Digital Signal Processing and its Applications
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Lecture 08 A
Review of Commutativity and Associativity

So a warm welcome to the 8th lecture on the subject of digital signal processing and its applications. Let us spend a couple of minutes recalling what we did in the previous lecture. And let us also take a couple of questions before we proceed to discuss further on the same theme. In the previous lecture, we had looked at the operation of convolution in its own right.

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So, let us just quickly review the previous lecture, we had looked at convolution as an operation in its own right. And we had seen that convolution is associative, and convolution is commutative. Now, a specific implication of these two properties is that because of the associativity of convolution, when I have two systems connected in cascade, they can be replaced by a single system with an impulse response given by the convolution of the impulse responses of the two systems in cascade that was a consequence of associativity

The consequence of commutativity was that you could interchange the order of the two systems, LSI systems of course, without affecting the input output relationship. That is right. So, these were of course specific implications of those properties of associativity and commutativity, on linear shift invariant systems. But we could look at these properties as properties of the operation convolution in its own right. You do not have to always interpret convolution in the context of linear shift invariant systems.

In fact, convolution is a very important operation in the context of multiplication of two numbers. I leave it to you as a challenge to come out with a connection between convolution and multiplication of two numbers represented in any, in any radix for example, the decimal radix or the binary radix.

That means numbers represented in binary, numbers represented in decimal, numbers represented in hexadecimal base 16 or numbers represented in octal, these are challenges, come out with a relationship between convolution and multiplication of two numbers with respect to any radix, any base.

Anyway, so much, so we saw that because of the associativity and commutativity of convolution, where I have a connection, cascade interconnection of several linear shift invariant systems, you may reorder those systems in any way without affecting the input output relationship.

In fact, I left it to you as an exercise to use commutativity and associativity together to prove this, but you could rearrange a cascade interconnection of LSI systems in any way without affecting the input output relationship. But you see, you must remember this is true only for linear shift invariant systems. If even one of those systems in the cascade connection is either not linear or not shift invariant, this cannot be said with certainty.

In fact, I also leave it to you as an exercise to construct examples of systems which are not linear and shift invariant that means they disobey one of the three properties, what are the three properties that additivity, homogeneity and shift invariance where they disobey one of the three properties and where when you connect them in cascade and interchange is not permissible. I leave it to you as an exercise to construct a few such examples, where such an interchange is not permissible. Is that correct?

Now, we also looked at the consequence of convolution or the convolution relationship between the input and impulse response in the context of the excitation of a linear shift invariant system with sinusoids or more appropriately with complex exponentials with rotating phasors. So, we concluded yesterday that if I excited the linear shift invariant systems, this was a part of the review again yesterday.

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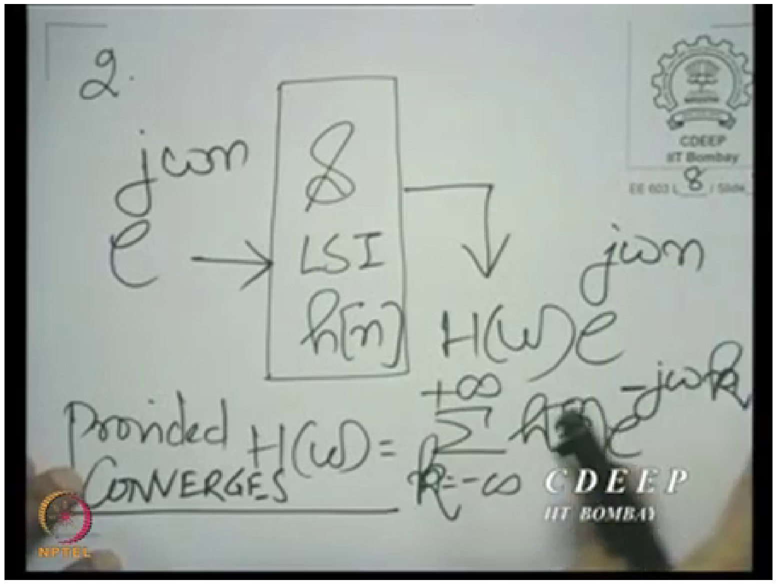

If we excited a linear shift invariant system of impulse response h[n] with a phasor $e^{j\omega n}$ what emerges is $H(\omega)e^{j\omega n}$, provided $H(\omega)$ convergence and $H(\omega) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$, of course, you can always make the index of summation k here if you like so, I can do that let us make it k, provided this converges and that was the million-dollar question, when would it converge? We were looking at that we said that a sufficient condition for this to converge is that the impulse response be absolutely summable.

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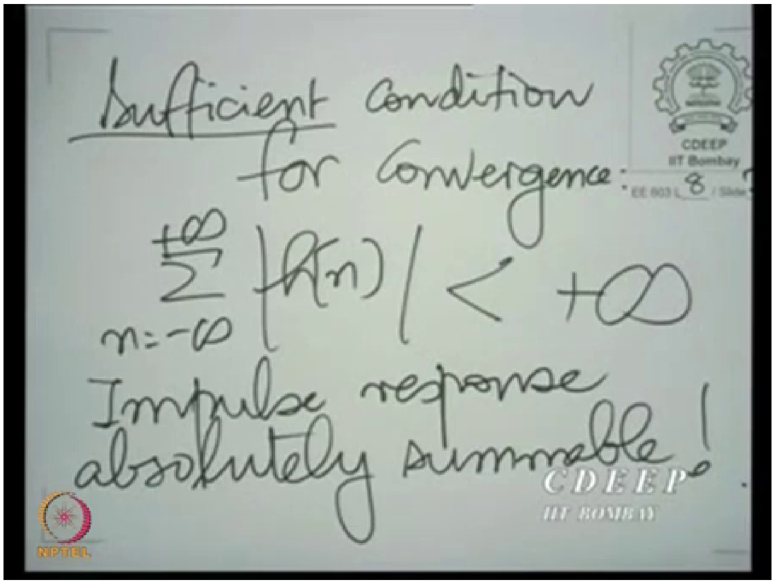

Is that $|h[n]|$ when summed over all is finite impulse response is absolutely summable. We were trying to give this a further interpretation. We said this impulse response thing is absolutely summable has a greater implication on input output behavior. We showed that it ensures that if you give the system a bounded input. By a bounded input, we mean an input each of whose samples can be upper bounded by a finite positive number, a finite nonnegative number.

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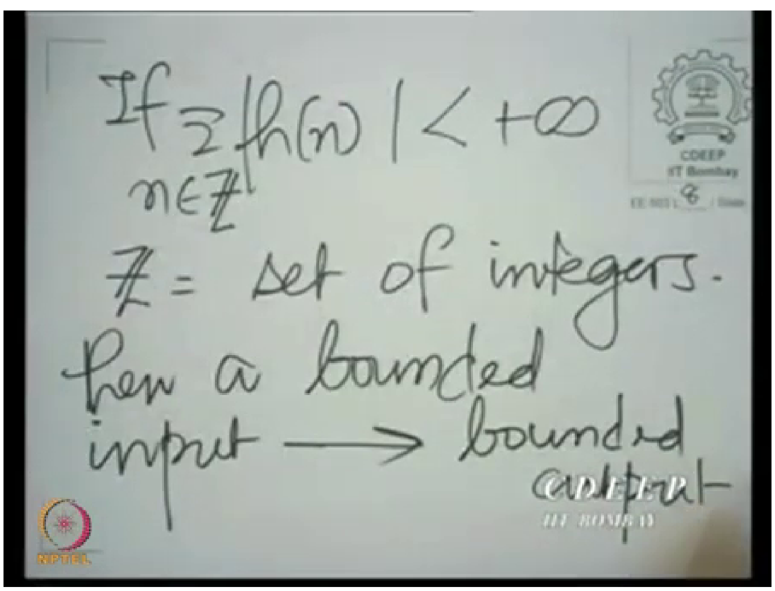

So, we said if this condition, if mod h n summed over all integer n, now, I am going to introduce a notation ℤ shall henceforth refer to the set of integers. And therefore, when we write summation $\forall\, n \in \mathbb{Z}$, we mean summation of n going from $-\infty$ to ∞ or summation over all the set of integers $|h[n]|$.

So, if $\sum_{n\in\mathbb{Z}} |h[n]| < \infty$, then we said a bounded input results in a bounded output, a bounded input leads to a bounded output and we have defined the bounded input, we had said the bounded input is one. Each of those samples can be upper bounded by a non-negative number. We call that nonnegative number M_x, the input x.