Digital Signal processing & Its Applications
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Lecture – 05 c
Shift Invariance and Characterization of LTI systems

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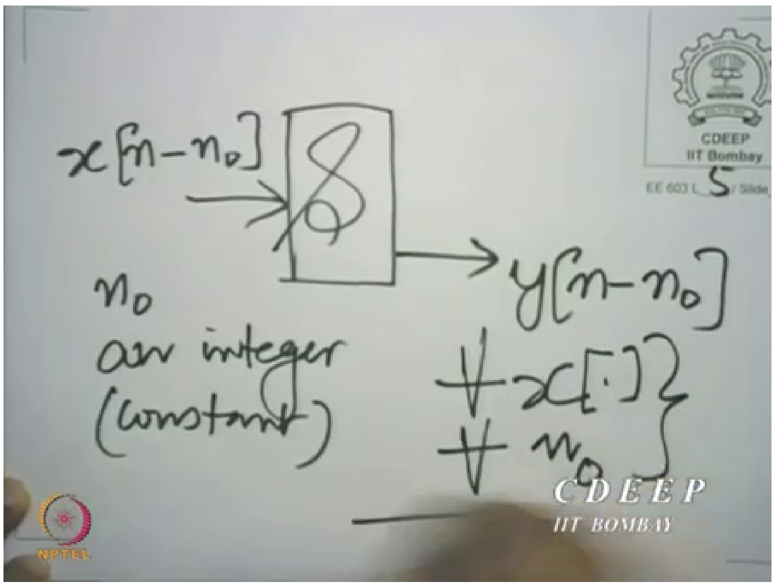

And the system is shift invariant if and only if the output is also $y[n-n_0]$. Now, here again some things need to be qualified. Firstly, that this needs to hold for all x[n] and this needs to hold for all n_0. This is very very very important. This is the statement formally now we need to understand.

You see the statement is that when I shift the input by an integer number of samples. Please note, we are only asking for n_0 to be an integer. So, when I shift the input by an integer number of samples the only consequence on the output is that the output is shifted and by the same number of samples. That is important. Not only that this holds for every possible input and every such shift.

So, when I wish to prove a system is shift invariant I must prove it independent of what input I have given it and independent of what shift I have given the input. This is important. Of course, when I want to disprove I only need to take a counter example. And now let us take an example of both let us take an example of a shift invariance system. In fact, we can go back to the very system that we had discussed as linear.

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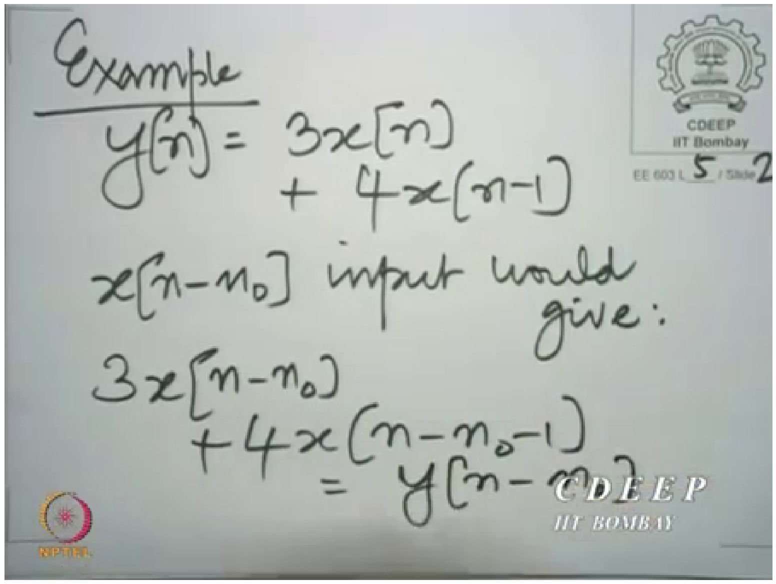

So, take the system $y[n] = 3 * x[n] + 4 * x[n - 1]$. That we had taken a minute ago. Now, if I were this is so you know what it means is when I put x n into the system I get y n according to this. Now, if I were to put $x[n - n_0]$ into the system it is a very small subtle point. But, what I would get you see how would I get the output. I would get the output by replacing n by n minus n0 in both places.

So, the output would be $3 * x[n - n_0]$ as expected plus $4 * x[n - n_0 - 1]$. That is easy and of course it is very easy to say that this is equal to $y[n - n_0]$. You see because when you replace y of when you replace n by n minus $n_0$here you get exactly the same expression. You know this is y of n minus n_0. So you know it is not too difficult to visualize. What is happening here?

At every point in the output you are taking 3 times the past, 3 times the present input sample and 4 times the past one. So, if I were to shift the say the whole input by $n_0$samples at every point I would be getting the same input but shifted by $n_0$sample. So, whatever was happening $n_0$samples before would now happen at this point.

So, its, I mean even intuitively it is not too difficult to see the system is shift invariant. In fact you know it looks like shift invariance is a trivial property. We expect this to happen rather too frequently. But, we will see in a minute that we can easily construct a counter example of a system which does not obey this. So, let us now take a.

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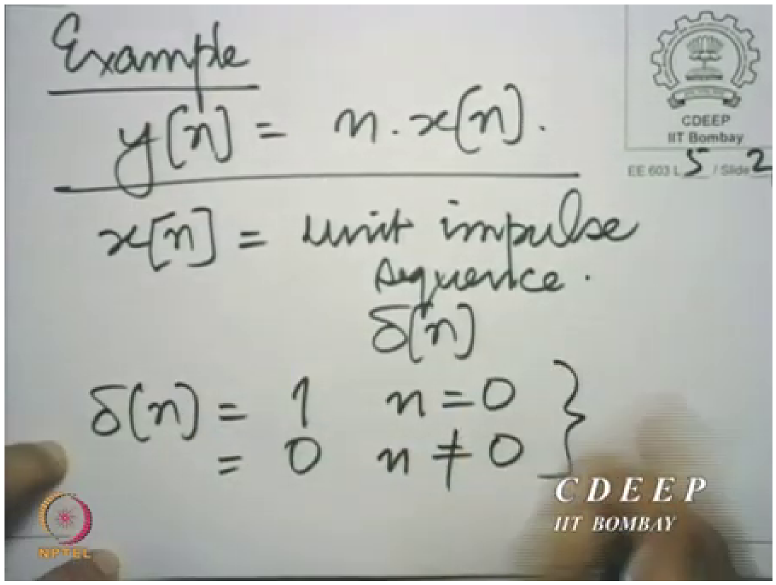

Very simple example of a system which does not obey this. So, let us take the system y[n] = n.x[n]. Now, you know one remark here typically shift variance that means disobedience of shift invariance tends to happen when you are looking at the clock in informal language. So, you know if a system looks only at the input and does what it wants to the system tends to be shift invariant.

But, when the system also takes a peak at the clock underlying the input then shift invariance is disobeyed. Now, in this system the output at a given point in time the output at the point n is the input at the point n and this system is all the time sneakingly taking a look at the clock. So, it looks at the sample instant and multiplies by the sample instead.

That is where the problem comes. So, now let us take two inputs let us take x[n]. now, we bring in some notation we bring in a very important sequence that we often use we call it the unit impulse sequence. We will use the sequence very frequently in future. We denote it by δ[n] and this sequence is 1 for n equal to 0 and 0 for n not equal to 0.

So, it is 1 only at 1 place it is 1 at n = 0 and 0 everywhere else. Now, I encourage you to work out what happens when this input is shifted by 3 samples forward. Find out the output. In fact, I ask you, what is the output in this case; when I give an input which is 1 at n equal to 0 and 0 everywhere else what output do I expect 0 everywhere.

Because, you know the place where it is 1 n is equal to 0 so it gets multiplied by 0 everywhere else the input itself is 0. So, it gets multiplied by 0 anyway.

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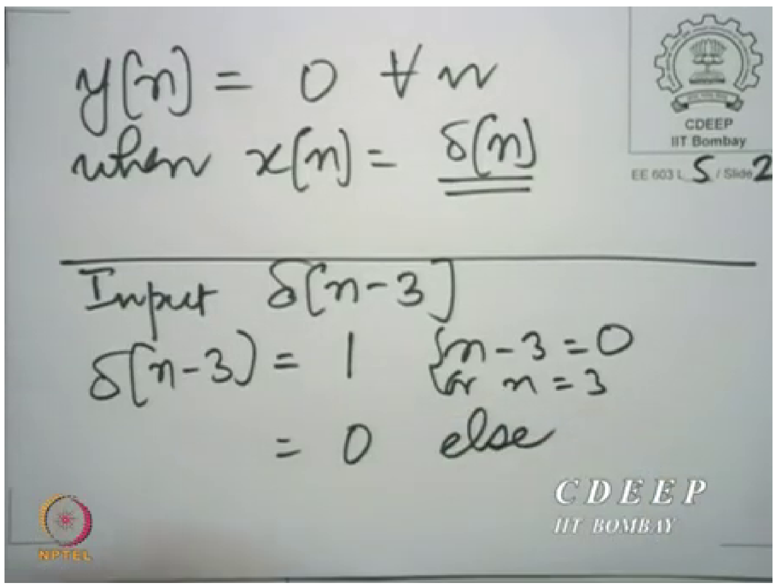

So, in this case y[n] is equal to 0 for all n when x[n] is equal to δ. This I repeat this sequence this unit impulse sequence is going to be very useful to us in future too. Now, let us find out what happens when we give the input δ[n - 3]. So, I ask you what happens when we give the input δ n minus 3. That means essentially now what is δ[n - 3], δ[n - 3] is going to be equal to 1 when n - 3 = 0 or n = 3 and 0 else.

So, it is 1 only at 1 point that is n = 3. Now, what is the output going to be the output is going to be at the point n equal to 3 it is going to be multiplied by 3 and therefore we can write the output. In fact, we can write the output down explicitly we can express the output in terms of δ.

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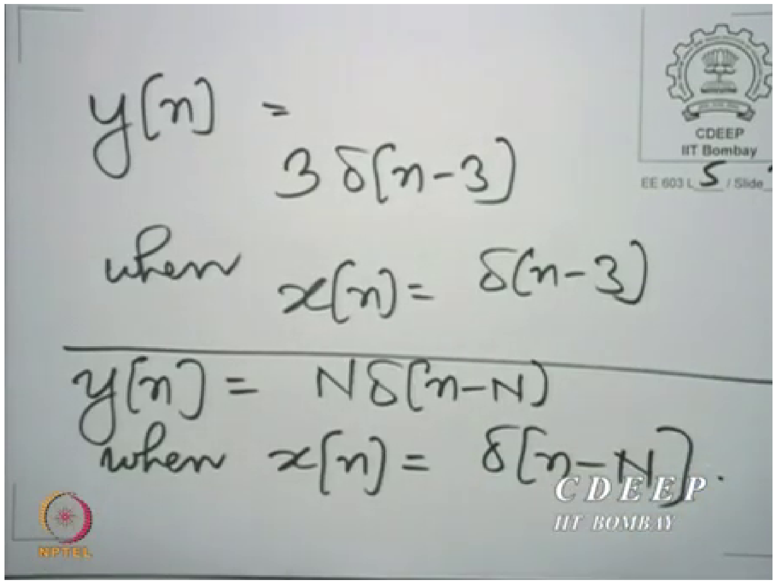

y[n] = 3 δ[n-3] when x[n] = δ[n-3]. In fact we can generalize this. In general, y[n] = Nδ[n-N] when x[n] = δ[n-N] for any integer n. And obviously this system is not shift invariant. If it were shift invariant then I should have got the output 0 here too.

But, I do not and you see why it is not shift invariant. The disobedience of shift invariance has come because the system is sneakingly taking a look at the clock when finding out. When you bring in the clock in your output there is a problem shift invariance is disobeyed. We need to take a break at this point to reflect on where we are.

We have dealt with two very important properties linearity and shift invariance. We now need to deal with others like stability. But, we will take a break as I said and look more deeply at just these two properties because, together they give us a great deal if a system is linear and shift invariant.

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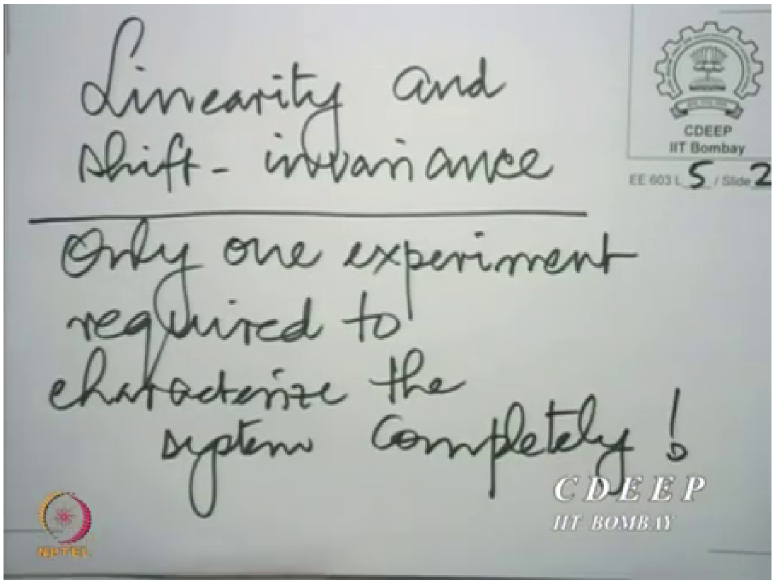

Then we will now prove a very important result. And the result is I need to perform just one experiment on that system to characterize the system completely. Only one experiment required. In fact that experiment will involve the very sequence that we mentioned a minute ago namely the unit impulse sequence. I told you that unit impulse sequence is very important. It tells us a great deal.

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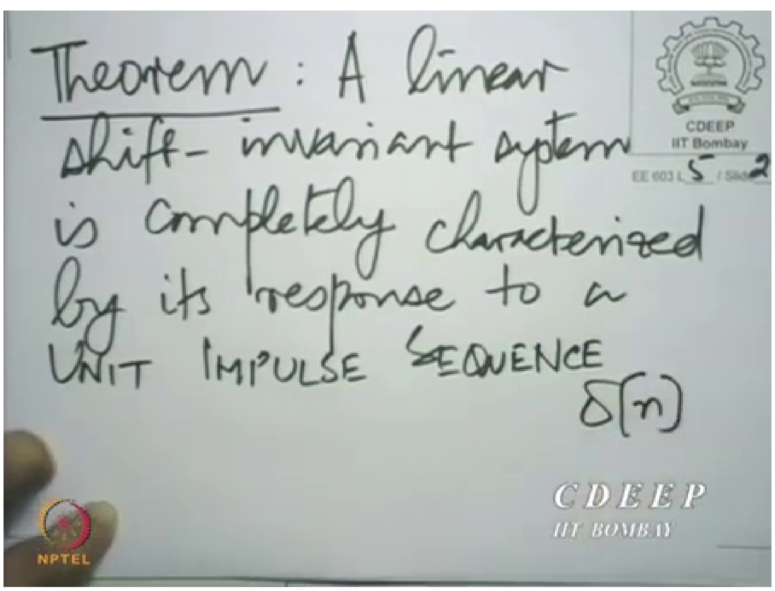

And it will tell us a great deal in context of linear shift invariance system. So, we will show that the following is true. A linear shift invariance system is completely characterized by its response to a unit impulse sequence δ[n]. We need to take a minute to reflect on what is meant by complete characterization. When would I say a system has been completely characterized?

Let me ask you when would you say a system has been completely characterized. You know what will be yes expected output. All the frequency components are excited, frequency. So, I will repeat some of the responses I have. One response says that I know the output whenever I know when I whenever I give it an input that is one.

The other response says that I am trying to excite all frequency components, any other responses. Yes, the relation between the input and the output can be determined. Yes, the same thing alright another. Transfer function can be obtained. Yes, yes you can predict the output for any input.

Yes, any input signal can you repeat that any input signal can be constructed from but how does that characterize the system. He says, the any input signal can be constructed by that does not tell me that the system is carrier. What do you mean by my question is what do you mean by characterizing a system. You know I will tell you a story and I think the story is relevant before we conclude the class. Because, very many of you must be studying the subject after possible exposure to some of the concepts in other contexts.

There is a story of a master and a disciple and the disciple was asked to bring a cup and a jug of water. So, you know the disciple was asked to fill the cup partially first by the master. And then you know the disciple was again asked to continue pouring water into the cup even after it reached its brim. Naturally, the water began to overflow. After a while the disciple wondered why this was being asked of him.

Why am I continuing to pour water into a cup which is already full? Master said that is exactly what I am trying to point out. If I need to explain certain things to you it is important that you have space in the place where you store what has been explained. If you come with that storage full it is very important to put something into it.

So for example, the question here I bring this up in this context because very often we already have our notions of what systems need to be like what system definition. For example, some people I am not trying to I am not trying to dissuade response.

But, I am just trying to tell you that it is important that you begin with the assumptions that have been made and the question that has been asked and the context that is being addressed and not try to bring in extraneous things when trying to understand concepts.

For example some people mention transfer function, some people mention frequency. Now, you see at this moment we are just asking what characterizes the system we have not said anything about transfer we do not even know if the system has a transfer function.

And if we do not even know the system has been given a sinusoidal input. We just know the system has an input and an output. And our answer naturally should be related to that. So, we do not bring in things.

And you see when we when we try and answer questions that are fundamental to the subject it is very important only to proceed from assumptions that are made there. So, here we have not made an assumption about any transfer function or you know we do not even know if a transfer function exists.

Similarly, we have not even made an assumption about any particular kind of input. So, characterization so you know in proofs in in I, I mention this as a word of caution because it is very easy sometimes we misled by our knowledge in other contexts.

For example some people think characterizing a system means being able to write an explicit relation between the input and output. No. Characterizing a system means just one thing it means that no matter what input I give it I know what the output is. And that is all that we can say because we are just talking about a general system at this point in time.

We have no knowledge of whether system. You see we are of course going to characterize a particular class of systems. But, then we the question that is before us is what is meant by characterization. Characterization means if I give it an input I can determine the output. Whether I can determine the output by an explicit relation or an implicit relation or by observation is a different issue.

In fact now we will soon show that the output can be determined constructively. Constructively means I can actually write down a process by which I can construct it. But that is an except that is a bonus that you get. It is not a part of characterization. So, let us make a statement about characterization.

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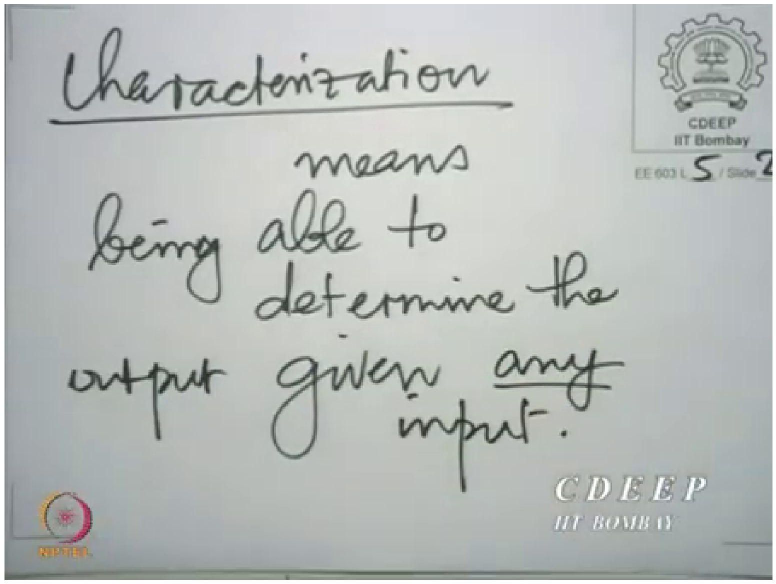

Characterization means being able to determine the output given any input. In fact there is nothing more to a system. If I know the output for every input the system is determined its characterized anyway. Now, we shall proceed to prove this in the next lecture. But, what I will do is to give you what I call a trailer of the proof. And the trailer of the proof is as follows. You see what will show is that.

As one of you said input can either ultimately be thought of as a combination of unit impulses appropriately shifted. Now, because a system which is linear in shift invariant has two properties one is since it is shift invariant if I know what it does to a unit impulse I know what it does when I shift the unit impulse.

Because it is homogeneous I know what it does when I scale the impulse. So, if I put a unit impulse and multiplied by a constant I know what the output will be. Further if I take two such impulses located at different points and if I add them I know what the output would be. If this were given so I use these 3 parts of the system description. The shift invariance, the additivity and the homogeneity by using these 3 properties together we shall prove this theorem as the first step in the lecture to come.

With that then we conclude this lecture will meet again for the next class.