Digital Signal Processing and its Applications
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Lecture 12 b
Visualisation of inner products

We need to think of this also as an inner product. Because we are, you see, if you look at it, what is the Discrete Time Fourier Transform really, let us, let us let us look back at the inverse Discrete Time Fourier Transform to fix our ideas.

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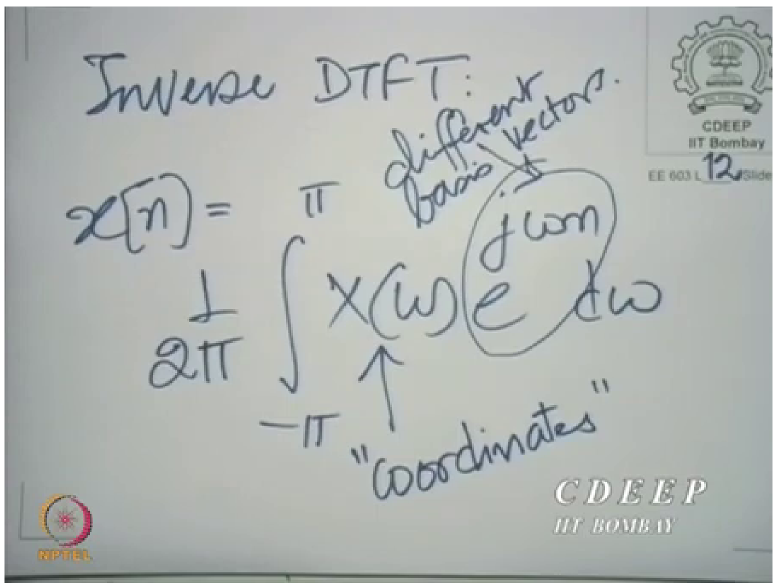

So, the inverse Discrete Time Fourier transform says that you can reconstruct x[n] from the Discrete Time Fourier Transform in the following way. And we have already given this an interpretation. These are like the coordinates with different directions, different ωs are different directions, so to speak, or different ωs give you different axes. And you see, these are different basis vectors.

I mean, there are different basis vectors for different values of ω. Now, lets get this idea clear. What we are saying is that for different values of ω over any contiguous interval of 2π, each omega gives you a different so called, I mean, you can call it a perpendicular direction. I mean, I do not know whether it is the correct thing to do. But you can call it a perpendicular direction. It is a different axis. So, you have as many like, like, for example, in the time domain, every point, every value of n gives you gives you 1 degree of freedom.

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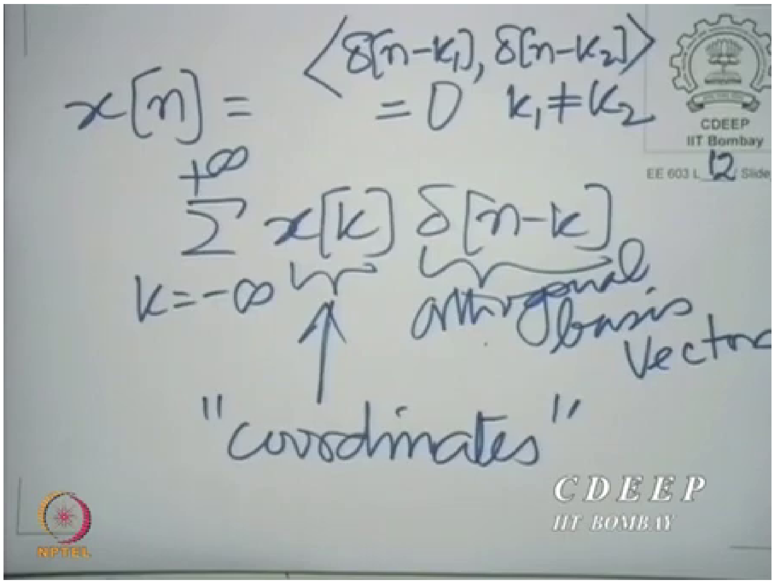

In fact, if you look back at the way you constructed a sequence from its samples, there was the idea of a basis there, you said that $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$. And here x[k] work like the coordinates if you recall. And these were like the orthogonal basis vectors.

Now, here, of course, this is exact, different $\delta[n-k]$'s for different values of k are truly perpendicular. In fact, if you were to take a dot product of $\delta[n-k_1]$, and $\delta[n-k_2]$, I mean, you have 2 sequences, it would be 0 for $k_1 \neq k_2$. So, if the impulses are located, if the impulse sequences have the non zero 1, located at a non zero sum located at different points, that dot product is automatically 0, that is obvious because the sample the non zero samples do not overlap.

So, these are all perpendicular directions, so to speak. So, any sequence can be constructed from all these infinite, countably infinite perpendicular directions. Now, you know, if we go to finite dimensions, we can recreate the situation to some extent, but as I want you in the previous lecture, one must not take literally the conclusions of finite dimension to infinite dimension.

However, we can get a good clue about what to expect in infinite dimensions, when we look at finite dimensions.

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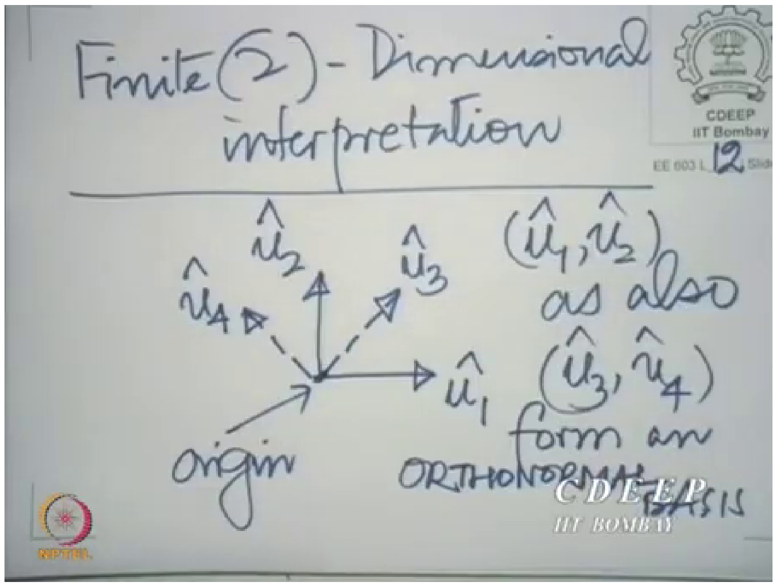

So, let us look at the very simple finite dimensional case of 2 dimensions. You see, let us consider this 2 dimensional space in which this paper lies in which the sheet lies I mean, you can visualize the sheet extending to infinity, constituting a 2 dimensional space. And lets put the origin here. Let us draw 2 pairs of perpendicular vectors. So, one pair is like this $u_1$cap and u_2 cap. I draw another pair of perpendicular vectors, $u_3$cap and u_4 cap. It is very clear of course that $u_1$cap, u_2 cap as also $u_3$cap, u_4 cap form an orthonormal basis.

What is an orthonormal basis? An orthonormal basis is a collection of vectors from that space which are mutually perpendicular, take any 2 of them they are perpendicular. And together these vectors span that space, the word basis means they span that space. Span means you can construct any vector in that space as a linear combination of these. So, of course, it is very obvious that you can construct any vector in 2 dimensional space as either a linear combination of $u_1$cap and u_2 cap or a linear combination of $u_3$cap, u_4 cap that is very obvious.

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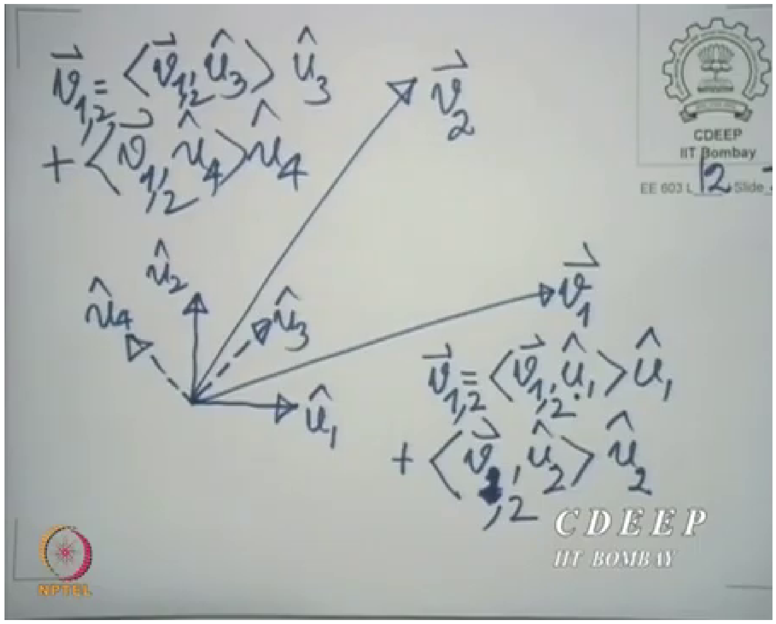

In fact, let us, to emphasize that point, draw a vector and illustrate what I am saying. In fact, we will not draw 1 but 2 such vectors. So, lets redraw these, you have a u_1 cap there, you have a u_2 cap here, you have a u_3 cap here and you have a u_4 cap there. And you have this vector row along one, v_1. And you have this vector v_2. And of course, it is always possible to write v_1 cap as a product of v_1 cap with u_1 times u_1 plus dot product of v_1 cap of v_1, sorry, not v_1 cap with u_2 times u_2.

You see this is the beauty of an orthonormal basis. In an orthonormal basis, you can find the components of a vector along one of the orthonormal basis elements by taking the dot product of v1 with that basis element. And this can be done for each of the basis elements. So, when you have an orthonormal basis, this is the boon; the coordinates are easy to find. When the basis is not orthonormal.

You can of course have a basis that is not orthonormal. What I mean is you can have a collection of vectors. For example, in this collection u_1, u_2, u_3, u_4, you can take the pair u_1, u_3; u_1, u_3 also form a basis, because you can express, you can express v_1 in terms of just $u_1$and u_3, or you can express v_1 in terms of $u_2$and u_4. And you can do it by using the parallelogram law. You can construct a parallelogram with sides parallel to $u_1$and u_3, and they will give you a linear combination of $u_1$and u_3, which gives you a v_1.

So, using the parallelogram law, you can always express v_1 in terms of $u_1$and u_3, or $u_2$and u_4. So $u_1$and u_3 together form a basis but not an orthonormal basis. Similarly do $u_2$and u_4 together. So, the beauty of an orthonormal basis is that finding the coordinates is very easy. You see, if you take $u_1$and u_3 it is an it is a basis but not an orthonormal basis. You can of course find the coordinates by using the parallelogram law.

But finding the coordinates is not a decoupled process. That means I cannot find the coordinates of along u_1 and along u_3 independently, I need to solve 2 equations for 2 coordinates. As opposed to that when I have an orthonormal basis, the job is very easy. I simply take the dot product of the vector along each of these orthonormal basis elements and there I am the coordinate comes is that clear to everybody any doubts on this?

So, now we have done this for v_1 we of course can do the same thing for v_1 with the basis u_3 and u_4. So, you know you can write v_1 is also dot product of v_1 with u_3 times u_3 cap plus dot product of v_1 with u_4 times u_4 cap and of course, if you like I can complete this by putting 1,2 here so I do this both for v_1 and v_2, please read this as v_1. So, there I have 4, I mean coordinates for v_1 the coordinates respect to u_1, u_2, u_3, u_4 and similarly 4 coordinates for v_2. Now, let us write these 4 equations down.

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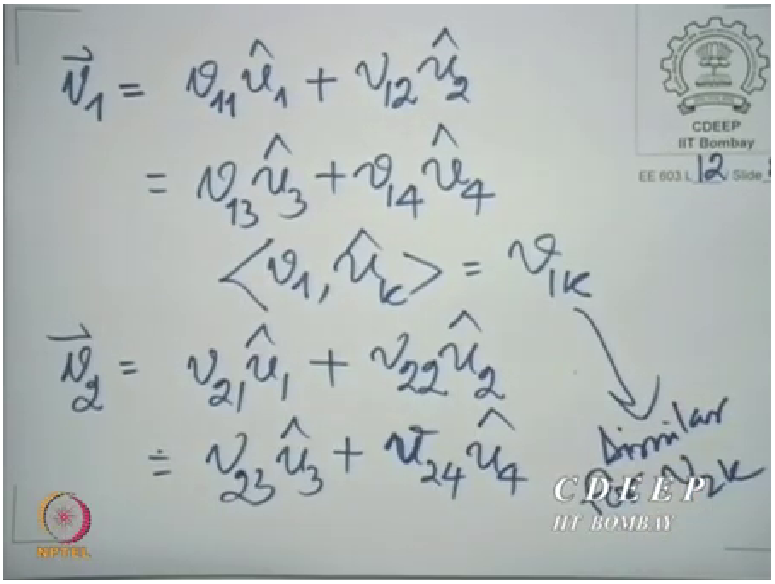

What we are saying is, v_1 is of the form v_{11} u_1 cap plus v_{12} u_2 cap which is alsov_{13} u_3 cap plus v_{14} u_4 cap where the dot product of v_1 with u_k is v_{1k} and similarly for v_2, v_2 is v_{21} u_1 cap plus v_{22} u_2 cap. Which is also v_{23} u_3 cap plus v_{24} u_4 cap. Similarly, for v_{1k}. Now, the dot product is very easy to calculate, because this is the perpendicular basis.

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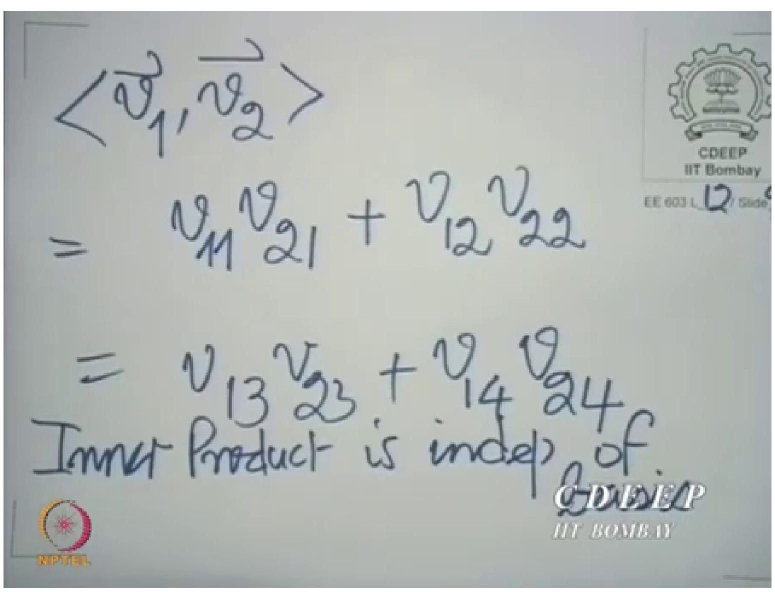

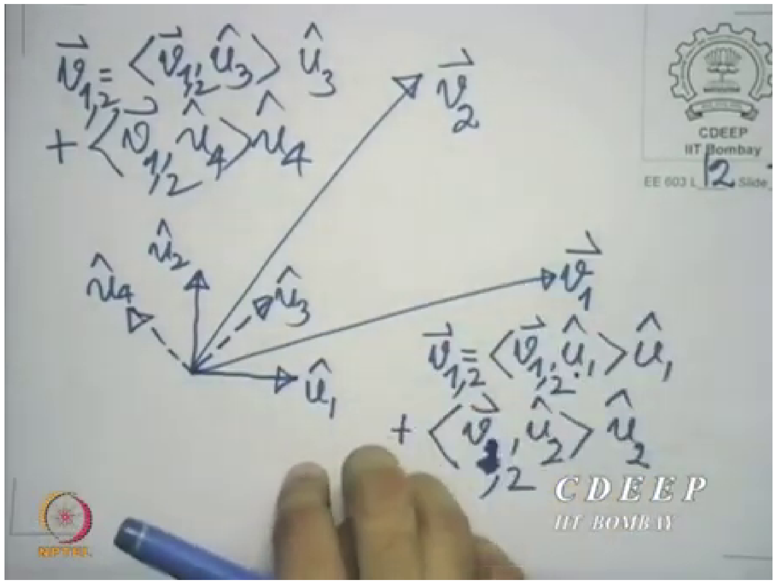

So, the dot product of v_1 with v_2 in the simple 2 dimensional space is $v_{11}v_{21} + v_{12}v_{22}$ and it is also, $v_{13}v_{23} + v_{14}v_{24}$. So, that is what I am saying the dot product has nothing to do with which basis you use, it is independent of the basis. The dot product or the inner product is independent of basis. And that is not very difficult to see. You can use if you if you use orthonormal basis, then calculation of the dot product is easy.

You just take products of corresponding coordinates and add. But the inner product or the dot product itself does not depend on which basis you chose. And that is not difficult to see at all. I mean, you know, if you go back to the drawing a couple of slides ago, if you were to take the dot product of v_1 and v_2, inherently, it has nothing to do with what basis you have used to represent v_1 and v_2.

The dot product is a property of the 2 vectors, not a property of its representation. However, given a representation, one can of course, calculate the dot product with convenience and ease. Now, this is exactly what that relationship is saying. And now let me put it back before you with this renewed understanding.

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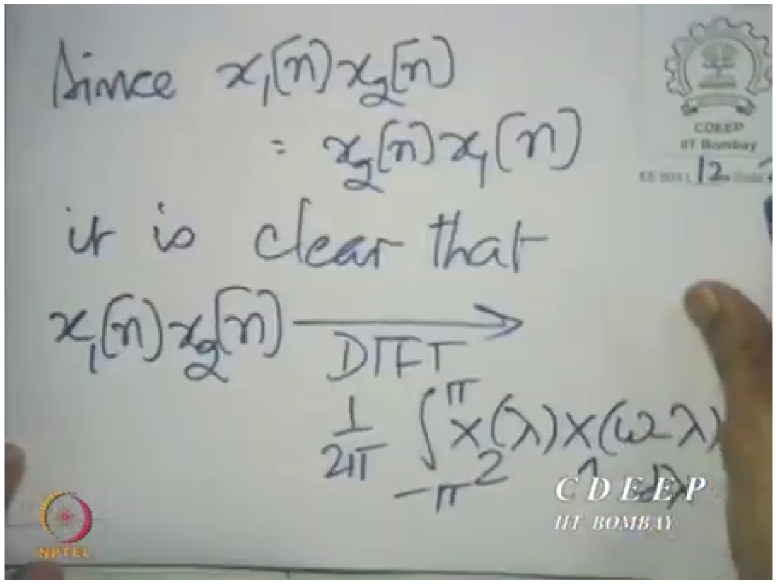

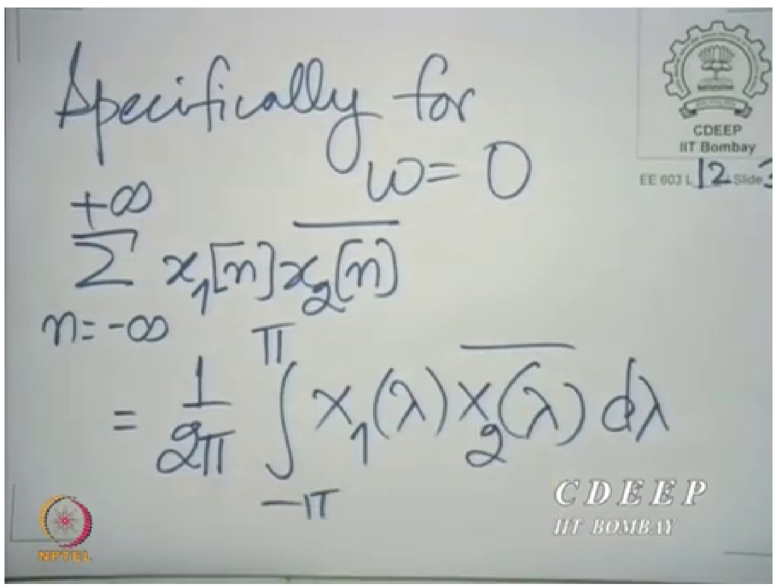

So, what you are seeing in this relationship, look at this relationship once again. Look at this relationship once again. We are saying the dot product of the sequences $x_1[n]$ and $x_1[n]$ is independent of the representation of those vectors. You could think of the vectors represented in the natural domain. Or you could think of the vectors represented in the frequency domain, the dot product is unchanged. And of course in the frequency domain, the dot product is defined in this way.

You need to multiply corresponding points from the frequency axis and integrate over all these points instead of add because the frequency variable is continuous. It is a very elegant and simple interpretation once you think about it.