Digital Signal Processing & Its Applications
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Lecture No. 10 c
Definition of sequences and properties of DTFT

Now, we need to see a few more properties of the discrete time Fourier Transform. You see we now need to take a minute to see where we are in our whole strategy of dealing with signals and systems, discrete systems and discrete sequences.

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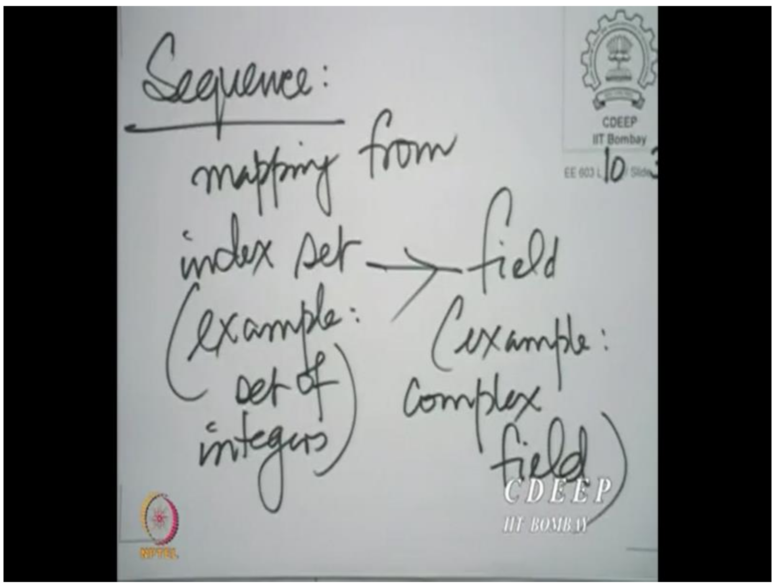

A sequence was a mapping. From an index set. So, it could be a set of integers. To a field actually here the field of complex numbers. So, typically we are talking about the index set of integers and the complex numbers as a field. We also appreciate why we have introduced complex numbers.

Because we do want to have rotating complex numbers. And physically we sometimes need to deal with electromagnetic fields where it is easier to deal with complex phasors to represent the fields. And there are many other situations where it is a natural choice to use the complex fields to deal with the situation anyway. This is a sequence. What is a system? What is a discrete system?

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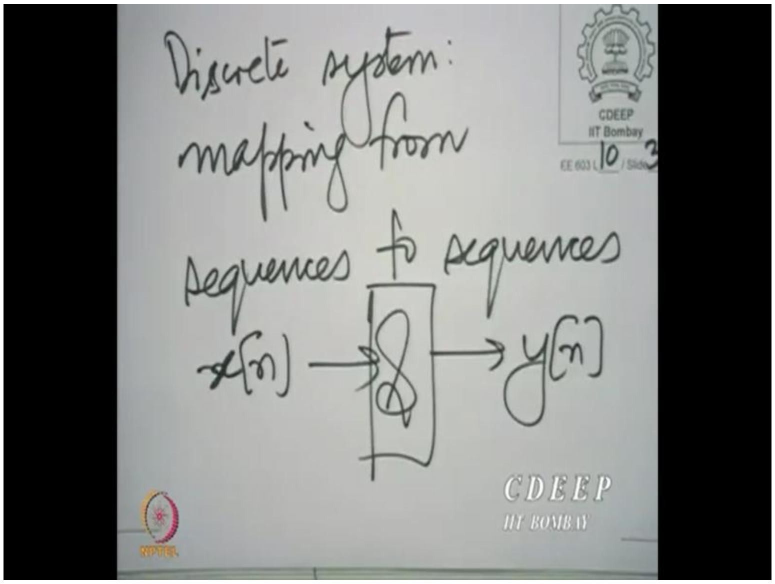

A discrete system, a discrete system is a mapping from sequences to sequences. So, you have a system where you have an x[n] input sequence an output sequence y[n] and this mapping between sequences, it is important to understand a system as a mapping from sequences to sequences.

Of course, if the system is more or less then it also becomes a point wise mapping. But it need not be a point wise mapping you must think of it as a mapping of the whole sequence x[n] to the whole sequence y[n]. And you do this for all the sequences, which are permissible inputs. Now, we have come to a transform. What is the transform?

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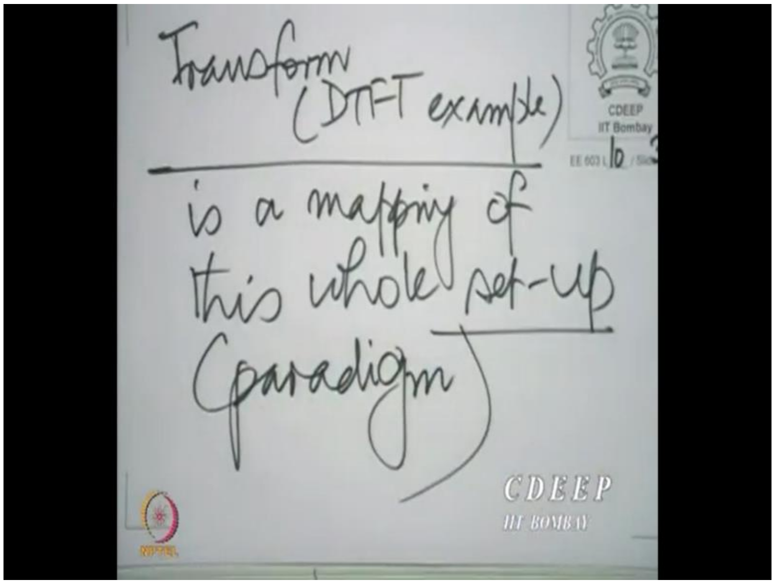

A transform at least the discrete time Fourier transform is an example even mapping of this whole paradigm of this whole setup, or this whole whole paradigm world view. It is a different way of viewing the world of sequences and systems.

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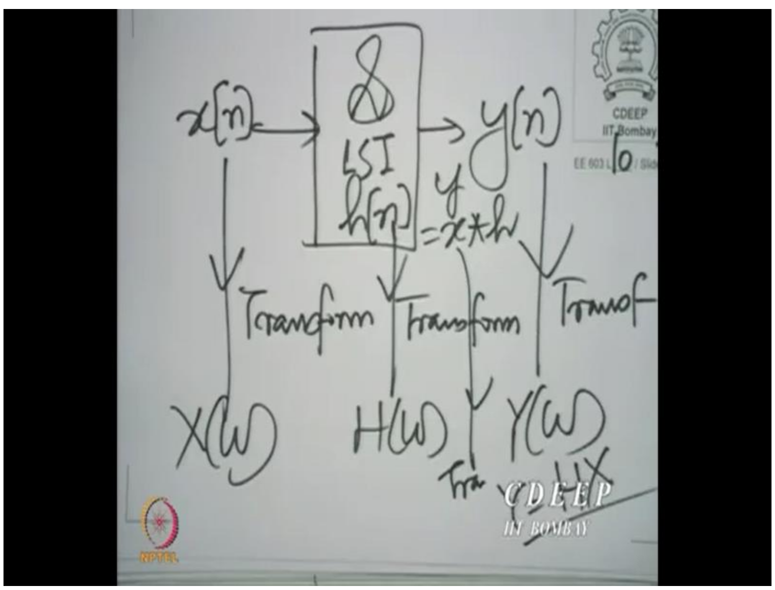

What I mean by that is when we take, for example, the LSI system. The transform acts on this, the transform acts on this and the transform acts on this to give you $X(\omega)$, $H(\omega)$ and $Y(\omega)$ here and the relation between x[n] and h[n] and y[n] gets transformed into a different relationship between $X(\omega)$, $H(\omega)$ and $Y(\omega)$.

So, relationship also goes under a transformation. y is equal to x convolved with h get transformed to Y is equal to H times X. And of course, this relationship is much more elegant. So, transform domain often offers us the possibility or the potential to deal with that same setup in a much more efficient manner.

The discrete time Fourier Transform is a strong example. We shall see another transform after a while. The discrete time Fourier Transform is admissible, only when x, h, and y all of them have discrete time Fourier Transform, that means physically you could think of it, that the dot product of the input, the inverse response and the output on each of those rotating phases e to the power j omega n converges for all those omegas between minus $-\pi$ andπ .

And you also see the transform in the inverse transform. This transform is invertible. You can go from the sequence to the transform and come back form the transform to the sequence. Now, what we need to do is to look for a few properties of this discrete time Fourier transform. Of course, we have derived one property right away. We have seen that when we convolve two sequences that discrete time Fourier Transform are multiplied, if the DTFT of the convolution converges. But now let us focus on the DTFT of one sequence and make some transform and make some changes to that sequence. So, let us begin with some properties.

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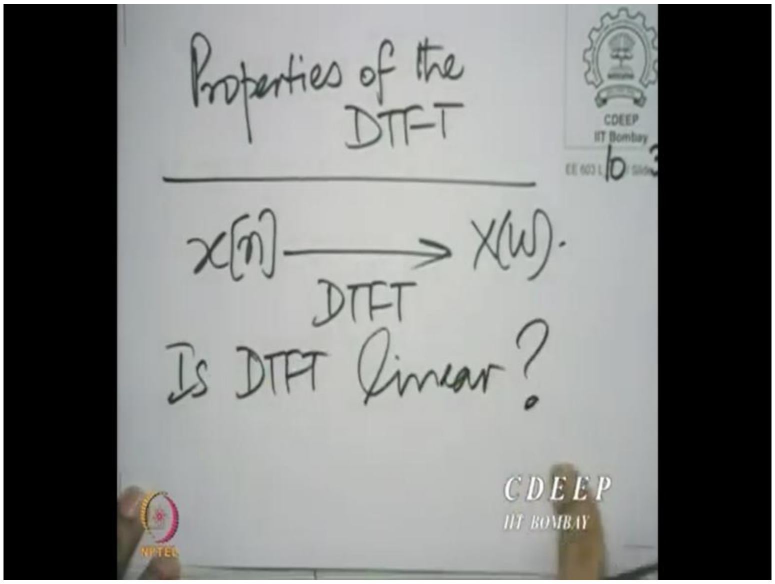

You see the DTFT can also be thought just as we thought of convolution as an operation in its own right. You can think of the DTFT as an operation in its own right. So, x[n] is operated upon by the DTFT to produce x omega. Of course, here the operation takes you from one independent variable to a different independent variable. So, in a transform the independent variable changes. And we say that the domain has changed from the actual domain to the transform domain.

In this case, the transform domain is the frequency domain and the physical interpretation of the frequency domain is it corresponds to the set of angular frequencies, normalized angular frequencies of the phasors that come into picture when that sequence needs to be constructed. Anyway, treating this as an operation in its own right we can ask is the DTFT linear in other words.

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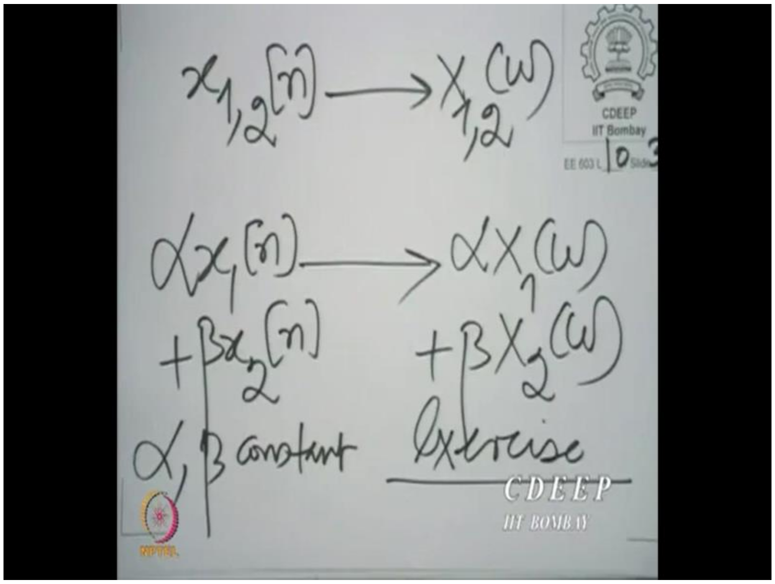

If I took $x_{1,2}[n]$ respectively, and had their DTFTs, $X_{1,2}(\omega)$. What will be the DTFT of$\alpha x_1[n] + \beta x_2[n]$ for any constants α, β and the answer is very simple. It would be just $\alpha X_1(\omega) + \beta X_2(\omega)$. I leave it to you an exercise to prove this. It is very easy to do.

You just write down the expression for the discrete time Fourier transform of this linear combination. It is easy to prove. So, the discrete time Fourier transform is a linear operator. What happens when we complex conjugate a sequence or before that, let us make life a little easier by doing something slightly simpler.

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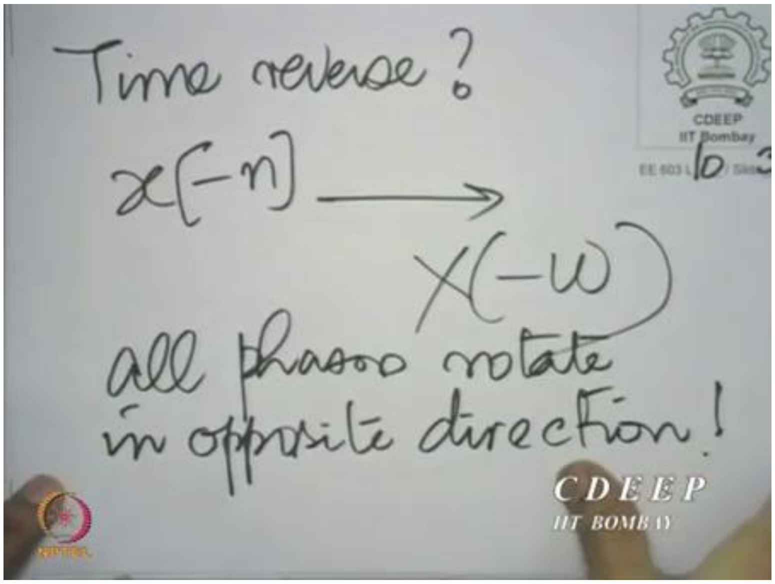

What happens when we time reverse. So, If I take $x[-n]$ into what I do expect will happen. Well, now you know here again we can answer the question by inclusion first and then prove it algebraically. If you time reverse. What happens when you time reverse a rotating phasor. If you time reverse a rotating phasor it rotates in the opposite direction. And therefore, if you essentially rotate all the phases in the opposite direction you get the time reverse sequence what it means is $x[-n]$ should lead us to $X(-\omega)$. All phases rotate in the opposite direction. But let us prove it algebraically. Let us also prove the same algebraically.

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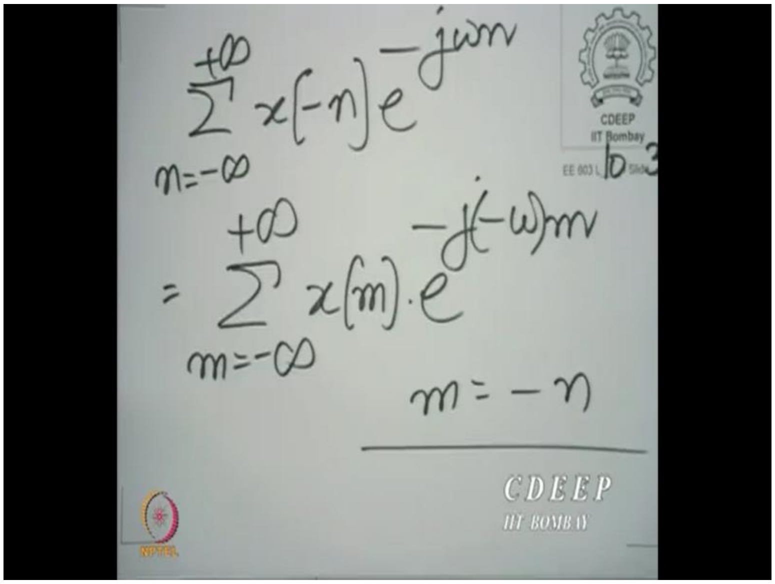

Indeed, $\sum_{n=-\infty}^{+\infty} \quad x[-n]e^{-j\omega n}$ can be rewritten as $\sum_{m=-\infty}^{+\infty} \quad x[m]e^{-j(-\omega)m}$. where m is minus n. You see it is very easy to see if you put m equal to minus n. when n runs over all the integers so does n, is that right. And this is simply x of minus omega. So, we have proved. We need to see several such other properties of the discrete time Fourier transform and we shall do so in the next lecture. Thank you.