Digital Signal Processing & Its Application
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Lecture 15c
Partial Fraction Decomposition

I strongly recommend that the class should review and revise the concepts of partial fraction expansion. In fact, let me write that down as a task to be done, it is a very important task that we should do when dealing with Z transforms.

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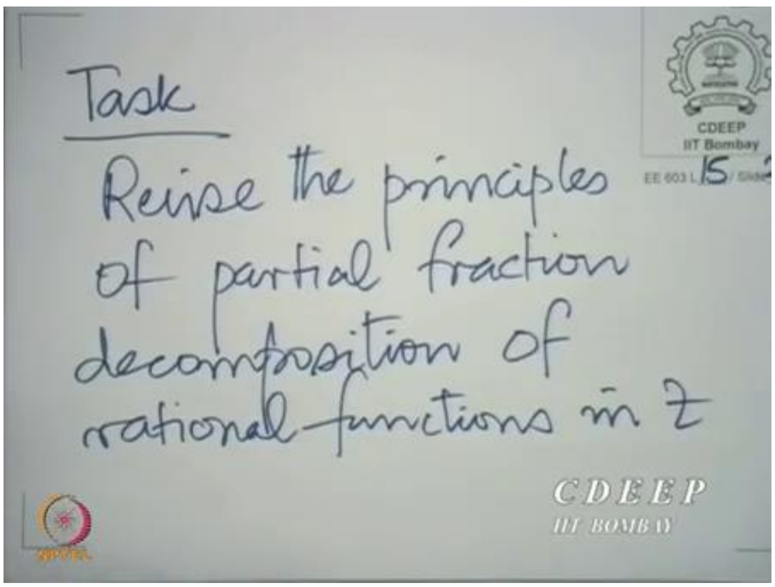

Revise the principles of partial fraction decomposition of rational functions. I would just like to point out some important things that need to be seen in this revision of partial fraction principles. One of the things is that when you have repeated poles, you must not forget that you cannot obtain the numerator simply by that principle of multiplication by that term, and then, you know, setting Z equal to that pole, you cannot do it, when there are repeated poles, at that point.

You need to use slightly more, you know, intelligent strategies there. So, you know, I recommend, since you know, I mean, 1 expects that partial fraction decomposition is done in basic engineering mathematics, I recommend that you review those ideas, it is important. Secondly, 1 must also know how to deal, how to obtain partial fraction expansions with, you know, by after looking at the degree of the numerator and denominator.

You see, if the, if in the variable with respect to which you are making the partial fraction expansion, the degree of the numerator is more than the degree of the denominator, you must first extract a series, 1 must not forget that. Let me illustrate what I mean.

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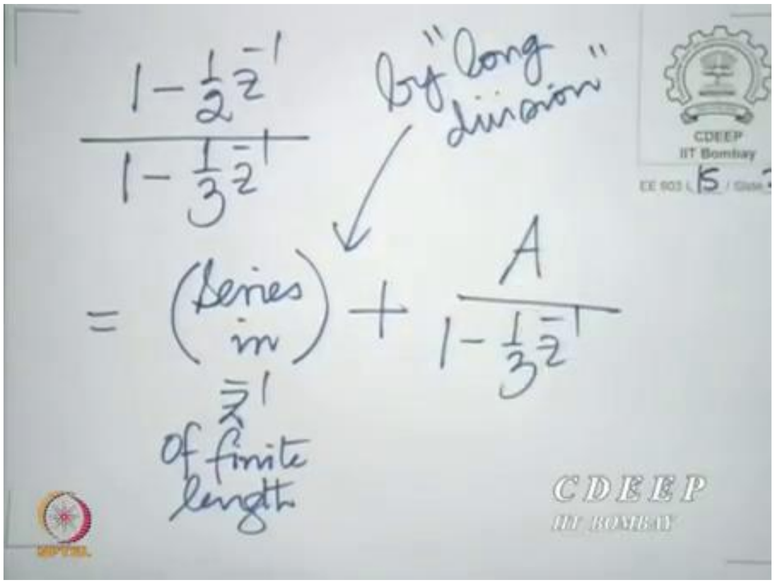

You see, I am saying, suppose you had 1 minus, let us take the very simple case, 1 minus half Z inverse by 1 minus one third Z inverse. Now this is not in the standard partial fraction expansion form. In the partial fraction expansion form, it should be written down, as, you know, a polynomial or a series in Z or Z inverse here, of finite length plus some constant divided by 1 minus one third Z inverse. And we can obtain the series by a process of long division. What do you mean by long division, let us illustrate it.

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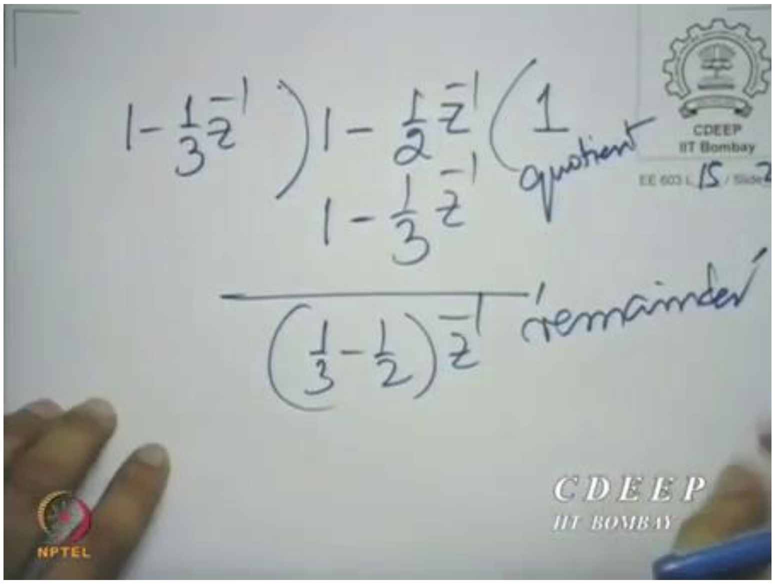

So long division is you know what we have conventionally studied even with integers, but here it is applied on the rationals. So, 1 minus half Z inverse long divided by 1 minus one third Z inverse. So, of course, it goes once and you remove 1 minus one third Z inverse, leaving you with one third minus half Z the inverse on top. This is the remainder, so to speak, and quotient.

Now you know, here the long division, actually, I intentionally did what is actually incorrect here. Incorrect in sense, not useful, it is not incorrect. But you know, the long division here needs to be done on Z inverse as the lead. So, if we do the Z that long, so this quotient and remainder would not help us here.

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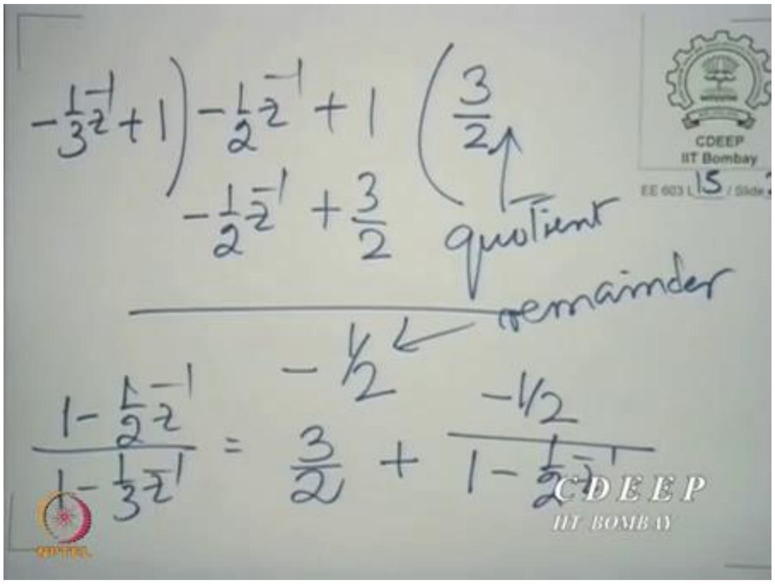

So instead, in fact, we should do the long division the other way. We should write this as minus half Z inverse plus 1 long divided by minus one third Z inverse plus 1. And you need to multiply minus one third by 3 by 2, that gives you minus half Z inverse plus 3 by 2. I mean you subtract; you get a minus half there.

So, therefore, 1 minus one third Z inverse 1 minus half Z inverse by 1 minus one third Z inverse is 3 by 2, this is the finite series, plus minus half divided by 1 minus half Z inverse. So, this is now the remainder and this is the quotient. I intentionally illustrated both forms of long division here to distinguish between the one that is useful here, and the one that is not.

When you are trying to express all your partial fraction terms in terms of Z inverse in the long division should be done by the lead being on Z inverse. So you treat it is a polynomial in Z inverse. So, you must put the highest power of the inverse first in the long division process. So, you know, I showed you both those approaches just to distinguish between which one should be used and should not be used in this specific case.

Of course, it depends on you, you can make an expansion with the, the rational function being treated as in Z inverse or being treated as in Z. Either way, you will finally arrive at the same sequence, the sequence cannot change, but you would arrive at it by slightly different processes, it is just a matter of, you know, slight adjustment of the process. And, you know, there is no serious difference between the two approaches.

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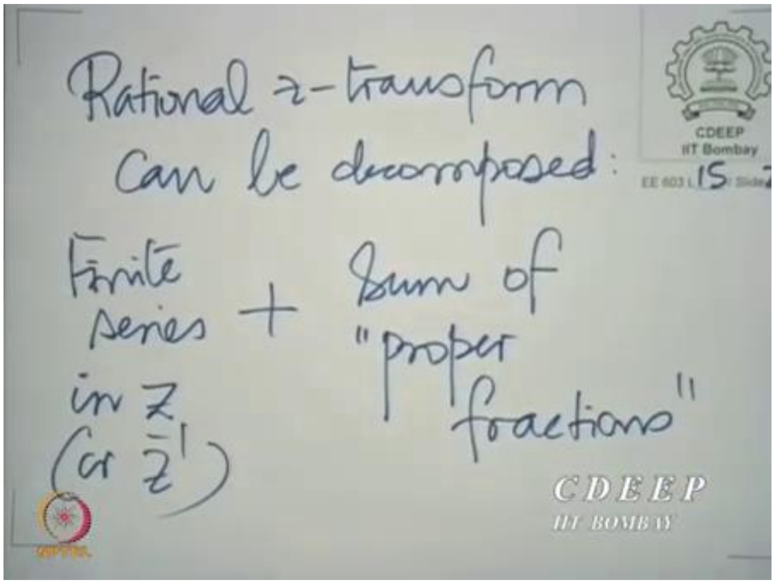

Anyway, so, so much, so, to illustrate that in general, I mean, you could you could generalize, so, you know, whenever you have a rational Z transform, any rational Z transform can be decomposed in this manner. A finite series in Z or Z inverse plus a sum of terms sum of terms, sum of what are called proper fractions, you know, proper fractions, I use the term proper fraction in a more generalized sense here.

What is a proper fraction, you know, as we understand in the context of integers, for example, 9 by 7 is treated as an improper fraction because the numerator is greater than the denominator. Now, here the greater or lesser is in the sense of degree. So, what we are saying is, we call a term a proper fraction if the degree of the denominator in the variable with respect to which you are expressing the series is greater, strictly greater than the degree of the numerator, an improper otherwise. Even if they are equal, as we saw in the example a minute ago, it is treated as an improper fraction.

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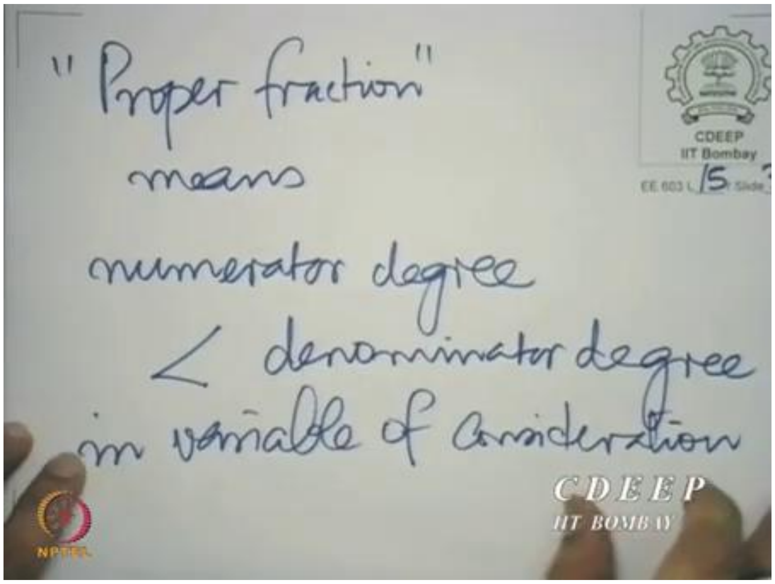

So, let us write down. Proper fraction means, numerator degrees strictly less than the denominator degree in the variable of consideration. Let us take a couple of examples just to drive the point home.

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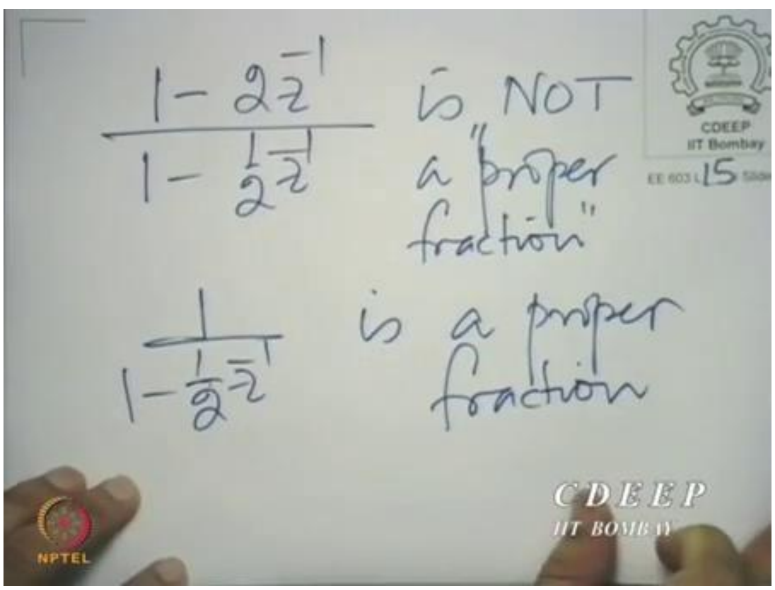

Let us take 1 minus 2 Z inverse by 1 minus half Z inverse is not a proper fraction. 1 by 1 minus half Z inverse is a proper fraction.

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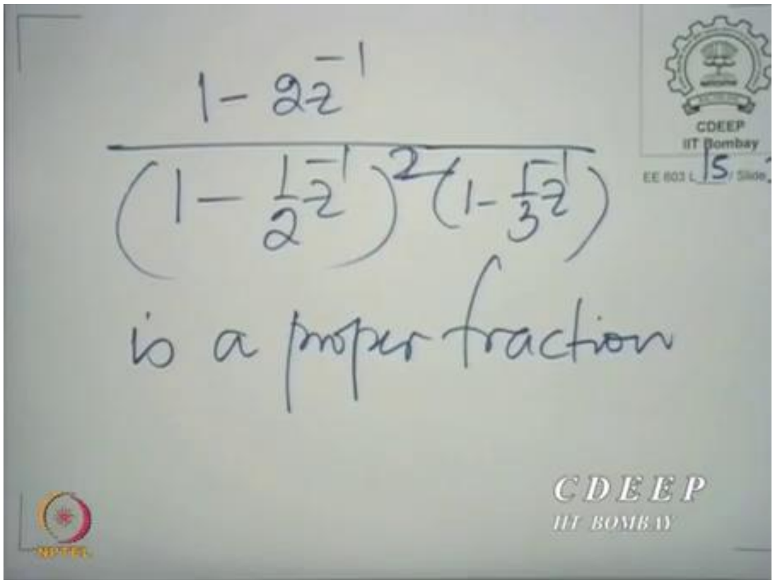

1 minus 2 Z inverse by 1 minus half Z inverse the whole square is a proper fraction. And in fact, we now need to qualify that proper fractions, see all these are proper fractions. In fact, this would be a proper fraction; this could also be a proper fraction. If I multiply this by 1 minus one third Z inverse would still remain a proper fraction. But we do not want something like this. You see, so we need to qualify a little more.

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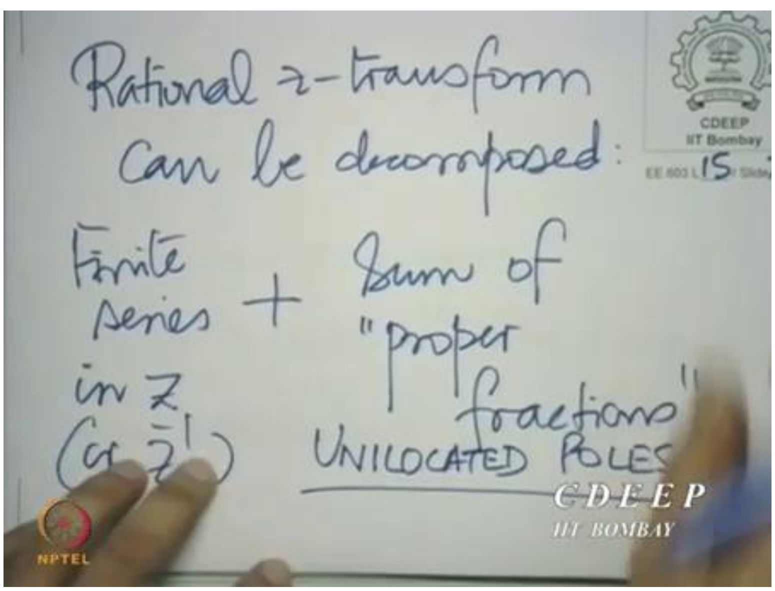

You know, let us, let us go back to that, to that slide where we said sum of proper fractions, we do not want a sum of proper fractions just not any proper fraction. We want proper fractions with poles all at one place. Some of proper fractions with uni-located poles. Now, uni-located poles does not mean there is just one pole, there could be multiple poles, but all at the same place.

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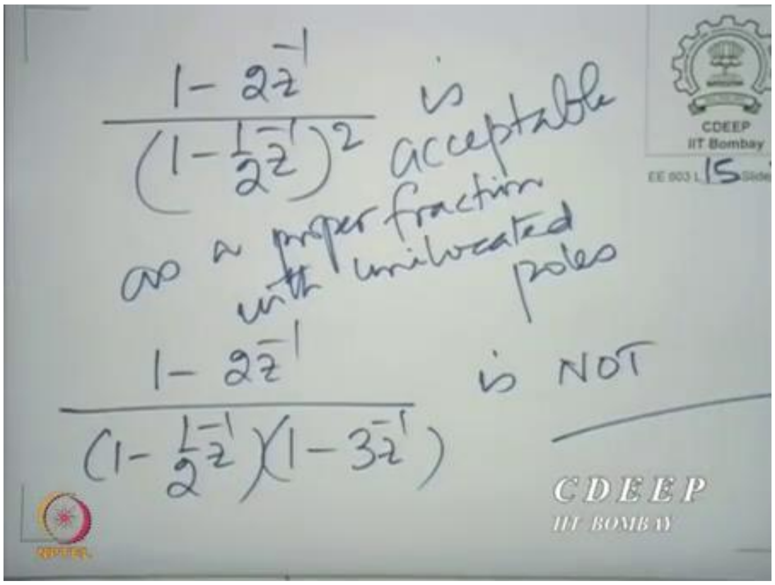

So, let us take examples of both. 1 minus 2 Z inverse by 1 minus half Z inverse the whole squared is acceptable as a proper fraction with uni-located poles. On the other hand, 1 minus 2 Z inverse divided by 1 minus half Z inverse times 1 minus 3 Z inverse, if you like is not, it is not acceptable. It is a proper fraction, but it is not a proper fraction with a uni-located pole. We want proper fractions with the uni-located pole and when we know why, we know how to invert such terms. We have seen that with a combination of the geometric progression and the property of differentiation, were are in a very convenient position to invert such terms.