Digital Signal processing & Its Applications
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Lecture 13 a
Discussion on Unit Step

So warm welcome to the 13th lecture on the subject of digital signal processing its applications. In this lecture we intend to introduce a new transform. But, before we do that we would like to ensure that we have understood the previous transform entirely. The discrete time fourier transform and therefore I would like to allow a couple of few questions. You know so that we clarify any doubts that might still be remnant. Before we proceed to discussing a new transform.

So, we will take a few questions from the class. Are there any questions, doubts or difficulties? I may mention that we also have a course module page one should make use of the technical discussion forum there to discuss concepts. So, any doubts or difficulties so far? No, not at all. So, the question is what can we say about the discrete time fourier transform?

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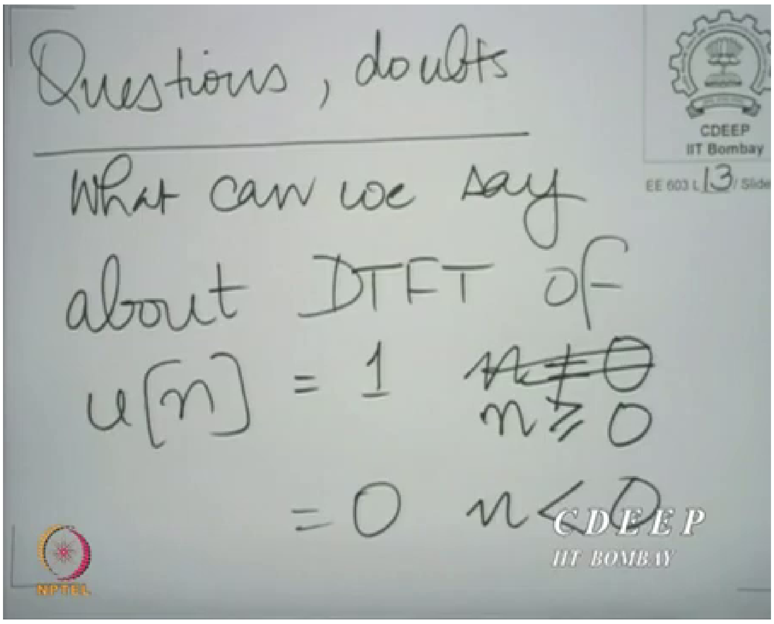

What can we say about the DTFT of $u[n]$. The sequence $u[n]$is very important in discrete time signal processing. It is a sequence which is *1* for $n = 0$, in fact for$n \geq 0$, you know, and *0* for $n < 0$. So essentially, it's one for the entire positive set of integers in positive and I mean *0* and positive set of integers. So, it is called a unit step because it looks like that looks like a unit step.

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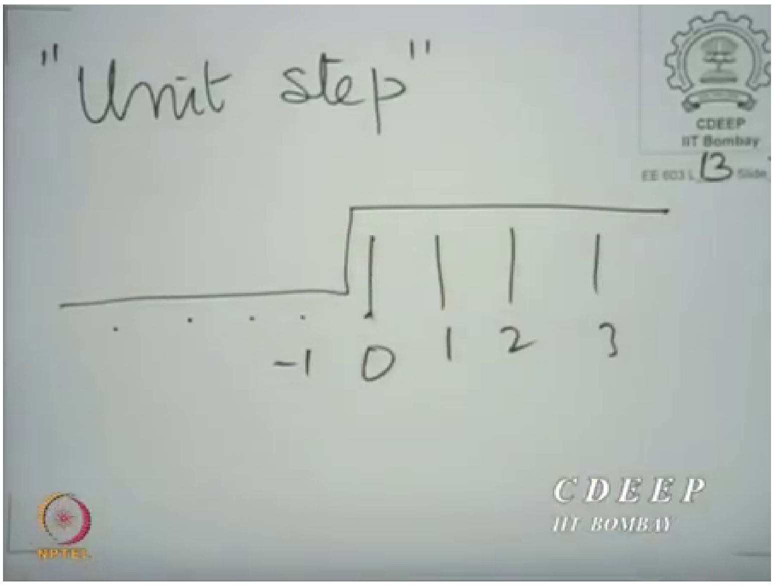

So, it looks like a unit step. Now, this unit step function is a very important function in discrete time signal processing. One importance is that it, when multiply, when you multiply the unit step by any function it retains the positive side of samples and destroys the negative side.

So, it's useful in concept. It is also useful in describing what is called the accumulator system.

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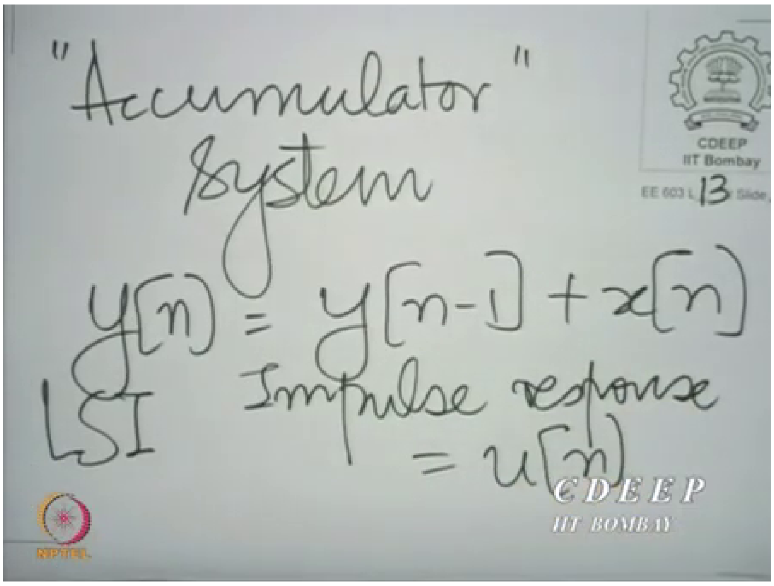

So, an accumulator system, as the name suggests accumulates the inputs. So, it keeps taking the sum of what it already has with the current input. So, if you consider this system and if you take the LSI system described by this equation where the output at a current time is equal to the output of the previous time plus the input at the current time.

So, it does it keeps accumulating whatever is coming now it accumulates in itself. So, it keeps taking a sum until that point in time this is called an accumulator system. Now, if we treat it in fact this can be looked upon as an LSI system if you treat the accumulator as having operated all the way from $-\infty$.

And the impulse response of this LSI system is $u[n]$that is easily seen. Because, if you have an impulse then when, if the impulse is given to an accumulator the output is one starting from 0 onwards. The question is one one way to ask the question is would the accumulator have a frequency response? That is another way to reframe the same question. The answer strictly is no.

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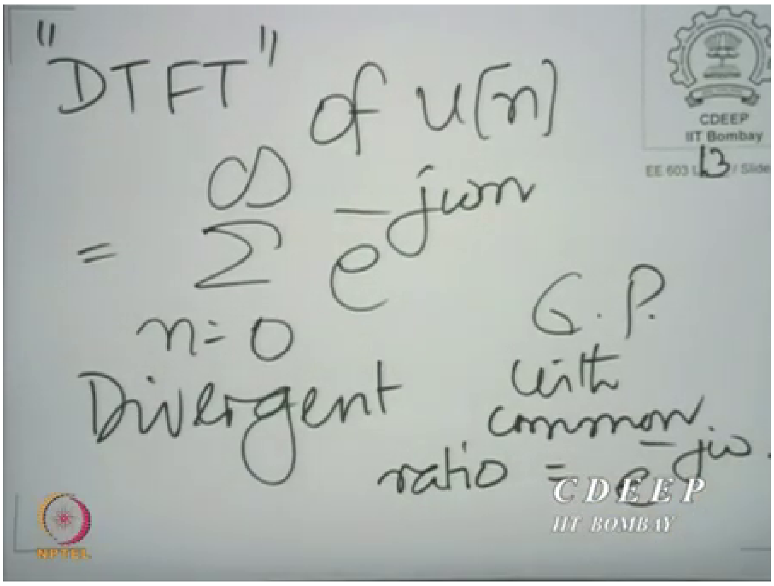

That is because if you look at the discrete time Fourier transform of the or so called DTFT and that does not exist really DTFT of $u[n] = \sum_{n=0}^{\infty} e^{-jwn}$ which is actually divergent because, the common ratio is a geometric progression with common ratio equal to e^{-jw}.

And the modulus of this common ratio is *1*. So, such a geometric progression is known to be divergent. However, some people like to write this as $\frac{1}{1-e^{-jw}}$ you know some people write it.

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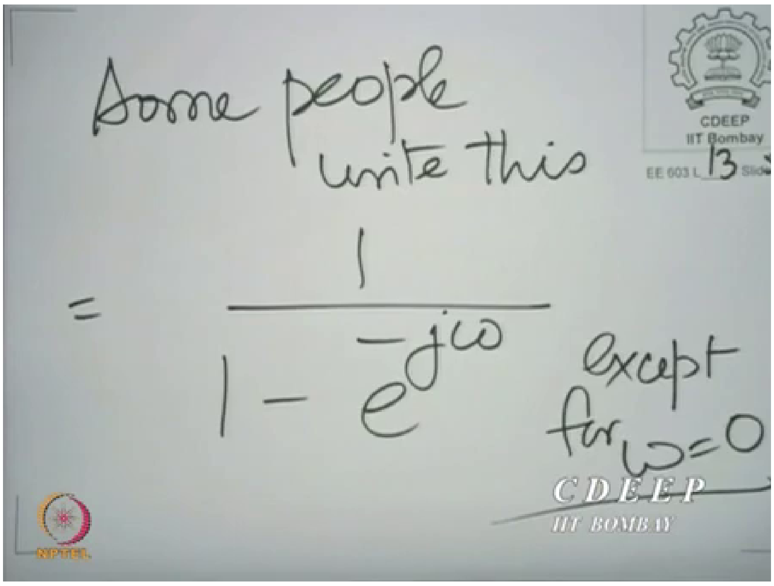

And say it should be accepted except for $\omega = 0$. Some people like to write that that is not entirely correct. Actually, this is divergent. However, I mean the way to interpret this is that if you happen to give the accumulator an input other than$u[n]$so among the complex exponentials if you happen to give the accumulator a complex exponential input other than $u[n]$ you could possibly hope for a non-unbounded or a bounded output.

That is the interesting part. So, although this system if you look at the LSI system its impulse response is not absolutely summable. So, it is very clear that the system is unstable, be the one be unstable. But then you know it is it exhibits an unbounded input for some selected bounded outputs. Sorry, it exhibits about an unbounded output for some selected boundary inputs. So, you know it's not unlike some other systems that will soon see.

This system is selective in its instability. So, some people like to call it marginally stable or marginally unstable. Of course, these are all informal terms they have no real formal meaning. Otherwise, the system is unstable. Strictly speaking it does not have a frequency response in the true sense. But, the question whether it produces a, an unbounded output for every bounded input has the answer, no. It does produce bounded outputs for several bounded inputs including some of the complex exponentials.

But you know which needs to be seen carefully. Alright. In particular when you take $u[n]$ itself if $u[n]$ is given as the input to the system the output is unbounded clearly. So, if you give a constant *0* frequency input so to speak. It has an unbounded output but if you give it for example the highest possible frequency that you can see in discrete time where you alternate $1, -1, 1, -1; \omega = \pi$. There actually the output is not unbounded.

The output is bounded by *1*. But the output is divergent still. So, these are the subtle points. So, in that sense the frequency response does not exist. Anyway, so that, so one does not one should not really say that it has a DTFT does not. Alright. Any other questions?