Digital Signal Processing & Its Applications
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Lecture 07 B
Response to LSI systems to complex sinusoids

But now, we have seen something very interesting, we can combine LSI systems in cascade, of course, it is a minor variation or a very easy question to answer when we ask, what happens to LSI systems in parallel? That is very easy to answer.

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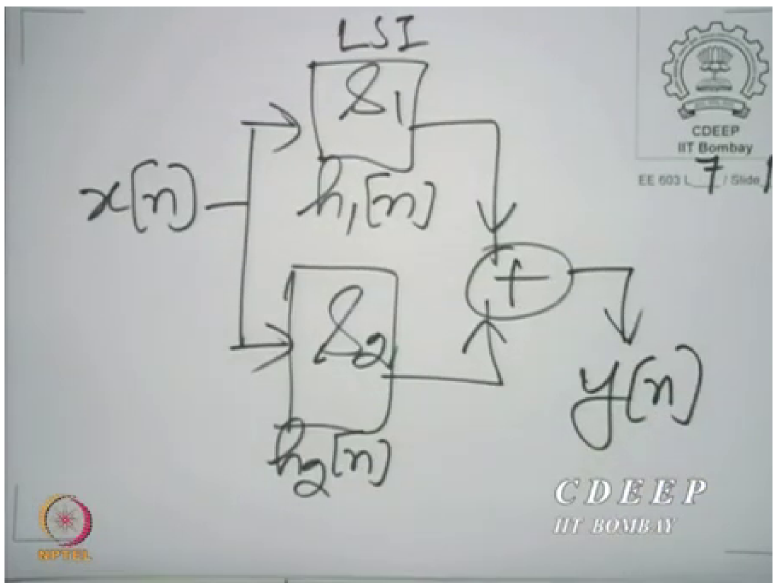

So, suppose you have the same x[n] being given to two LSI systems in parallel with impulse responses, $h_1[n]$, and $h_2[n]$. By parallel, you mean, you apply the same input, and then add the outputs. The answer is very easy. Can we find an equivalent system for this? And if so, what is the equivalent system?

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$$y[n] = \left(x[n] * h_1[n]\right) + \left(x[n] * h_2[n]\right)$$

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$$= \sum_{k=-\infty}^{+\infty} x[k]\, h_1[n-k] + \sum_{k=-\infty}^{+\infty} x[k]\, h_2[n-k]$$

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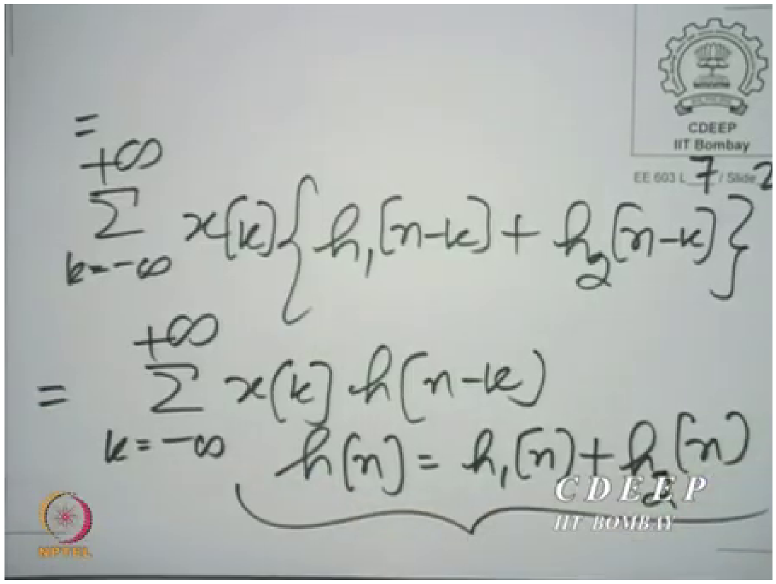

Well, it is very easy to see that

$$y[n] = x[n] * h_1[n] + x[n] * h_2[n]$$

And of course, this can be written as

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h_1[n-k] + \sum_{k=-\infty}^{\infty} x[k]h_2[n-k]$$

which of course, is very easy to combine. This does not require any great knowledge of, just a simple distributivity property of multiplication that we know very well, where h[n] is equal to $h_1[n]$ plus $h_2[n]$.

So, in fact, when we have two LSI systems in parallel, there is an equivalent LSI system whose impulse response is the sum of the impulse responses of the individual LSI systems, simple enough. So, now we are well in a position to deal with any combination of LSI systems in cascade or in parallel, or a combination of cascade and parallel, is that right? We can always find an equivalent LSI system for them.

The only problem is, if you wish to find an equivalent LSI system for a cascade, it is hard work because you will have to do a very cumbersome operation convolution. So, this is another reason why we might want to see if you can go to some other domain or go into some other mode where

I can carry this operation out more easily. And in fact, it is quite beautiful, how several questions in signal processing converge to one answer.

Now, recall that the whole reason why we started discussing linear shift-invariant systems was to deal with sinusoids. We come back to the same story again. There we had those sinusoids. We had complex exponentials because we did not want sinusoids. And the reason why we wanted complex exponentials was that, although a change of amplitude could be represented as multiplication by a constant change of phase could not, but they could if you dealt with complex exponential.

And therefore, we asked what is the system which leaves a complex exponential as it is in frequency but changes only the amplitude and phase and we said that system must be LSI. Now, we need to justify that. So, why have we worked so hard to build all these properties of convolution unless we can show that we have indeed got where we wanted to?

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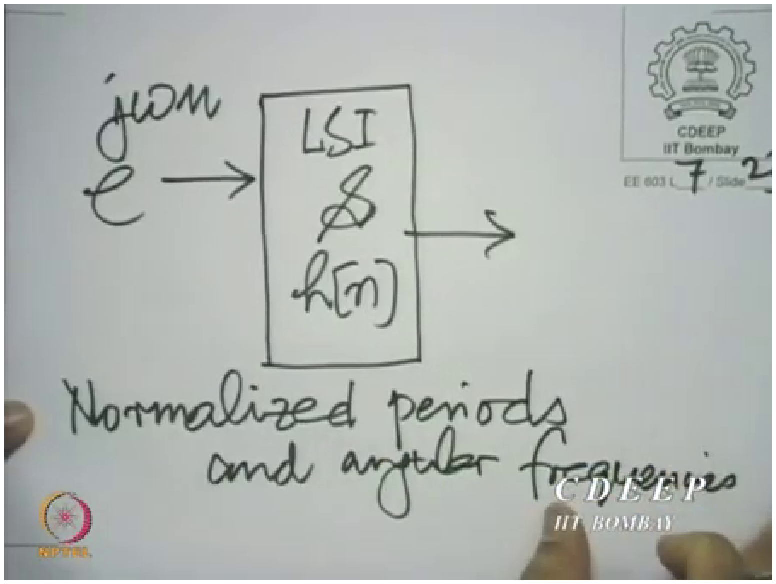

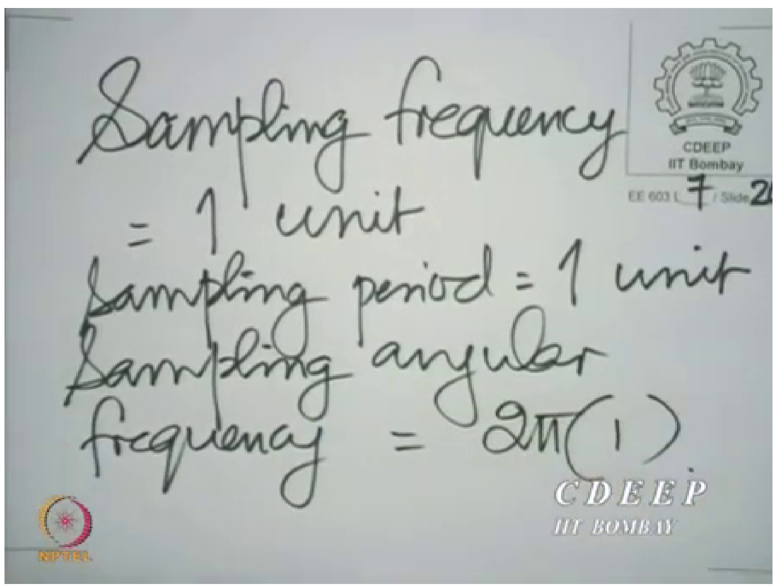

So, now let us ask the question, what happens when we feed a complex exponential of angular frequency ω, we use small omega, and let me now bring in some notation here. So, what happens when we feed in a complex exponential of angular frequency ω into an LSI system? Let us call it S with impulse response h[n]. Now, a few remarks about the angular frequency omega.

Henceforth, we are going to use what are called normalized periods and frequencies. And what we mean by that is that we shall assume, that the sampling angular frequency or that well, let us first take the sampling frequencies. We will take the sampling frequency to be 1 unit, the unit is our choice ultimately, whatever it is, if it is 10 kilohertz, we will say 10 kilohertz is 1 unit. If it is 1 megahertz, we say 1 megahertz is 1 unit.

So, we say the sampling frequency is 1 unit is the unit frequency, whatever, so you can always choose your unit. And therefore, of course, the sampling period is also 1 unit, obviously, it is the reciprocal of the frequency. Of course, these are units of different quantities, sampling frequencies, as units of frequency, whatever they might be in sampling periods have units of time. Therefore, the sampling angular frequency becomes 2π times 1 unit. So, whenever we were dealing with a discrete system, we will assume that we have chosen the unit so that 1 unit is equal to the sampling frequency. And therefore, the sampling period is 1.

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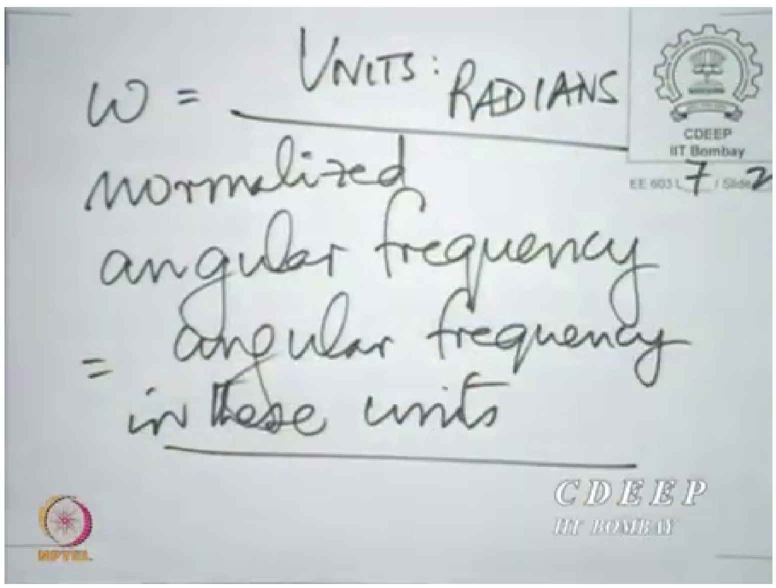

And thereby, we shall use ω to denote what is called the normalized angular frequency, or the angular frequency in these units. Now, we need to reflect for a minute, what the use of the normalized angular frequency would be? You see, when we normalize, we divide. So, for example, if you say you normalize the angular, or you have normalized the frequency of sampling, you have actually divided the sampling frequency by the actual sampling frequency.

So, for example, if your sampling frequency were 1 megahertz, you have divided all frequencies by 1 megahertz to get the normalized frequency. Similarly, if you have divided the angular frequency in a similar way, to get the normalized quantity, then, in fact, what you have done is to replace radians per second, radians per second was the unit of angular frequency as it were, but you have divided this by the sampling frequency in actual value.

So, radians per second divided by per second, or hertz, leaves you with radians. And therefore, the units of normalized angular frequency here are radians, not radians per second. Similarly, the units of period or units of frequency are null, there are no units because you have divided hertz by hertz, which is just numbers.

Student: Sir,

Professor: Yes, there is a question.

Student: Sir, it is only, it is not a unit only, it is only an angle.

Professor: Okay, so the question is, is it appropriate to think of this as a unit or to think of this as an angle? Well, both are correct. You see, what we are saying in a way, what ω denotes is how much of angle is covered in a unit sample time, in a sample time. So, you see, when ω is equal to 2π, that means when you use the sampling frequency itself, then you have covered angle of 2π in a sample time.

So essentially, ω is a measure of the angle covered in one sample time. That is another interpretation. Yes, that is why the unit is radian. All right, then, we agree then to use the normalized angular frequency because, you see, we are going to use the integer n to denote the sample number and also to denote the sample time since the sampling time is unity.

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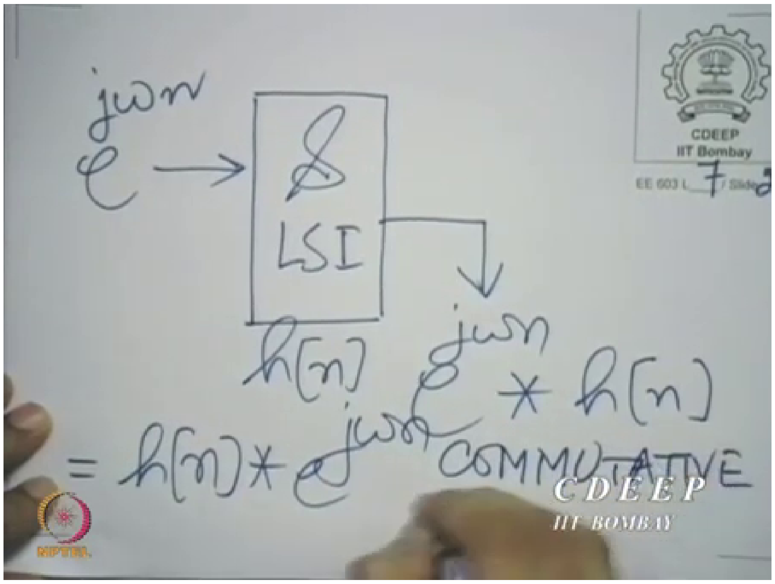

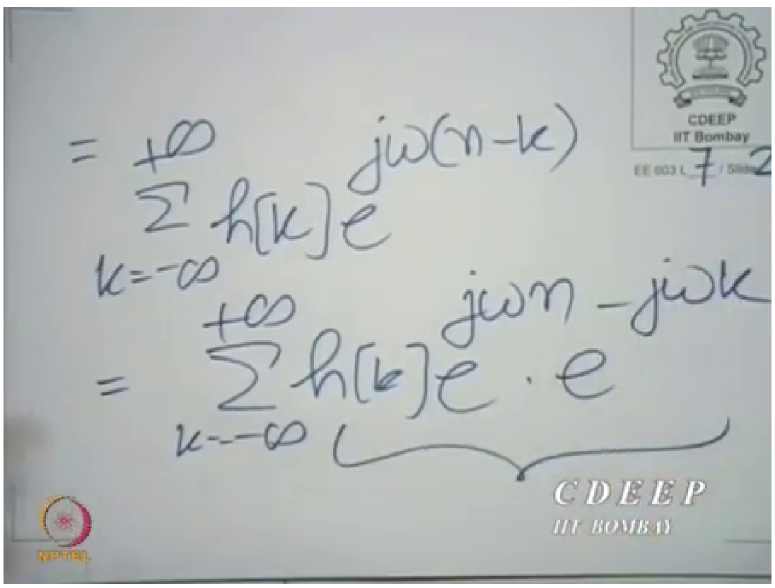

So, therefore, we have $e^{j\omega n}$ as the complex exponential, otherwise, we will have to write down $e^{j\omega nT_s}$ and so on. So, we do not need to do that now. So, when we feed$e^{j\omega n}$ to the LSI system S with impulse response h[n], we know what what we get out? We will get out $e^{j\omega n} * h[n]$.

Now, here we are going to invoke the commutativity of convolution. And that will give us $h[n] * e^{j\omega n}$ equally well, from commutativity. So, if you use that expression, then it gives us.

$$\sum_{k=-\infty}^{\infty} h[k]e^{j\omega(n-k)}$$

And that is very easy to break up. It is $\sum_{k=-\infty}^{\infty} h[k]e^{j\omega n}e^{-j\omega k}$

Now, note that here $e^{j\omega n}$ is independent of k. So, I can draw it out of the summation. And I am left with an infinite summation on k. The infinite summation on k is in fact a function of omega and a function of h.

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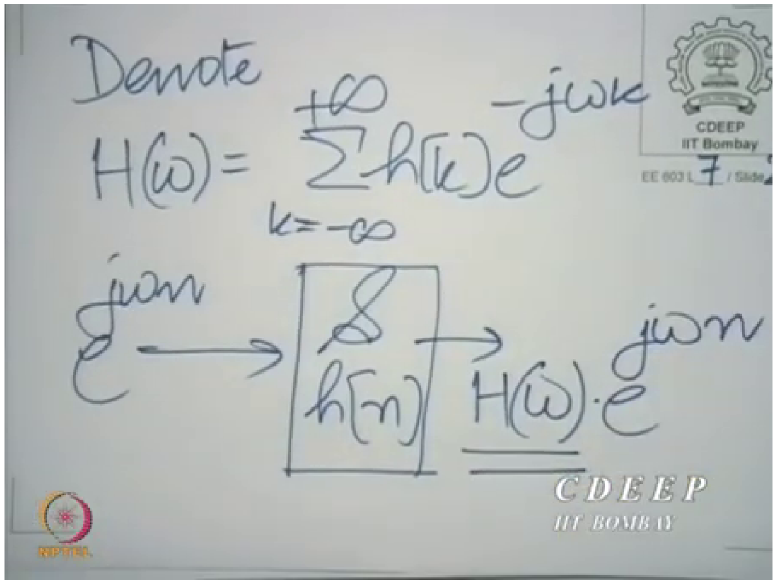

So, let me denote this summation,

$$H(\omega) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

Where upon, what we have said essentially is that $e^{j\omega n}$ going into the system S with impulse response h, and has led to $e^{j\omega n}$ coming out but multiplied by a complex constant capital $H(\omega)$.

Please note that $H(\omega)$ is a complex constant. But of course, at the moment, we have kind of brushed something very important under the carpet. Let us look back at this expression here,

$$H(\omega) = \sum_{k=-\infty}^{\infty} h[k]e^{-j\omega k}$$

there is an infinite summation here.

Now, infinite summation is not guaranteed to converge. An infinite summation can diverge. In fact, this is a problem with convolution in general and we had brushed this issue under the carpet even the last time, we had conveniently ignored that issue altogether. That is because we had dealt with finite length sequences. So, convergence was never an issue. But in case our sequences happen not to be of finite length, there could be a problem in the summation, whether it is in the context of convolution, or it is in the context of this response that we see here.