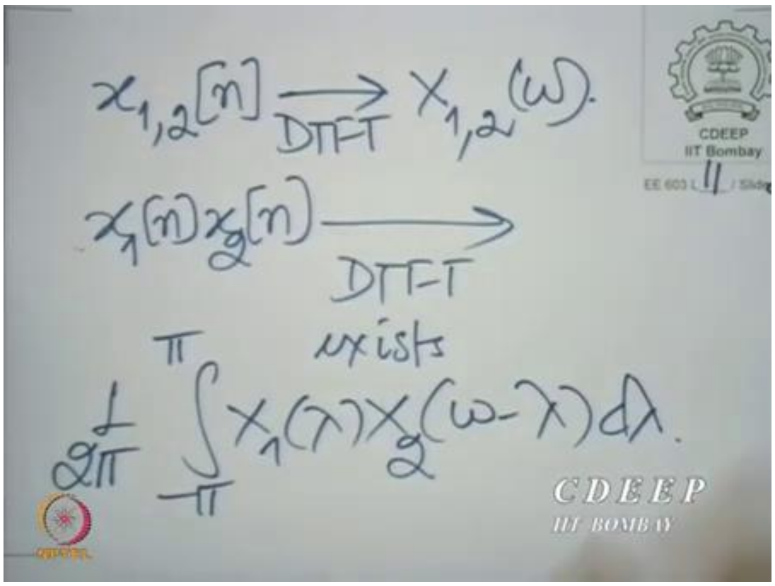
Digital Signal Processing and its Applications
Professor Vikram M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Lecture 11 c
Premise for Parseval's Theorem

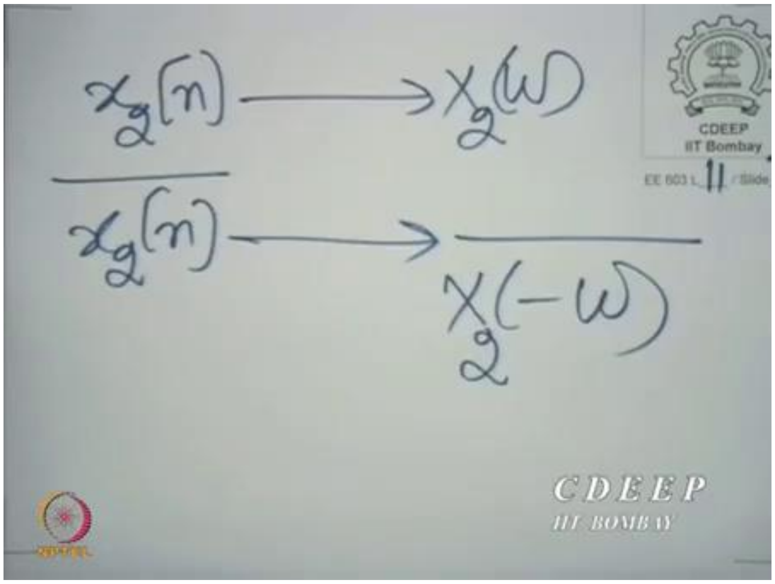
Anyway, as I have said let us come to one very important special conclusion of this so called multiplication principle and that is as follows. Let us look at what happens when we put ω = 0. So, we have $x_{1,2}[n]$with the DTFTs $X_{1,2}(\omega)$.

(Refer Slide Time: 00:48)



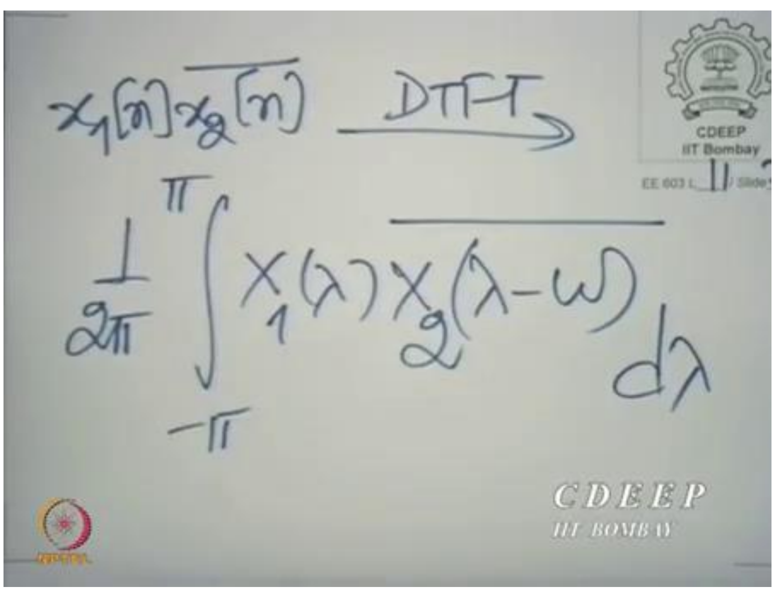
And we know that $x_1[n].x_2[n]$if it does have a DTFT it would be essentially $1/2\pi\int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega-\lambda)d\lambda$.

(Refer Slide Time: 01:34)



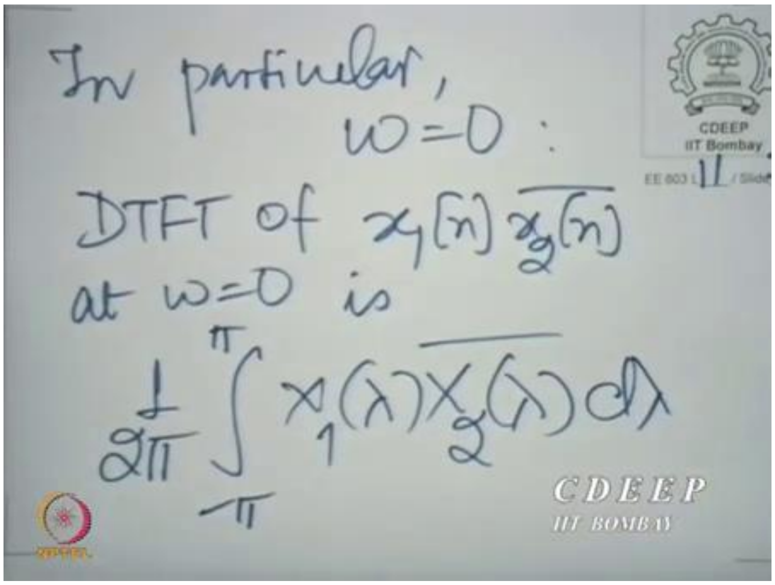
Now, let us in particular take x complex conjugate n you see. So, if $x_2[n]$ has the DTFT $X_2(\omega)$, what is the DTFT of $\underline{x_2[n]}$and we know what it is it is $\underline{X_2(-\omega)}$ and therefore, if you choose not to multiply $x_1$and $x_2$but $x_1$and $x_2$conjugate, what do we have?

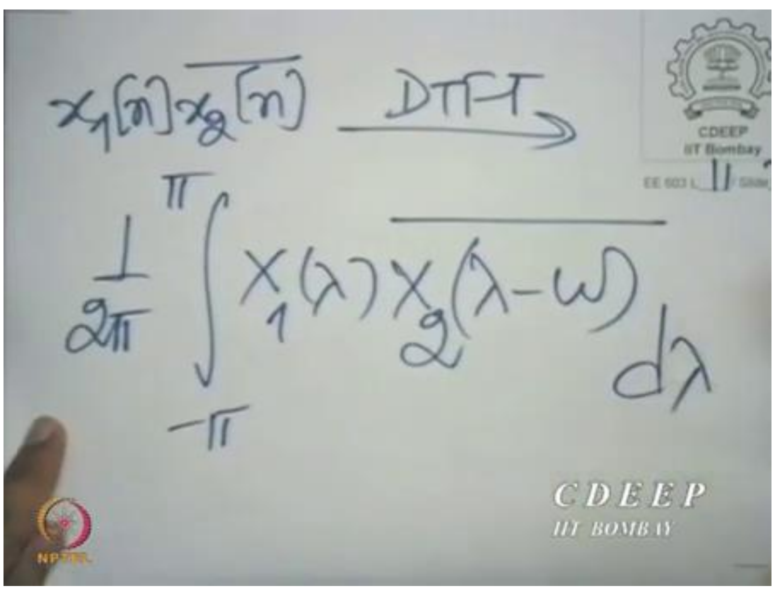
(Refer Slide Time: 02:03)



$x_1[n].\underline{x_2[n]}$would have the DTFT $1/2\pi \int_{-\pi}^{\pi} \quad X_1(\lambda)\underline{X_2(\lambda-\omega)}d\lambda$. Now, here you would have to write $X_2(\omega-\lambda)$, but it would be λ - ω complex conjugate dλ. Because you reverse or you change the sign of the argument and you also complex conjugate, that correct. Now, in particular, look at the consequence of this ω = 0.

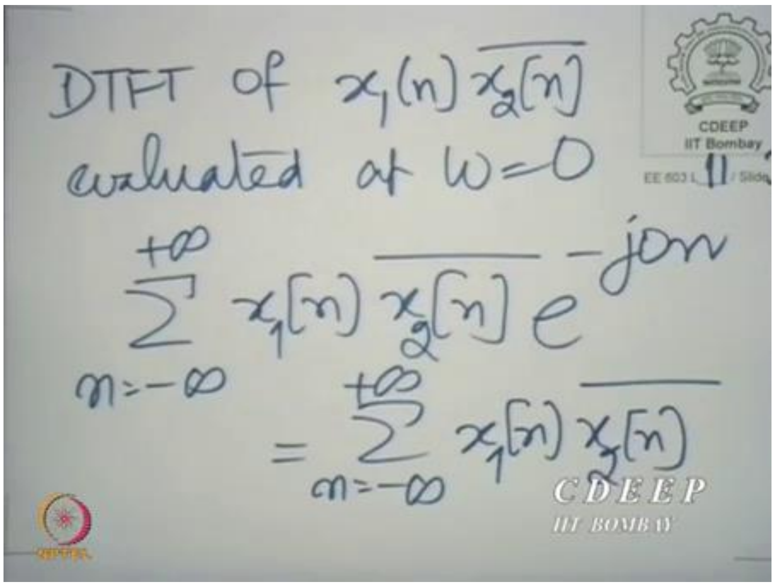
(Refer Slide Time: 02:47)





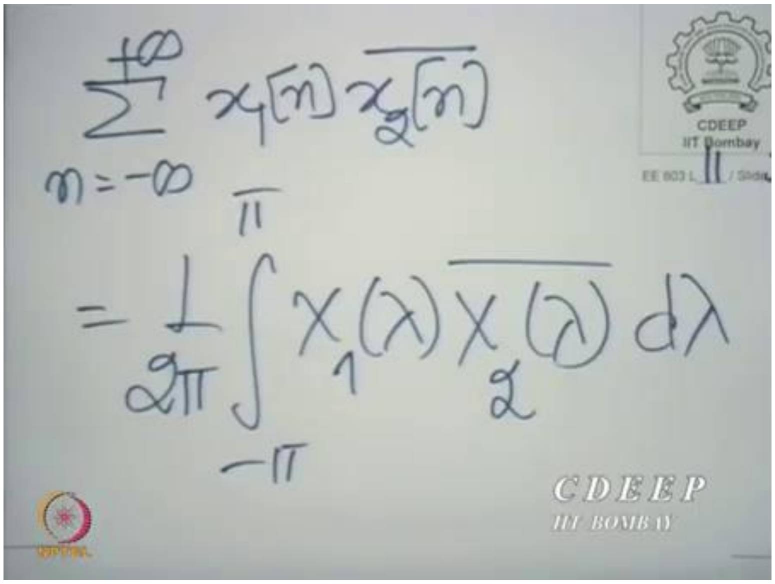
Look at the consequence of this at ω = 0, it says that the DTFT $x_1[n].\overline{x_2[n]}$ evaluated at ω = 0 is $1/2\pi \int_{-\pi}^{\pi} X_1(\lambda)\overline{X_2(\lambda)}d\lambda$ is not that true? Because in this expression all you are doing is putting ω = 0. Now, we have a very very interesting statement what indeed what do you mean by the DTFT of $x_1[n].\overline{x_2[n]}$ evaluated at ω = 0.

(Refer Slide Time: 03:52)



The DTFT of $x_1[n].\overline{x_2[n]}$ evaluated at ω = 0 is simply $\sum_{n=-\infty}^{\infty} x_1[n].\overline{x_2[n]}e^{-j0n}$ which is summation $\sum_{n=-\infty}^{\infty} x_1[n].\overline{x_2[n]}$. And this is known to us we are familiar with this. This is our familiar inner product of the sequences $x_1$and x_2. Very interesting result. Why so interesting? Let us put them down together then we will see why it is so beautiful.

(Refer Slide Time: 04:54)



The same solution n going from minus to plus infinity $x_1[n].\underline{x_2[n]}= 1/2\pi \int_{-\pi}^{\pi} X_1(\lambda)\underline{X_2(\lambda)}d\lambda$. Now, notice the similarity in these 2 sides, the left hand and the right hand side. Similarity in spirit, if not exactly an expression.

What do I mean by the similarity in spirit? Look at the left hand side, on the left hand side, we are saying multiply corresponding samples. But of course, the second 1 is complex conjugate an add over all sub samples. On the right hand side, we are saying multiply corresponding points on the frequency axis, but the second point is complex conjugate. On the left hand side, because we are dealing with a discrete independent variable, we are saying add.

On the right hand side because we are dealing with a continuous independent variable we are saying integrate. On the left hand side because we are dealing with an independent variable, which runs all the way from minus to plus infinity, the same add from minus to plus infinity. On the right hand side because we are dealing with periodic functions, so the discrete time Fourier transform, we are integrating only over the principal period.

But on both sides, we are multiplying corresponding components the corresponding points are the 2 functions. The second one is complex conjugated, and we are integrating on the right hand side adding on the left we are putting them together we are putting together all such sums. In other words, we are bringing out an equivalence between 2 inner products. And now we need to reflect for a moment on this. In fact, we shall we have seen something very interesting in this lecture and we shall dwell more on this as we begin the next lecture. Thank you.