Digital Signal processing & Its Applications
Professor Vikram M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Lecture – 13 b
Z Transform Introduction

Alright. So, there are no other questions we will then take this very example further. In fact it was good that this question was asked. Because, it leads us to the next limitation of the discrete time fourier transform. The discrete time fourier transform as you see does not exist for all sequences. In fact, the discrete time fourier transform exists when the summation corresponding to the discrete time fourier transform converges.

What if the summation does not converge obviously does not exist. So, then what do we do? Do we have some other transform domain where we can study the same sequence or do we have to make do with natural domain studies or natural domain processing. It turns out that we do have a more general transform which allows us to deal with sequences which are not absolutely summable as well. Let us take an example. So, let us take a more troublesome sequence.

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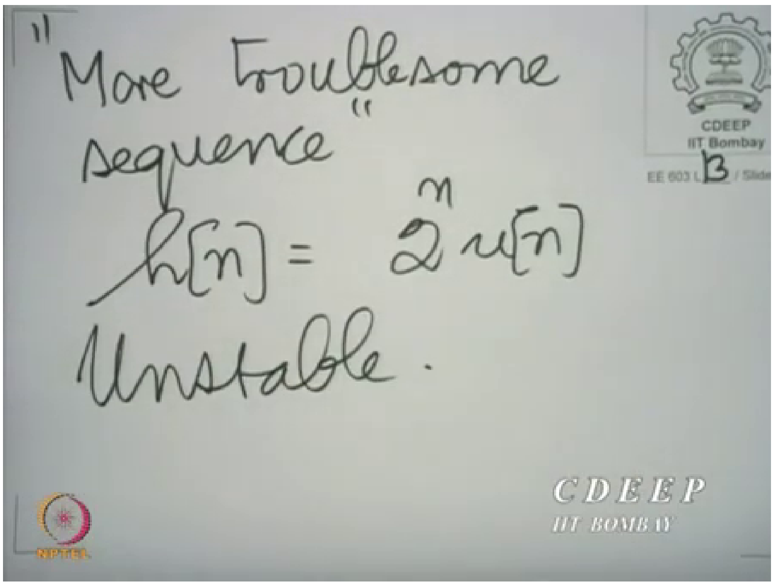

I say more troublesome because you know as we said the $u[n]$ sequence the unit step sequence at least does not produce an unbounded output for all bounded inputs. But, the sequence that we soon see if it becomes the impulse response of an LSI system it more or less always wishes to produce an unbounded output. Except for some very chosen bounded inputs that is, the interesting thing. You know in, in its, the system is, you know so there are, I mean instability also has different grades. The accumulator is an unstable system.

But, unstable you know it is like saying the person is insane but you know there are only certain points or certain situations where he exhibits insanity. But, here you have a system which normally behaves insanely, its very rarely the system exhibits sanity. And that system is let, let us let an LSI system have the following impulse response $h[n]$ is $2^n u[n]$. Now, this LSI system would at the slightest provocation exhibit an unbounded output.

In fact, it is very clear that this impulse response is not absolutely summable so it is an unstable system. And you see, I mean, when you have an unstable system, what do you? Do you want to study its frequency response?

Now, you know I liken the frequency response of a system to the behaviour under training. So, when you give a complex exponential what does it do? You know you have trained the system and you.

So, you know I would like then to take an analogy. Here, you could think of a system like this like a ferocious tiger and a system where the DTFT converges. Maybe, even $u[n]$ perhaps at the extreme or definitely $\left(\frac{1}{2}\right)^n u[n]$ like a little dog. Now, it is not too difficult to train a dog you know you can just hold a dog in open space and open, you know, in front of you and slowly by using rewards train it. On the other hand trying to train a ferocious tiger is dangerous both for the trainer and for people around.

Is that right? And therefore the only way to train a ferocious tiger is to encapsulate the tiger in a cage and then of course use rewards or punishments to train. Now, this system that we have here is like a tiger and we need to encapsulate such systems first, in a safe cage, and then train it. Train it means, see what happens when you give it specific inputs. How do we encapsulate?

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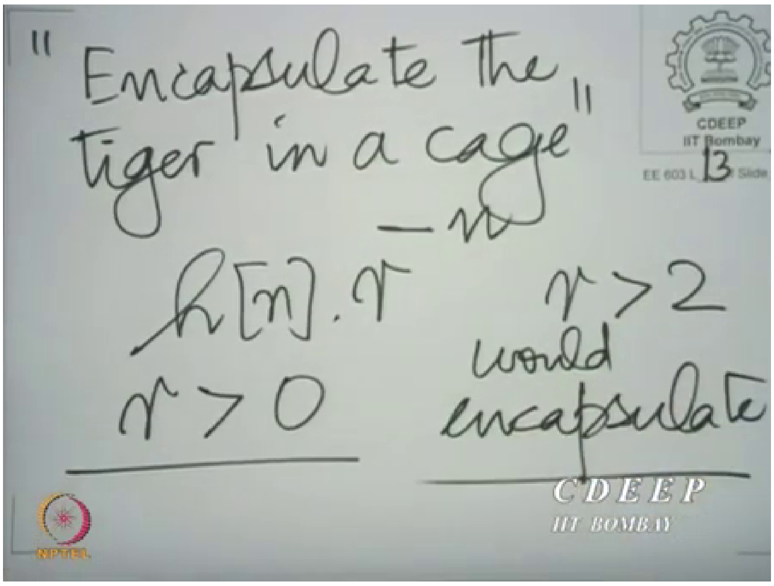

The tiger in a cage? We encapsulate the tiger in a cage by multiplying it, you know what is a cage? A cage is stronger than the tiger. So, we encapsulate the tiger in a cage by forcing an even more troublesome sequence on that system. So, multiply $h[n]r^{-n}$, r is a number greater than 0. So, for example here if you happen to multiply this sequence by any such exponential where $r > 2$, it would get encapsulated. Isn't it?

Here $r > 2$ would encapsulate, would encapsulate the tiger. And now you can of course, you can take the DTFT.

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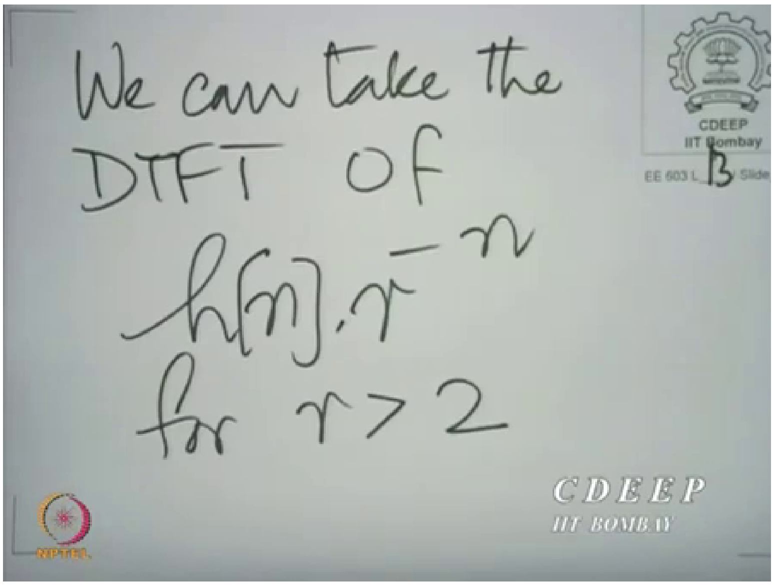

So, we can take, we can take the DTFT of h the encapsulated tiger, you can train the encapsulated tiger, that is obvious. For example take $r = 2.5$, then $h[n]r^{-n}$ would certainly be you know it you know the DTFT of that sequence exists. Now, you see that is what we are now going to do in our effort to deal with such unstable systems. We are going to encapsulate. We are going to first capture their naughty behaviour and therefore instead of multiplying by e^{jwn} we multiply.

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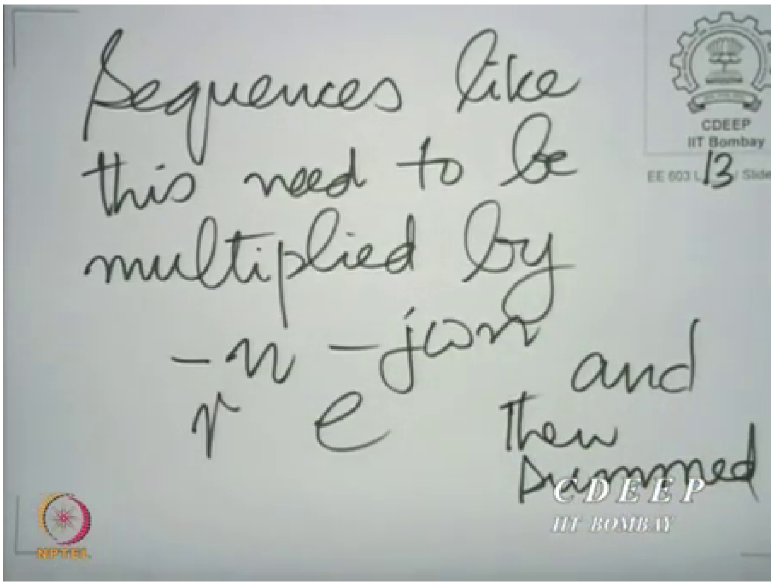

So, sequence is like this you see sequences, sequences like this need to be multiplied by r^n, $r^{-n}e^{-jwn}$ and then sum it. r is like a radius and e^j so, re^{jw} is the complex number that we are talking about here. And now you know the role played by r and e^{jw}.

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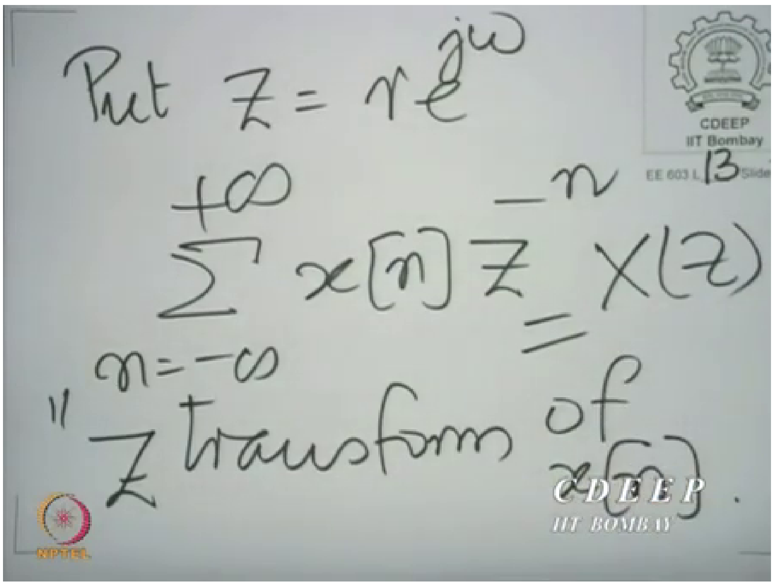

Put $z = re^{j\omega}$. So essentially, we are saying consider $\sum_{n=-\infty}^{+\infty} x[n]z^{-n}$. We call this the Z-transform of $x[n]$. And we denote it by capital $X(z)$for obvious reasons. So, of course a Z-transform unlike the discrete time fourier transform is a function of a complex variable. The discrete time fourier transform was a function of real variable ω.

In fact in a way there is a relationship between the Z-transform and the discrete time fourier transform.

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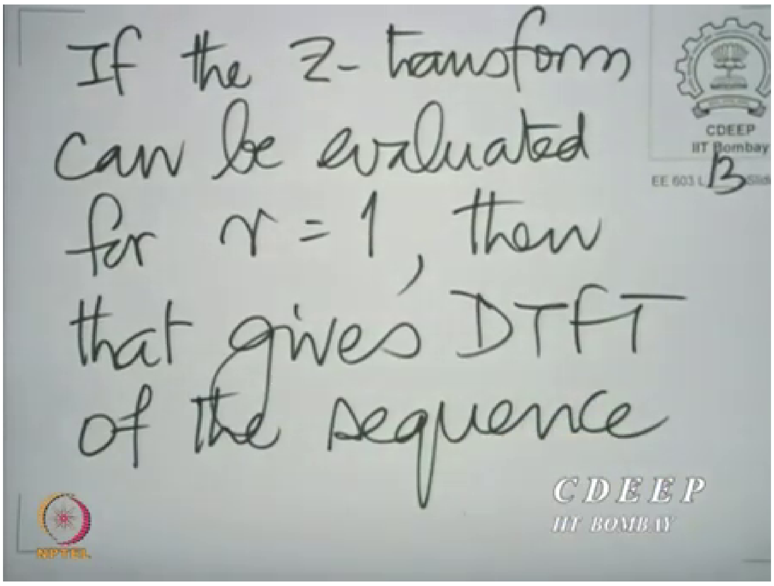

If the Z-transform can be evaluated for $r = 1$ then that gives its DTFT. So, Z-transform for $r = 1$ is the DTFT if that can be evaluated. It may not be possible to evaluate it, it may not converge.