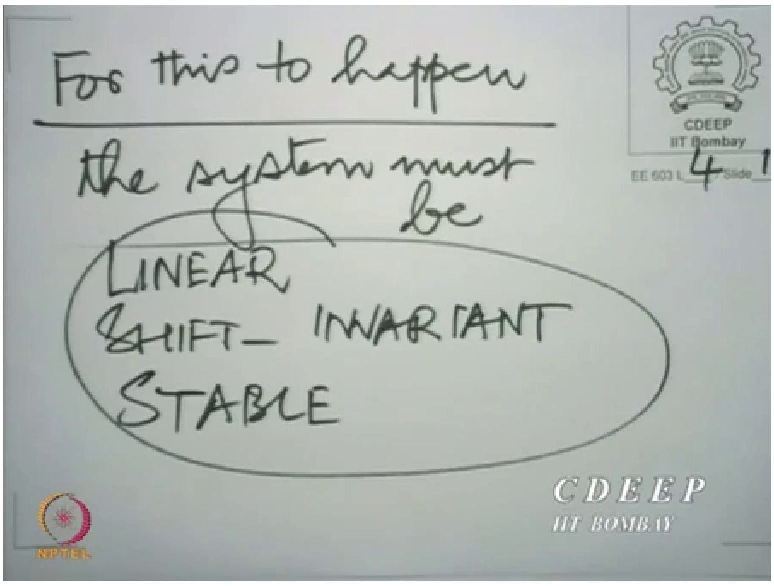

So, for this to happen. The system must be linear, shift-invariant, stable. These are only term, the only names at this point in time. I need to qualify what these means. Do you see? I mean, just to make a remark here. I am sure that many of us who might have taken a primary course in signals and systems would have heard these terms before. But what is important is to understand, why we bring in these terms, it is often that we start talking about system properties, without understanding why we should discuss them in the first place.

Why we need to discuss them in the first place is now clear. I want to build systems which have that requirement on phasors being obeyed. And if that system must obey those requirements, the only way is when it can be linear, shift-invariant, and stable. And I need to understand what these three terms mean. We will do that in the next lecture. Thank You.