Digital Signal Processing and its Applications
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Lecture 09 A
Introduction and Recap of Stability

A warm welcome to the 9th lecture on the subject of Digital Signal Processing and Its Applications. In the previous lecture we had put down the properties of systems and we had also just begun our discussion of going into an alternate domain.

We said that if we look at the expression which relates the input to the output, with an input equal to a complex exponential in an LSI system, the relation involves a constant which depends only on frequency. We saw that the constant could be viewed as an operation between the impulse response and the sequence $e^{j\omega n}$, and we had concluded the previous lecture by noting that we could perhaps give it an interpretation which relates to projection of the impulse response or taking a dot product of the impulse response in some way. So we now need to make these ideas more concrete. That is the objective of the lecture today, and also to take us to the first transform domain that we are going to discuss. In this lecture we shall also identify what we mean by a transform domain in general, and in particular we shall be talking about the discrete time fourier transform.

Student is questioning: The process of selecting the troublesome input is a bit confusing. We are choosing some troublesome input which gives rise to some transients like step input and others, which is also a bounded input… *student continues the question, but it is not clearly audible.*

Professor: Okay, so the question is that in the previous lecture we had looked at the proof of the necessity and sufficiency of the condition for stability, that is if an LSI system is stable then its impulse response was necessarily absolutely summable.

We had proved sufficiency with great ease but to prove necessity we needed to use what we called a troublesome input. So the question is that the choice of troublesome input and the reason why we proceeded with that input was not very clear. So let us quickly identify the main steps and the main reasoning that went into the so-called troublesome input.

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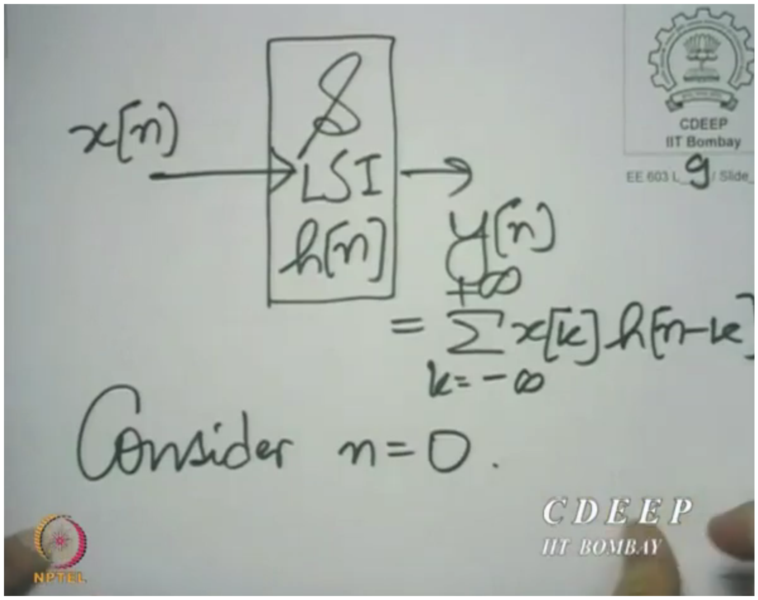

This is essentially a review and clarification of points from the previous lectures. The first point is regarding the necessary condition for stability. What we did was to take an LSI system with impulse response $h[n]$. We gave an arbitrary input $x[n]$ and noted that the output $y[n]$ in general is:

$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\, h[n-k] = \sum_{k=-\infty}^{+\infty} h[k]\, x[n-k]$$

In particular we said consider $n = 0$.

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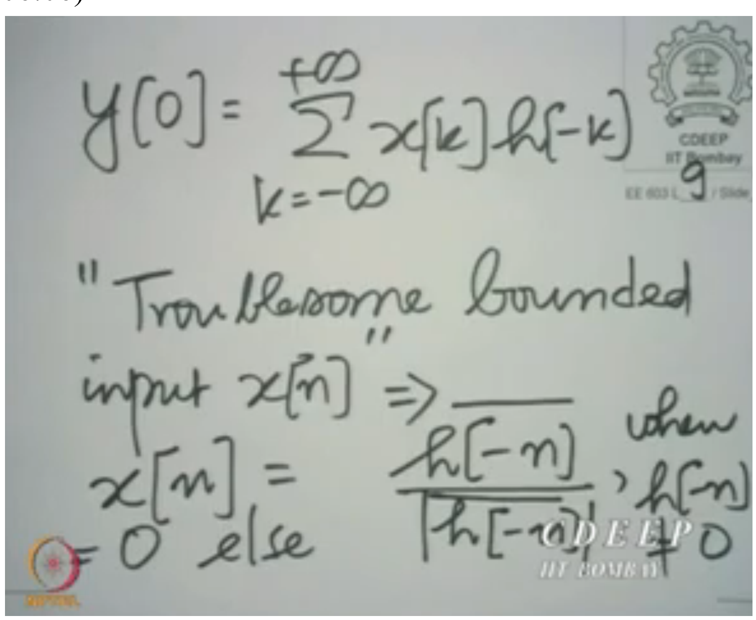

$$y[0] = \sum_{k=-\infty}^{+\infty} x[k]\, h[-\, k]$$

Now our reasoning was that we can choose a troublesome bounded input $x[n]$ and we chose that troublesome bounded input as:

$$\begin{aligned} x[n] &= \frac{\overline{h[-n]}}{|h[-n]|}, \text{ if } h[-\, n] \neq 0 \\ &= 0, \qquad \text{if } h[-\, n] = 0 \end{aligned}$$

Now, one point which perhaps may not have been so clear in the previous lecture is that this input is actually bounded, maybe that point was not emphasized enough. In fact, the modulus of $x[n]$ for any n is 1.