Digital Signal Processing & Its Applications
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Lecture – 06 b
Introduction to Convolution

So, here we conceive of the output as a sum of many shifted impulse responses that is one way to look at it. But often, that is not what we want to do. We do not want to sit and put so many sequences down and add them, we would much rather find out the output point by point. And for that, we need to carry out a little more work on the expression that we have just got. Therefore, we will introduce some terminology now.

You see, this output *y[n]* has been obtained by an operation between the input sequence and the impulse response sequence here. And we will give that operation a name. It is an operation not between two numbers, but between two sequences.

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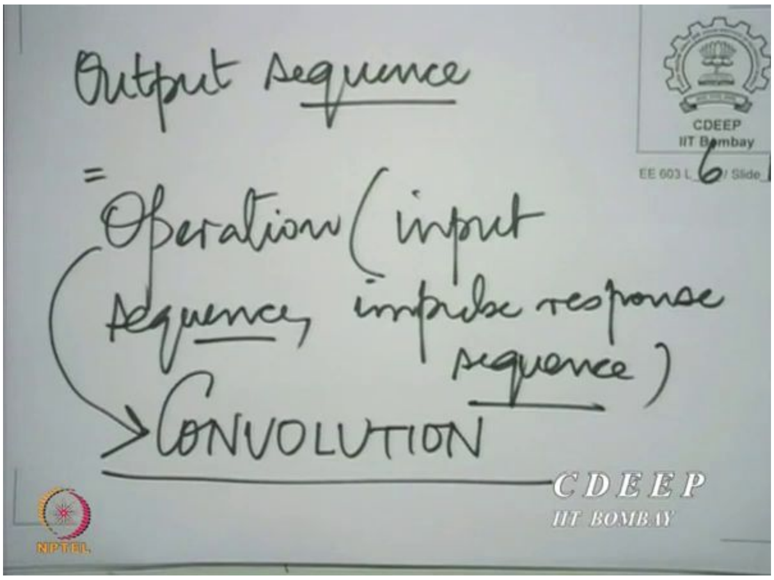

So, we are saying that the output sequence, some operation is obtained by some operation acting on the input sequence and the impulse response sequence. It is an operation between two sequences. And therefore, let me underline sequence everywhere. It is an operation between two sequences; we will give this operation a name.

We will call this operation convolution. And there is a reason why we give it that name. You see, we typically use the term convoluted, in common discussions, or in colloquial terms, where something is complicated, or where something twists and mixes.

You know, one, sometimes refers to a convoluted argument, to denote a set of, you know, statements or a set of utterances, or a set of justifications, which are very complicated to understand and which intertwine together in some complicated way, and perhaps lead to a conclusion, but it is not at all straightforward to see how they start from the beginning and come to a conclusion, we say it is a very convoluted argument.

Convoluted in general, tends to refer to twisting and mixing together in a highly intertwined, in a highly interactive way. And in a minute, we will see that that is exactly what we are doing in a way between the input and the impulse response here. If you think of the input as a string of samples, and if you think of the impulse response as a string of samples, what we are doing is to intertwine those strings together in a very complicated and a very fine manner to obtain the output.

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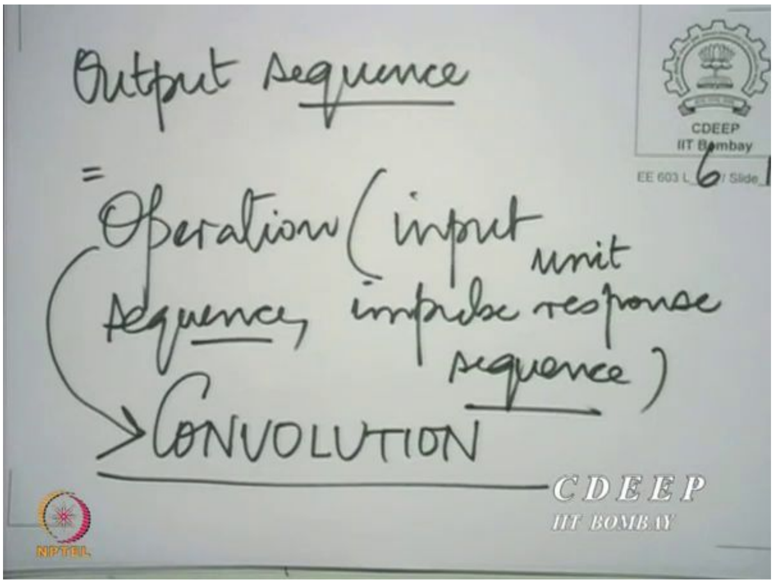

Yes, there is a question.

Yes, the question is that, should we be writing the operation between the input sequence and unit impulse response sequence? Well, the answer is yes. You know, we have sort of ignored the word unit, but we will write that to be clear here. So it is the response to a unit impulse. That is correct. One should in a way one should emphasize that it is the response to a unit impulse, but otherwise, it could be a scaled impulse too.

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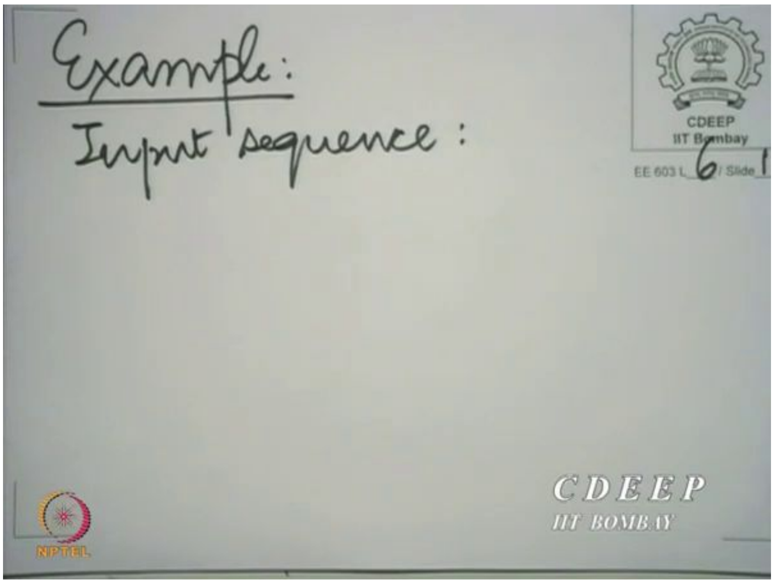

Now, you know, we will try and understand both of these interpretations with a specific example. We will take a very simple input sequence and a very simple unit impulse response sequence. So we will take the input sequence. And here, I am going to introduce some notation once again for convenience, when we have a very short input, an input which is non-zero only for a few samples, we tend to use this notation, where we denote the location of one of the points and then we denote all the other points around it.

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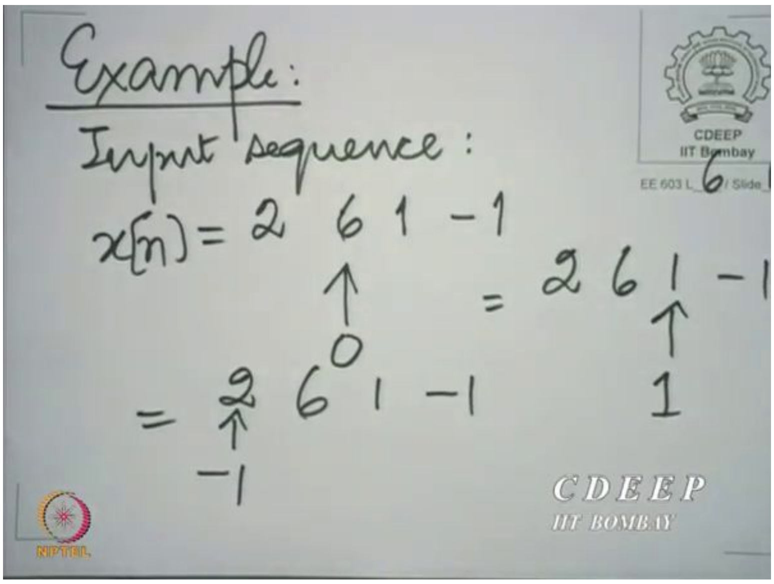

So, we indicate it for example, by showing that at point *0*, I have the sample let us say *6* here and then I have *1*, (-*1*) and *2*. Obviously, *1* occurs at the point $n = 1$, (-*1*) at the point $n = 2$, and *2* at the point $n = -1$.

Needless to say, I mean just for the sake of notation, I am trying to explain this notation. This is the same thing as writing. Once the same thing is writing, all of them are the same. So we can write them in any manner that we please. But what is important is to mark one point and all other points follow from there.

Yes, so here, the question is, you see the sequence is *2, 6, 1,* (*-1*) respectively at the points *(-1), 0, 1,* and *2* okay. So therefore, you could see that this sequence takes the value *1* at the point $n = 1$ and the other marks, of course, follow and similarly, it could be marked at (*-1*). And the other points obviously, follow.

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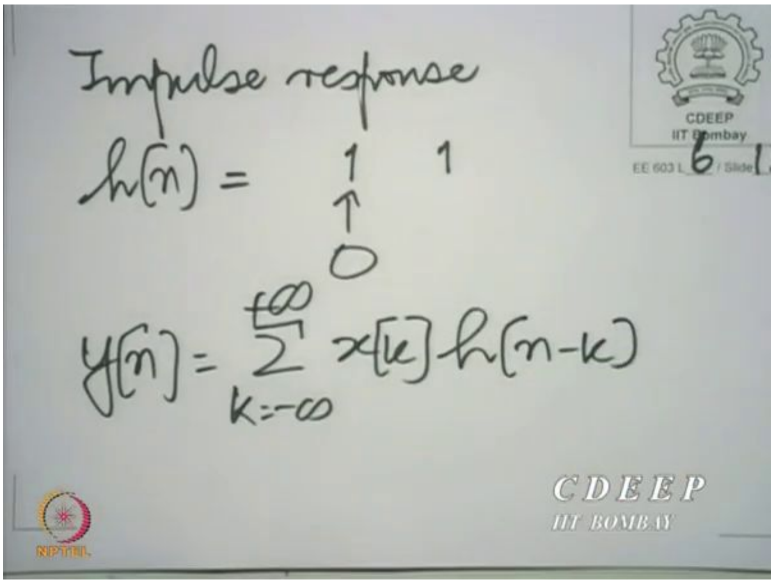

Yes. So similarly, let us take a very simple impulse response. And, of course, a unit impulse response, if you please. But henceforth, when we say impulse response, we will mean the unit impulse response, we will not want to keep repeating unit every time. The unit, the unit impulse response, the impulse response is very simple.

It is just *1* and *1* respectively at *0* and *1*. Now, I shall obtain the output in two different ways. And we will explain therefore, the operation of convolution, why it is called convolution? One way, as I said, is to use this expression with the interpretation that we are shifting and adding the impulse response several times. So, what would this mean?

This would mean, I have taken the impulse response, I have shifted it by each *k*, at scale that shifted impulse response by the value of the input sample at the point *k*, and I have summed up all these shifted versions of the impulse response. So let us sketch that output. Let me remind you of the input, once again, what it would mean is I must shift the impulse response backward by *2*, keep it as it is, shift it forward by *1*, forward by *2* and respectively, multiply by *2, 6, 1*, and (-*1*). So here I am.

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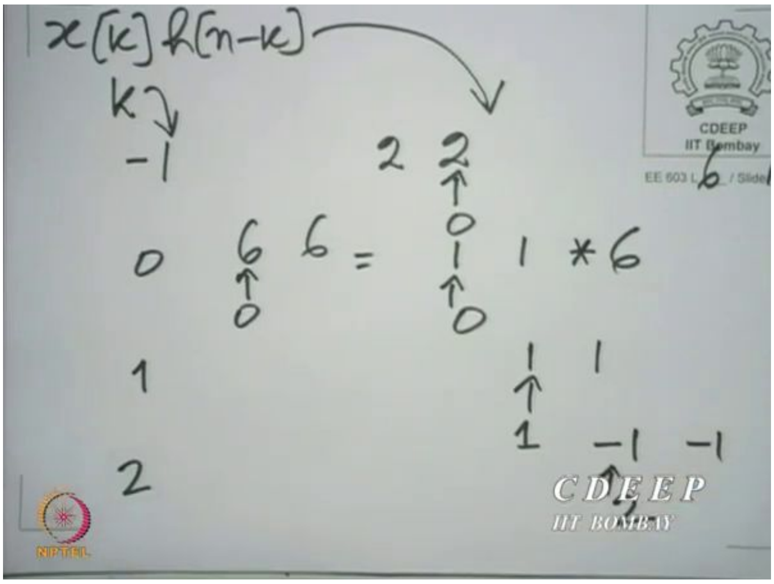

So you know, *x[k] h[n-k]*. So *k* = , I will write *k*, and I will write this sample here. So *k = -1,* and I have *2, 2,* and this occurs at *0*, *k = 0*, and I have *1, 1,* here. For convenience, I will put the *0* in at *k = 1*, I have, I am sorry, so this should be multiplied by *6*. That is *6* and *6*. So, let me you know, let me complete this. So 1 at *k=1* of course, I have *1* and *1* occurring at 1 and *k = 2*, I have (-*1*), (-*1*) at *k = 2.* So, these are the four *x[k] h[n-k]* ‘s. (refer to video)

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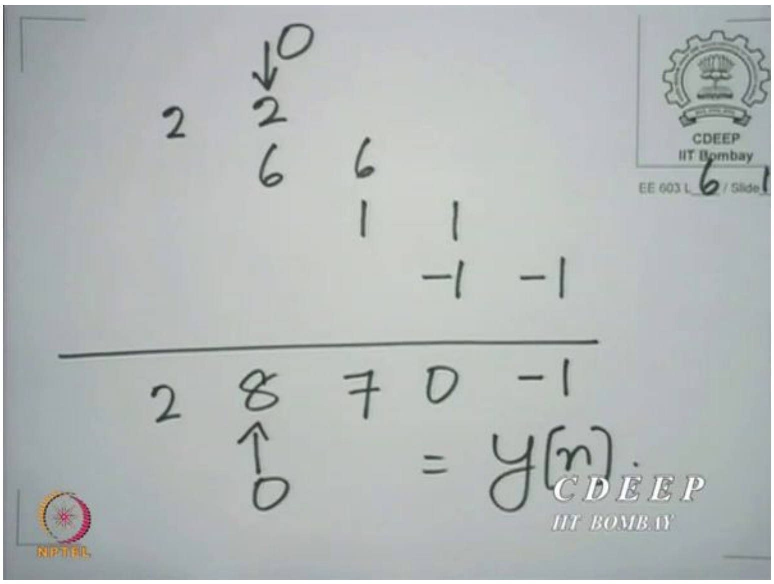

And now of course, I add them. So I will have to, you know, let me draw it again. Let me put them down. Since this is the first time we are solving such an example, *2, 2,* we have *6, 6, 1, 1,* and we have *(-1), (-1)*. So of course, and this is the point I will mark it from above, this is the point *0* here. So I have of course to *2, 8, 7, 0,* and (-1).

This is the output sequence *y[n]* Is that clear? Yes? Has everybody followed this, any doubts on what is going on here? However, what is very clear from this example is that you can obtain the output point by point, unlike what we are doing here, where we are putting sequences and adding them up, by looking at what is expected at each point. You see, what I mean by that is, we re-interrupt it, and let us do that.