Digital Signal Processing and its Applications
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Lecture No. 09 C
Interpretation of Frequency Response as Inner Product

Now, let us interpret the so-called frequency response that we wrote the last time in the language of the dot products.

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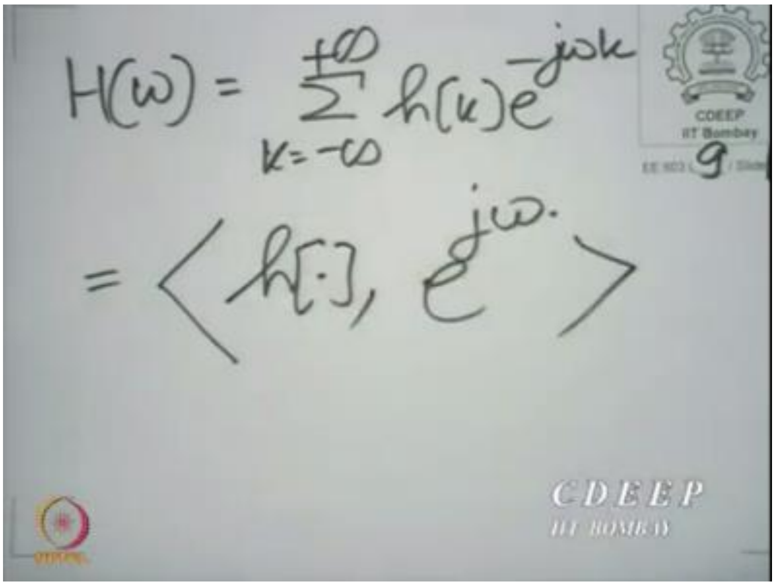

So, we have H(ω) that we wrote the last time is $\sum_{k=-\infty}^{+\infty} \quad h[k]e^{-j\omega k}$ it is obvious that this is the dot product of the sequence h[n] with a sequence $e^{j\omega n}$. Now, this dot denotes the independent variable in the sequence. So, when we write a dot like this what we mean is that we are treating the whole sequence as an object.

We are not taking an individual sample, otherwise if you write n, we could write n if we are careful, but when we write n the tendency is to think of the n^{th} sample only and here we are not talking about a specific sample we are talking about the whole sequence as an object that is very important and that is why we put a dot there. More than the question of notation it is the philosophy behind this that matters, that is why we have to be careful with notation.

It is not so important that we should be very fussy about notation all the while, but the thought behind the reasoning should not be lost. So, when we take a dot product it is not enough to look only at a part of the sequence, a dot product involves the whole sequence that is what is being emphasized here.

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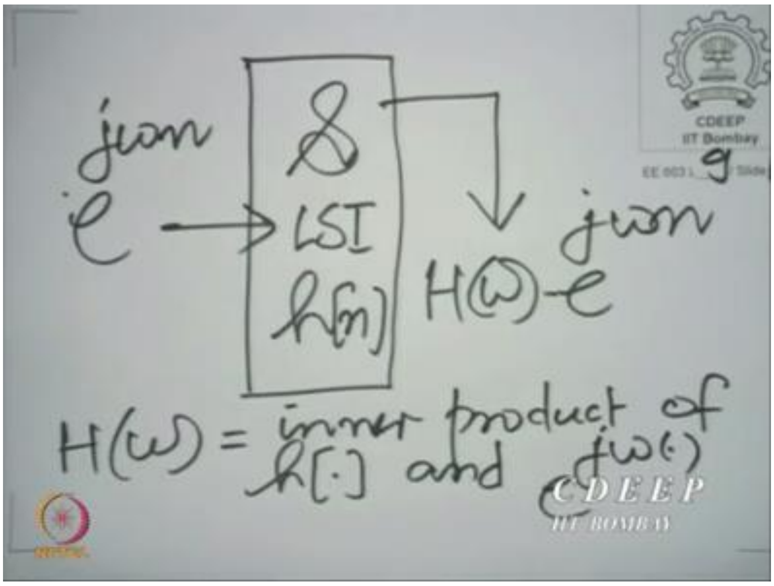

Now, you see it is a very interesting interpretation that we have just given this whole concept of frequency response. What we said is that you have this LSI system S with an input $e^{j\omega n}$ so we have given a complex exponential at the input. You get $H(\omega)e^{j\omega n}$ coming out and $H(\omega)$is essentially the inner product, the dot product or the inner product of h[n] and $e^{j\omega n}$.

Now, we already know what an inner product does in conventional small dimensional physics or engineering. When you take the dot product I told you of one force in a certain unit vector; in a direction chosen so if you have chosen a certain direction and if you take the dot product of a force vector with a unit vector in that direction what you have done is to resolve the force in that particular direction, you find the component of the force that acts in that direction. Now, what we are saying here is that $H(\omega)$ is like the component of the impulse response that acts in the direction of $e^{j\omega n}$. In fact it is an exact interpretation, it is not just very vague or approximate, it is exactly what we are saying. We are saying that if you give $e^{j\omega n}$ to an LSI system, and if $H(\omega)$ converges that means if the $\sum_{k=-\infty}^{+\infty} h[k]e^{-j\omega k}$ converges, then this quantity $H(\omega)$ is like the dot product of the impulse response in the direction of that complex exponential. The dot product is a number, it is a complex number and the physical interpretation of that complex number is that the magnitude of that complex number multiplies the amplitude of the complex exponential and the angle of the complex number adds to the phase of that exponential.