Digital Signal Processing and Its Applications
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Lecture No. 11 a
Review of properties of DTFT, IDTFT

So, warm welcome to the 11th lecture on the subject of digital signal processing and its applications. We recall that the previous lecture was devoted to an introduction and a discussion on the discrete time Fourier Transform. Let us quickly recapitulate one or two important conclusions that we drew in the previous lecture and then proceed to look at some more properties of the discrete time Fourier Transform. We had seen that if you have an arbitrary sequence x[n].

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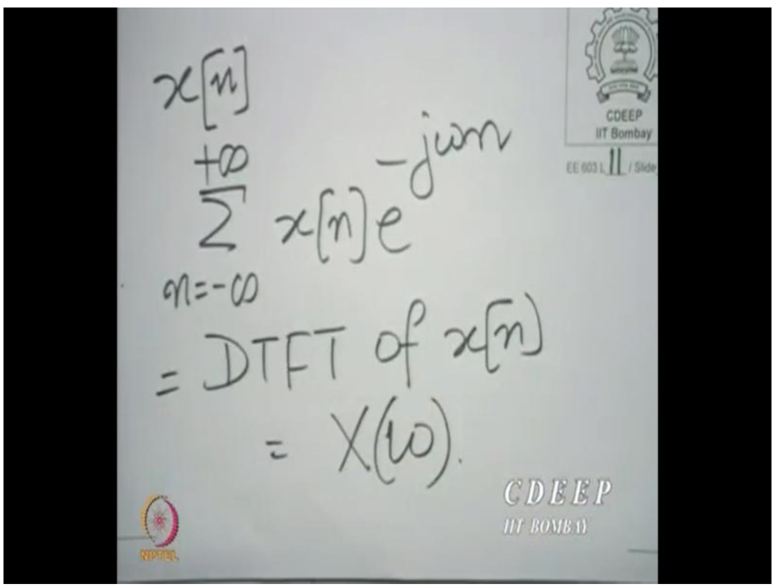

And if the summation n going from minus to plus infinity x[n] e raised the power minus j ω in converges, we call it the discrete time Fourier Transform of the sequence x[n] denoted by Capital X as a function of ω. Here, the dependence is on the variable ω, and you will recall that ω is a continuous variable. It can take on values continuously all over the real axis. However, there is periodicity in X(ω) right. So, you will recall also that we derived.

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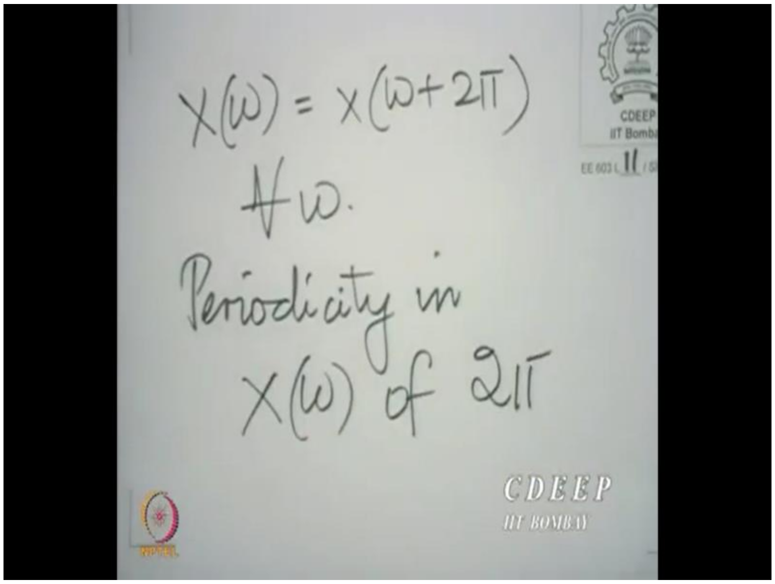

That X(ω) is equal to X (ω + 2π) for all ω. And therefore, there is a periodicity in X(ω) of 2π. We also saw that the prime interval over which X(ω) needs to be considered is the interval from -π to π. So, what we call the principal interval.

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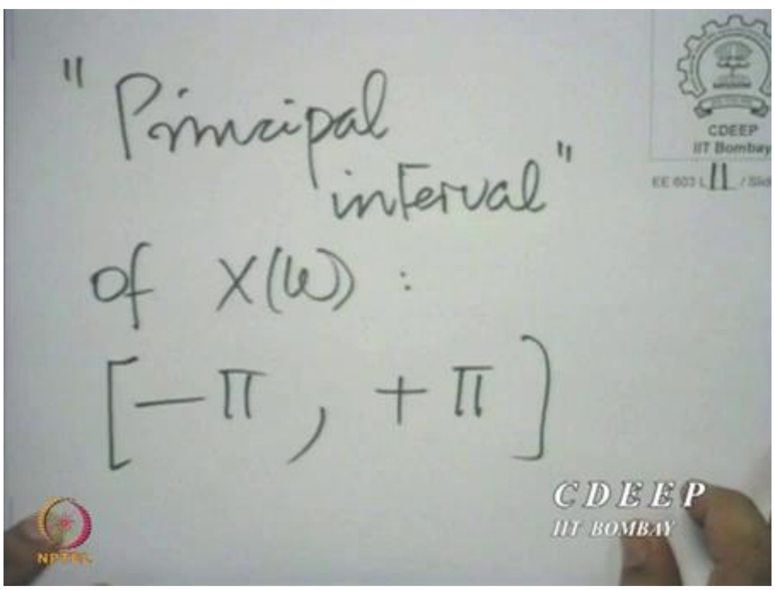

It is adequate for us to look at the discrete time Fourier Transform, over this principle interval of -π to π, right. Because it would then be repeated at every multiple of 2π. What is more is we had also seen the inverse discrete time Fourier Transform.

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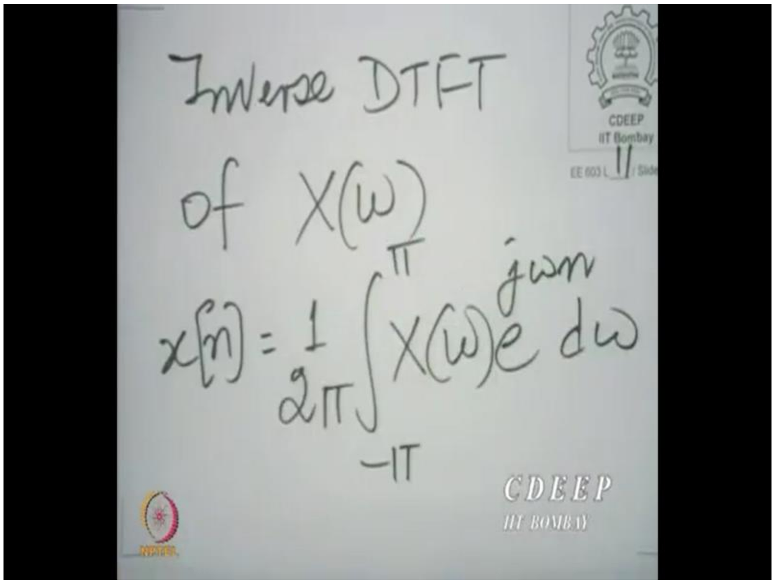

We had seen that you can reconstruct x[n]. It is from X(ω). In the following way. Further we had also looked at a very important property of convolution, we saw that if you convolve two sequences, each of which has a discrete time Fourier transform.

And if the convolution also has a discrete time Fourier transform. And there is a beautiful relationship between the convolution and the two sequences in the Fourier domain or in the frequency domain, namely when you convert two sequences their discrete time Fourier transforms are multiplied. All right.

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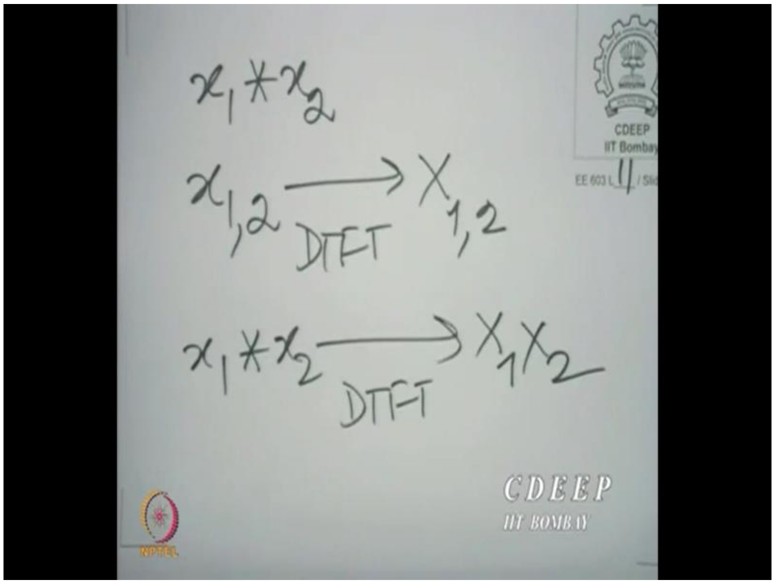

So, we showed that if x_1 is convolved with x_2. And if $x_{1,2}$ respectively, have the DTFT's Capital $X_1$and capital X_2, then X_1convolved with X_2has the DTFT x_1 into x_2 that is what we proved last time. We also started looking at some of the properties of the discrete time Fourier transform. We saw that the discrete time Fourier transform, viewed as an operator is linear right. So, we saw that. If you think of the DTFT as an operator.

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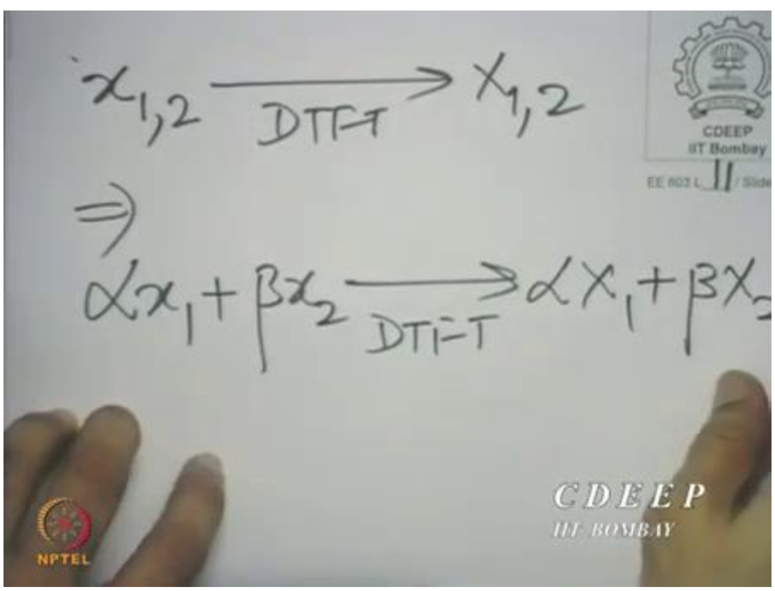

Then $x_{1,2}$ being operated upon by the DTFT to give capital $X_{1,2}$ implied that $\alpha x_1 + \beta x_2$ to have the DTFT $\alpha X_1 + \beta X_2$. What is more, we have also seen that if x had the DTFT capital X.

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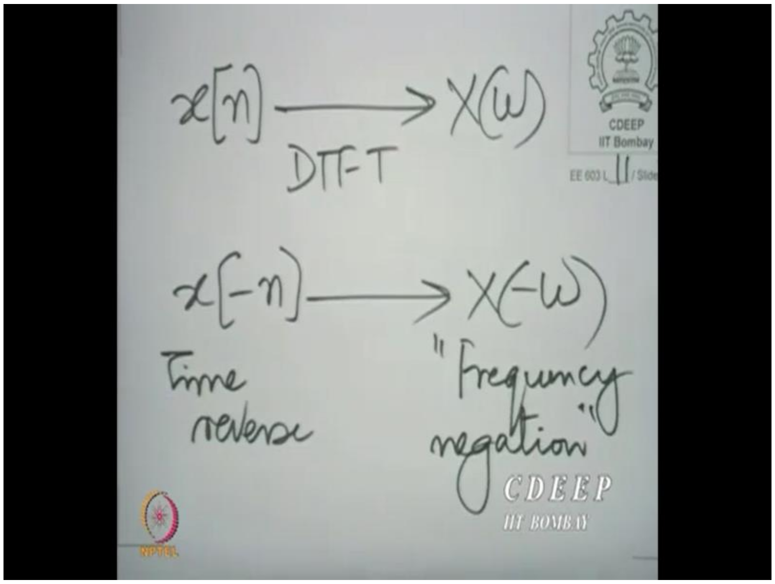

Then if we time reverse. It leads to a frequency reversal too. Or essentially frequency negation. That means every positive frequency becomes a corresponding negative frequency, and that is obvious because time reversal amounts to rotating in the opposite direction and therefore every ω is replaced by minus. Of course, we also came to the same conclusion algebraically.

Now, we take further the discussion of some properties, for example, we ask what happens when the complex conjugate? So, before that, let me ask if there are any questions at this point before we proceed. Are there any doubts or questions that need to be clarified before we proceed to discuss a few more properties? Yes, there is a question. That is a good question. So, the question is, we had said that, you know, the whole idea of reconstructing xn, in fact, let me go down to the inverse DTFT.

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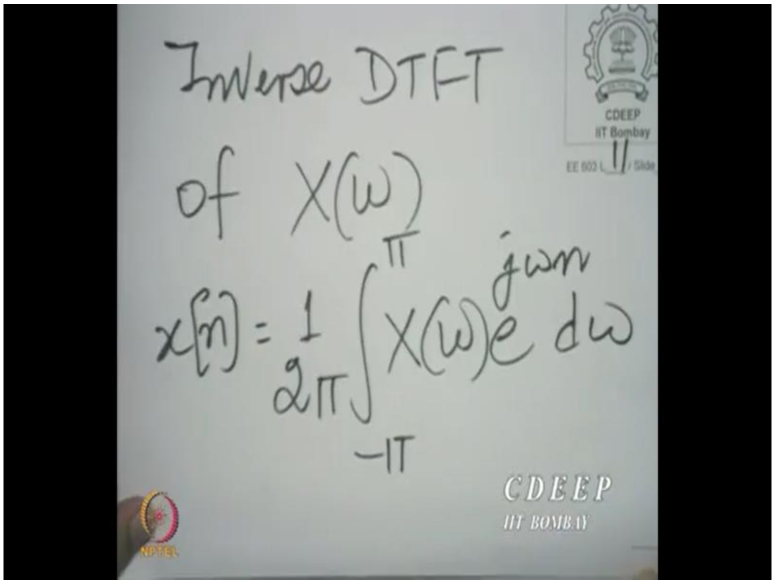

The question is, when we discussed this idea of inverting the DTFT, we said the idea is that you take each component, multiplied by a so-called unit vector in the direction of that component. And instead of adding now, you integrate overall components because these components are a continuum ω is a continuous variable.

And the question is, how could we all know, how could we talk of a unit vector in the context of $e^{j\omega n}$since $e^{j\omega n}$ does not have magnitude one of that matter, it cannot be made to have magnitude 1. Now let me clarify exactly what we are saying here. What we are saying is that when we use this idea of multiplying a component by a unit vector, it is of course true for a finite dimensional space.

But here we are talking about an infinite dimensional space so we can take inspiration from that idea, but we may not be able to use that idea exactly. There has to be a slight modification. And the modification is that here. What we assume happens is that, you see, we are assuming all the while, remember that X(ω) converges, that means there is an infinite summation that converge that converges that need not happen. So, under convergence I mean, under X(ω) converging.

What we are saying is all the information has been retained in the X(ω), ω going from -π to π and the approach that you would use to reconstruct a similar to what you would do for finite dimensional space that means take the component and multiply it by a so called unit vector in that direction and add overall such components.

Now, unfortunately here, multiplication of $e^{j\omega n}$by $\frac{1}{2\pi}$does not really make it a unit vector. But what we are doing here is akin to what you would do in a finite dimensional space, namely multiply a component by a unit vector and add over all such components.

So, the idea is similar, but the idea of a unit vector cannot be taken exactly from a finite dimensional space, to an infinite dimensional space here. Although what he is saying is that essentially $e^{j\omega n}/2\pi$really a unit vector its magnitude, or if you take the sum squared of the magnitude of all the samples, it does not converge.

But the idea that you can multiply components by vectors in different directions suitably normalized is being employed here. And what we did later was to, you know, to take inspiration from that idea, but to derive the factor 1/2π exactly by algebra. Later on, we used algebra to come exactly to the conclusion that you needed a factor of 1/2π there.

So, it is not correct really to call it a unit vector. Rather, it may be more, more appropriate to say that we take inspiration from that idea of multiplying components by unit vectors and adding overall components. But the conversion of a vector to a unit vector essentially involves normalization by a constant. We allow that normalization by a constant using the factor 1/2π here. That is the way to understand this.

So, when you go from finite to infinite dimensional spaces, there are certain patches of this kind which you need to deal with. So, for a greater or deeper understanding of this, I would recommend going into functional analysis. But that is not the objective of this course. What we are doing here is to take inspiration from that idea.

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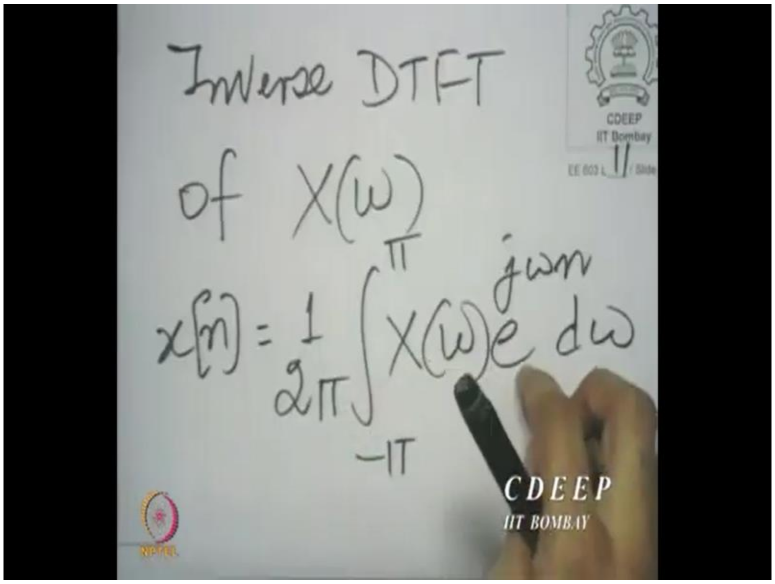

And we know that you probably would require a factor κ_0 here. We saw from algebra that factor turns out to be 1/2π. We later on prove this relation exactly. We later on derived this you know inverse exactly. And there we saw that the factor of 1/2π was required, right. That is the way to understand this.

So, in general, it is a good idea to use geometric insights to understand several signal processing ideas, but when using geometrical insights, one must be careful to distinguish between certain things that happen, obviously in finite dimensional spaces, which do not generalize exactly to infinite dimensional spaces.

But what you know, in finite dimensional spaces gives you an idea of what to expect in an infinite dimensional space. And you do not have to tune or to correct what you expect by looking at the basic algebra. Is there any other questions? All right, so in that case, we will proceed then with the discussion of some more properties of the discrete time Fourier Transform namely. Let us look at what happens when the complex conjugate.

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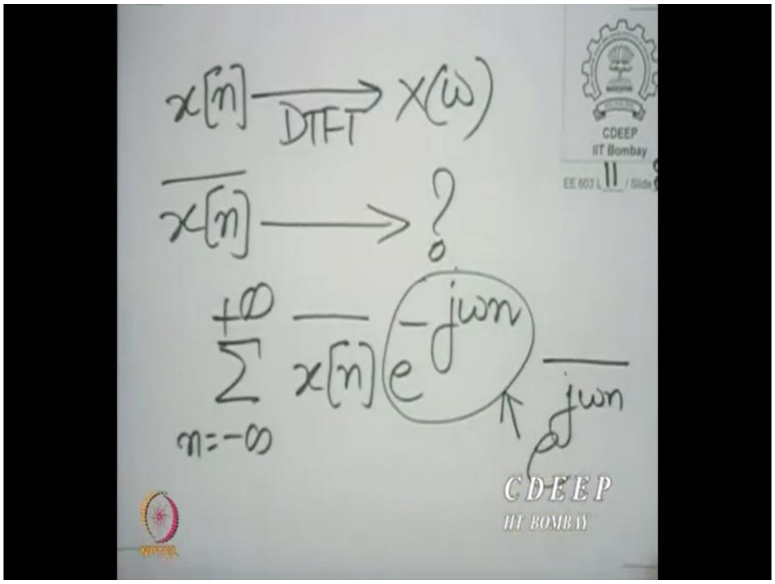

So, if you know that the discrete time Fourier transform of x[n] is capital X(ω), what is the discrete time Fourier transform of $\underline{x[n]}$is the question that we would like to ask. Now, that is easy to do. Let us calculate the discrete time Fourier transform of this it will be$\sum_{n=-\infty}^{+\infty} \underline{x[n]} e^{-j\omega n}$.

And of course, we can rewrite this. See, what we do is essentially we want to take the bar all above here. So, you know how would we do that? You could, of course, remove the minus sign and take the conjugate here. So, this is the same as $e^{\underline{j\omega n}}$ and therefore, I have.

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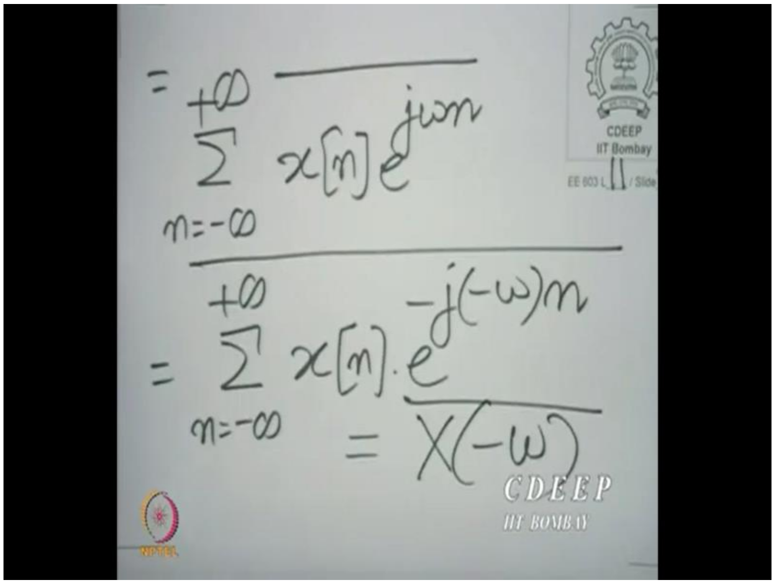

Now, of course, when you take a sum of complex conjugates, it is the complex conjugate of the sum. So, I have x[n]. Now here I can rewrite this as $\underline{\sum_{n=-\infty}^{+\infty} x[n]e^{-j(-\omega)n}}$, which turns out to be$\underline{X(-\omega)}$. So, complex conjugating without time reversal. Amounts to taking a negation on the frequency axis and complex conjugating once again. And this leads us to a very interesting conclusion.

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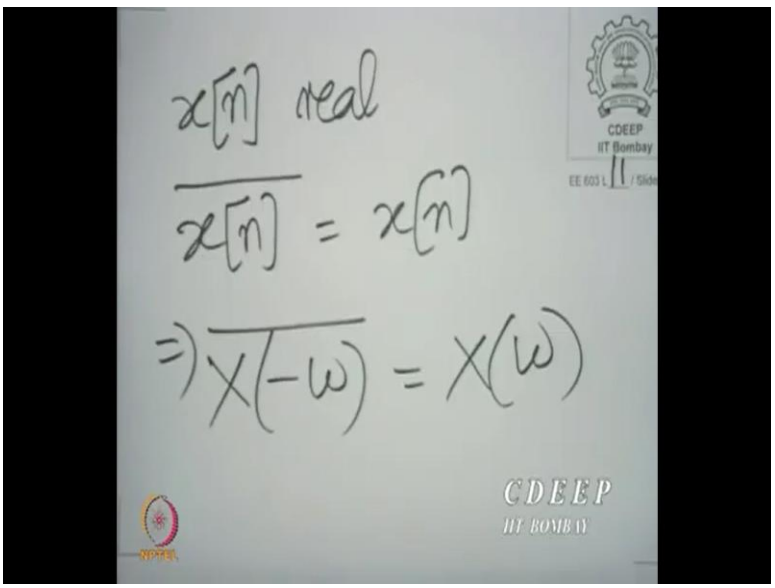

If I take x[n] to be a real sequence. Then $\underline{x[n]}$ is equal to x[n], that is what you mean by real, and that means $\underline{X(\omega)} = X(\omega)$. So, what we are saying is if you take corresponding frequencies, ω and minus ω, they are related by complex conjugation. That means on the ω axis, if I take corresponding positive and negative frequencies, the discrete time Fourier transforms are the complex conjugate of one another. Now, what is the physical interpretation of this? You see, if you look at how the signal is formed, so to speak, if you look at, look back at the inverse DTFT.

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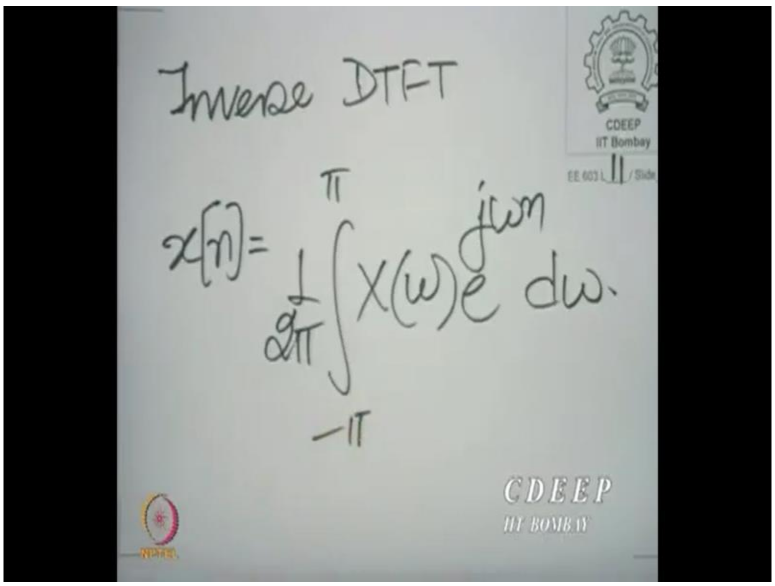

It says that $x[n] \ = \frac{1}{2\pi}\int_{-\pi}^{\pi} \quad X(\omega)e^{j\omega n}d\omega$. Now, of course we reinterpret this as before as saying that we take so many different ω's actually all the continuum of ω’s for ω going from -π to +π for each of these ω’s you can think of a pair of $\omega_0 + \omega_1$.

The other minus ω_0. They come together. So, X[ω_0] contributes $X[\omega_0]e^{j\omega_0 n}$ and $X[\omega_0]e^{j\omega_0 n}$ X(ω_0) contributes $X[-\omega_0]e\ -^{j\omega_0 n}$. So, you have a corresponding complex, rotating phasor, one rotating clockwise and the other rotating anticlockwise with respective frequencies ω_0 and $-\omega_0$. And what we are saying is that the corresponding components are complex conjugates. They have the same magnitude but the opposite phase or the opposite starting angle. In other words, what we are saying is.

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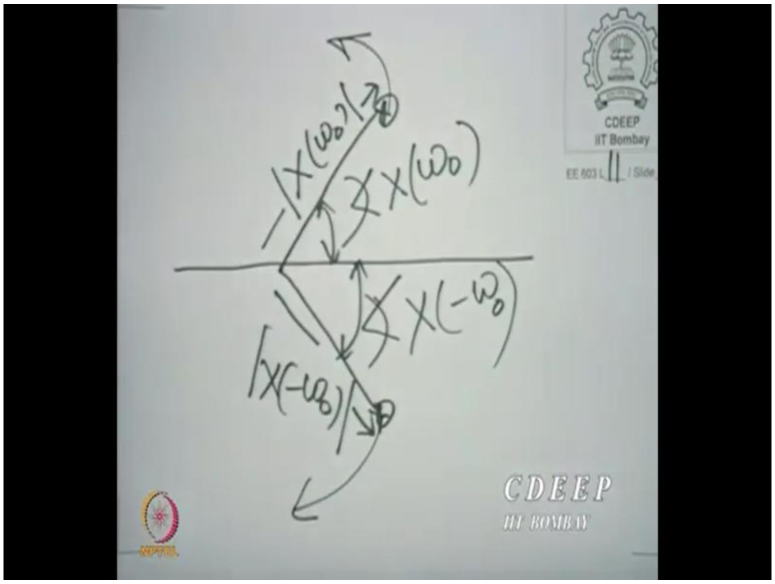

If you plot X(ω_0) and X($-\omega_0$). They would look like this. They have the same magnitude. And they have exactly the opposite starting angle. This is the symbol for. Angle or starting angle of phase. You can visualize. If there are two vectors if there are two rotating phases like this beginning here at t = 0 and here, and if this one rotates in a counter clockwise direction. And this one in a clockwise direction, they will always add up to a sinusoid, a cosine wave.

Their imaginary paths would always cancel and their real parts would enhance or be doubled. That is what we are saying here, so every property of the discrete time Fourier transform also has a corresponding interpretation that we must understand. It means every pair of corresponding positive and negative frequencies have the same magnitude, but opposite starting angle. And that is illustrated very clearly from how we see them coming together to form a real signal.