Digital Signal Processing & Its Applications
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Lecture No. 03 a
Review of Sampling Theorem

A warm welcome to the third lecture on the subject of Digital Signal Processing and Its Applications. Let us take a couple of minutes to review what we did in the previous lecture. In the previous lecture, we had initiated our discussion on sampling, we try to answer the question, when can we retain information even after sampling?

To expand a little bit, when can we consider values of a signal only at sampling instance and yet not create any confusion or not lose information? We have not completed our answer to that question. We are only got thus far and we shall now summarize that. So, let us put down the theme of our lecture today.

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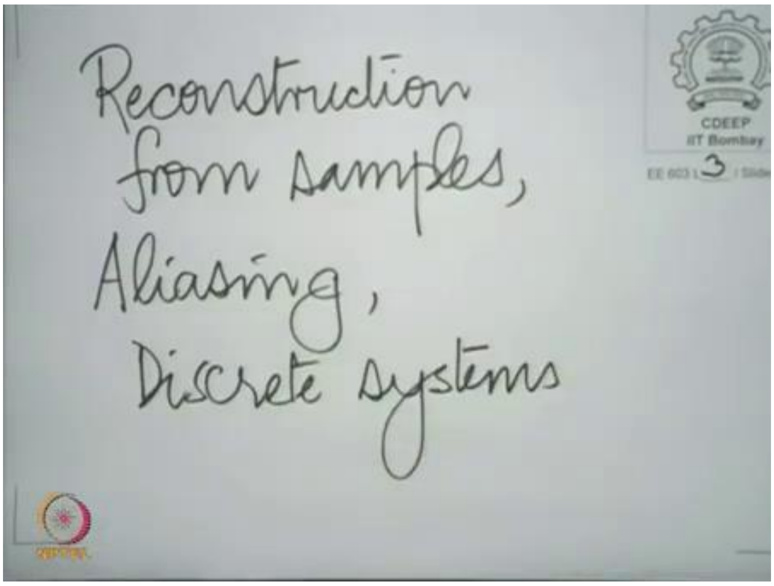

Reconstruction from sampling or from samples to be more appropriate, aliasing, and discrete systems. This is the theme of our lecture today. Well, what we have seen the last time is that.

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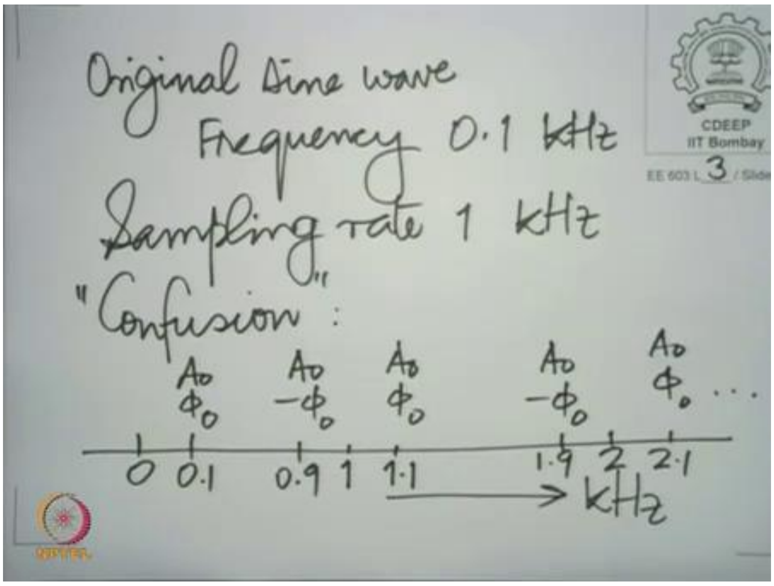

If you have an original Sinusoid of frequency 0.1 kilohertz, and if you have a sampling rate of 1 kilohertz then you land up with a confusion of sine waves and the confusion is as follows. On the kilohertz scale, the samples could have come either from a 0.1 kilohertz frequency or from a 1.1 or a 0.9 kilohertz frequency.

And of course, if the amplitude of this is A_0 and the phase ϕ_0, the amplitude of this is A phase ϕ_0, the amplitude of this A_0 – phase ϕ_0, the amplitude of this at 2.1 is A_0 phase ϕ_0, and we can keep doing this, amplitude A_0 -phase ϕ_0 we can continue. So, at every multiple of 2 kilohertz we have these confusing sine waves if you want to call them that, which contribute to the same samples.

Now, I want to say a little more about this observation. You see, what we have shown in the previous lecture, was that all these sine waves generated the same samples at those instances. However, we need to show something more, we need to show that if I indeed add up these sine waves as I have shown here, with amplitudes A_0 and phase either ϕ_0 or - ϕ_0, depending on whether it is after the sampling frequency or before if I add up all these sine waves, I generate what I really call a train of samples and what do you mean by a train of samples?

In fact, this is a challenge which I put before you. It is a practice that I follow in courses that I teach, to challenge your imagination right from the beginning, to put before you exercises which will make you think and work certain things out on your own, so that you get a much better grasp of the ideas that are being taught. And we are beginning with the first challenge.

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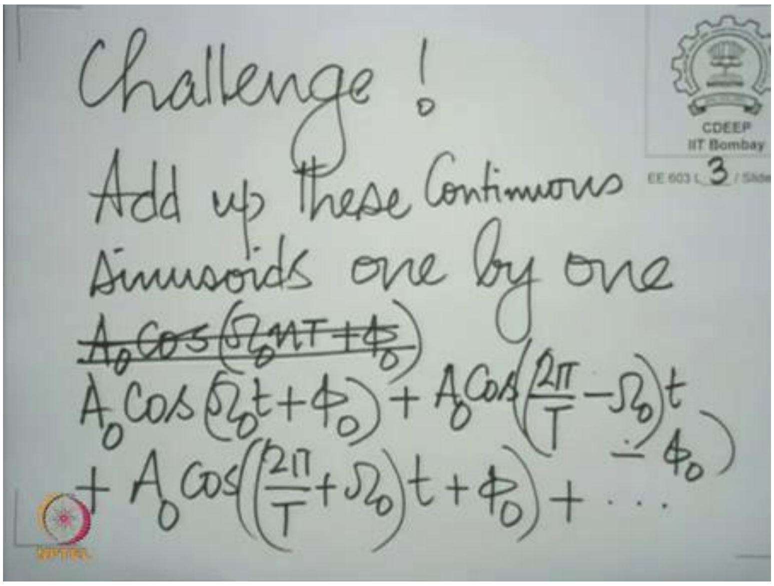

Take these sine waves and add them one by one. What I mean by that is take the sum, A_0 (cos Ω_0 n T + ¢$_0$).Or in fact, let us put sample, at n T of course, you will get infinity there is nothing much to be seen, but let us instead add up the continuous.

So, you know,we should say not these Sinusoids' at the samples, but Sinusoids' the continuous Sinusoids'. So, 2π/T as we would have it(-Ω_0 * T - ¢$_0$)+ A_0 cos 2π/T + Ω_0T + ¢$_0$ + now what will come next? Let me do it for a few terms for you.

So, what I mean is, you see you have taken the original Sine wave as a continuous Sine wave here. You have taken the next one, the next one is the sampling frequency in radians per second 2π/T - ¢$_0$ and then the phase would be - ¢$_0$, sampling frequency in radians + Ω_0 + ¢$_0$ Now, the next term would be 2 times the sampling frequency in radians - ¢$_0$ and the next term after that would be 2π/T *2 +¢$_0$. Let me write down two more terms to make the concept clear here.

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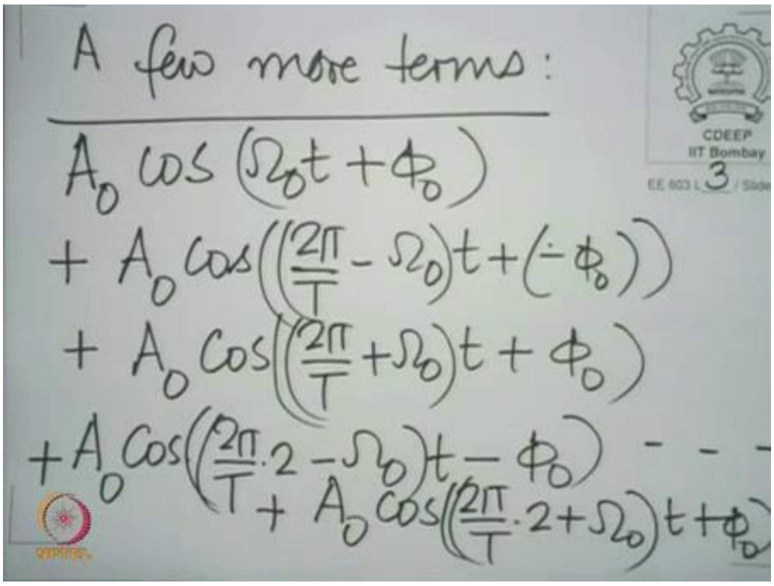

Let me write down completely, you can keep on writing more and more terms. Anyway, we got the basic idea. Now, the challenge before you is to prove that as you add more and more terms, you have a constructive interference at the point of sampling and a destructive interference at all points other than that of samplings. In other words, what I want you to prove in the challenge, let me write that down.

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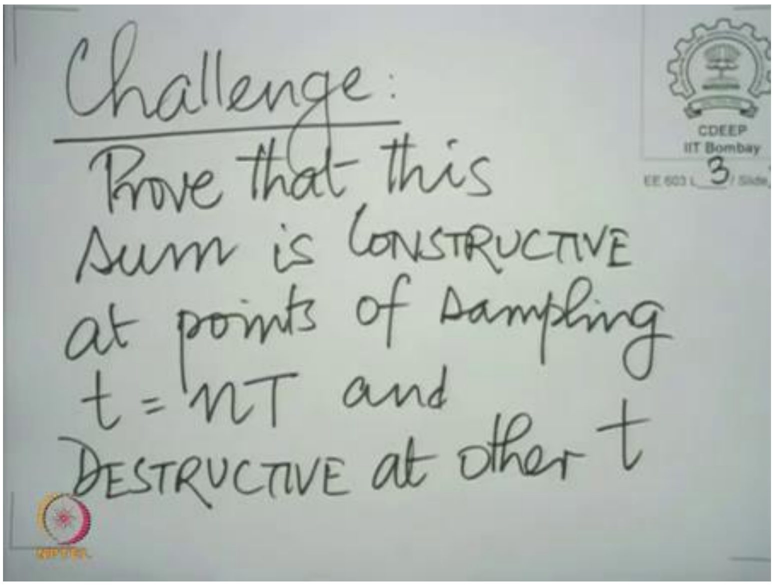

Prove that this sum is constructive at the points of sampling, that means T = N T, and destructive at others. So, what is going to happen? As you add more and more terms, essentially, this sum is going to become localized near the points of sampling, it is going to rise higher and higher. In fact, it is very easy to see that at the point of sampling, the sum is divergent, it would go towards infinity.

We knew right in the beginning, from the previous lecture that the samples are the same when they come from all of these Sine waves. So, at the points of sampling, of course, constructive interference is obvious. Because all the samples are the same. All the values, all the Sine waves take the same values.

The key thing is to prove that at other points, the sum is destructive. That means as you add more and more and more and more terms, they all compete and cancel one another out. I leave this as a challenge to you. And I give you a hint, you will have to use trigonometric identities.

You are summing up Sine waves of the same amplitude of different frequencies, use trigonometric identities, and show and try and see what they are converging to what that sum converges to. If you like, you could decompose a Sine wave into a sum of two oppositely rotating complex numbers as you often do. So that is also acceptable. We will be dwelling on that idea in more detail later, called phasers. Anyway, for the moment I am putting this before you as a challenge. Why I am saying this is because it tells you what happens when you sample.

When you sample what you are doing is essentially to create small pulses. At instance of sampling of value or strength equal to the sample value of the waveform at that point. At all other points, you essentially have a 0 signal. So, sampling means to replace a continuous signal by a train of pulses.