Digital Signal Processing and its Applications
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Lecture 12 a
Review of DTFT Convolution

So, warm welcome to the twelfth lecture on the subject of Digital Signal Processing and its Applications. We continue in this lecture with our discussion of the Discrete Time Fourier Transform its properties. And we also establish examples of some Discrete Time Fourier transforms, we also establish when we do not have a Discrete Time Fourier transform, and that will lead us to another transform that is more generalized.

So, lets recall the property that we had briefly discussed in the previous lecture, namely, where we looked at the multiplication of 2 sequences and find out what happened in the Fourier domain. We also saw something very interesting when we evaluated that expression at 0 frequency, and we were discussing that and coming to some conclusions from that point. So, let us recall what we were doing the previous time.

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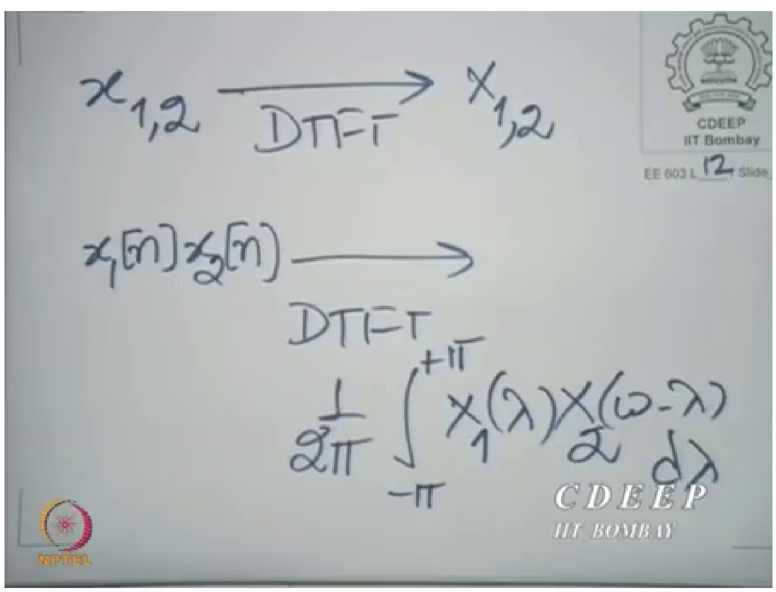

We saw that if you have 2 sequences $x_1[n]$ and $x_2[n]$, both of which have Discrete Time Fourier Transforms $X_1(\omega)$ and $X_2(\omega)$, then $x_1[n]x_2[n]$ has the discrete time Fourier Transform $\frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)X_2(\omega - \lambda)d\lambda$ which is essentially like a convolution of $X_1(\omega)$ and $X_2(\omega)$ save for the fact that this convolution is evaluated only on one period. I must here emphasize that this is commutative. So, of course, $x_1[n]x_2[n]$ is the same as $x_2[n]x_1[n]$ that is obvious.

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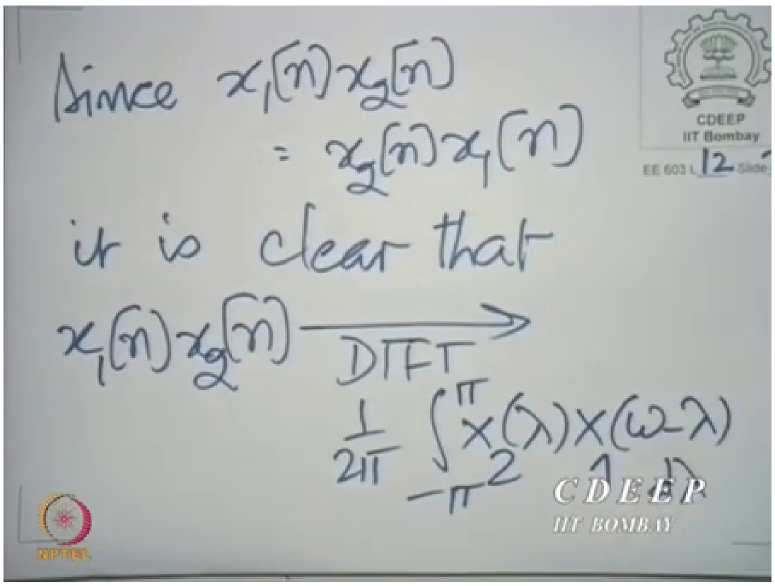

It is clear that this can also be $x_1[n]x_2[n]$ also has the DTFT $\frac{1}{2\pi}\int_{-\pi}^{\pi} X_2(\lambda)X_1(\omega - \lambda)d\lambda$. So, it does not matter which one you fix and which one you move. This is not a very important observation when you have very similar DTFT. But there are situations where it might be preferable to fix one of the DTFT and move the other and therefore, you have full freedom which one to fix and which to move, that is what is being emphasized.

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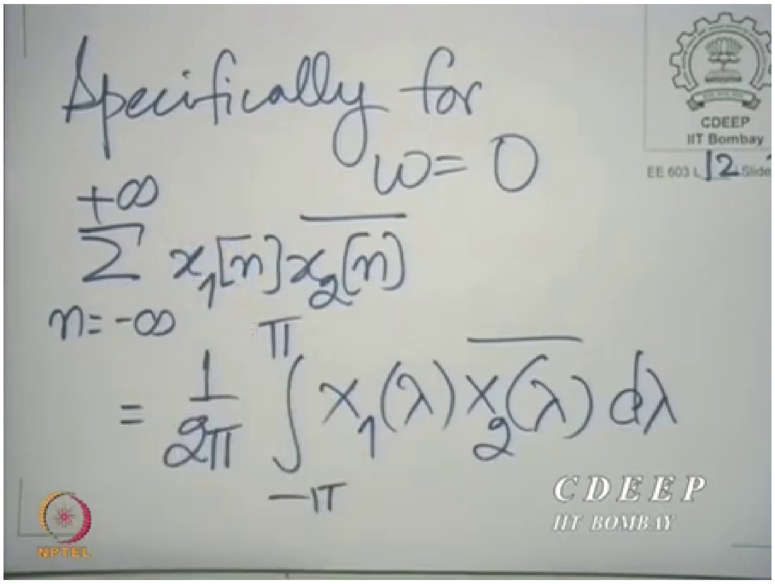

Anyway, we have taken the specific case of omega equal to 0 and we came to a very interesting conclusion, we said $\sum_{n\in\mathbb{Z}} x_1[n](x_2[n])^* = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\lambda)(X_2(\lambda))^* d\lambda$ we had come to this conclusion. Now, we were trying to interpret this conclusion and of course, we had an easy interpretation for the left hand side, the left hand side is the dot product or the inner product of the sequences $x_1[n]$ and $x_2[n]$.

The point was to interpret the right hand side and of course, not just to interpret the right hand side, but also to give it you know a corresponding meaning to bring parity and congruity in the meaning from both sides. You see on the right hand side you have a product of corresponding points on the Discrete Time Fourier Transform. Of course, the second one is complex conjugated like it is for the sequence here you multiply the sequence by the by each point of the sequence by the complex conjugate of the corresponding point on the other sequence and you add up over all such points.

Here, you multiply corresponding points on the discrete time Fourier transform. That is, you multiply the point on one discrete time Fourier transform by the complex conjugate of the other. And you integrate because here, the variable omega is continuous. It is not discrete.