Digital Signal Processing & Its Application
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Lecture 15d
LSI System Examples

So, in a way, we have more or less equipped ourselves very well to deal with systems, LSI systems, where the Z transform of the impulse response is rational. And the input also has a rational Z transform. Let us take an example.

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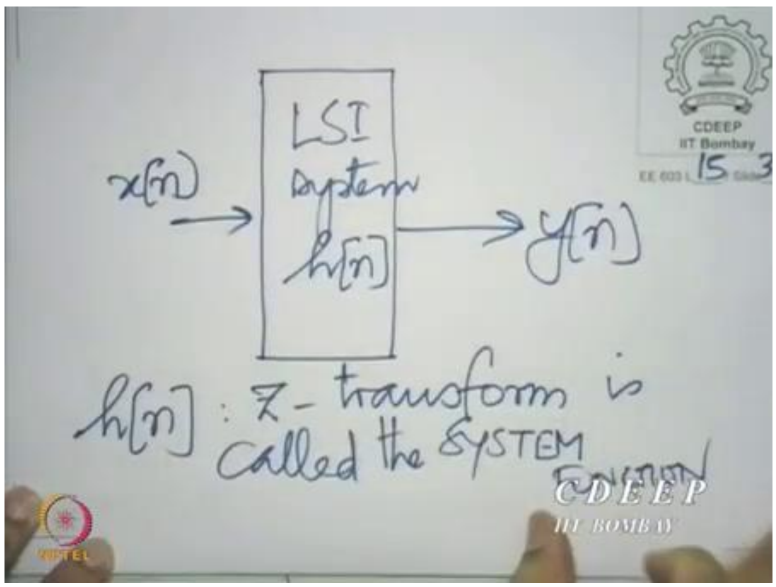

You see, let us take an LSI system, I work straight away in the Z domain with impulse response h n. Now introduce a very important idea. The Z transform of h n, if it exists, is called the system function. I must point you to a couple of important warnings here. A system function is not just an expression there; a system function is an expression with a region of convergence. This is often forgotten in the literature. And it is and justifiably so because the context makes the region of convergence clear. We shall also see very soon how the context can tell us a lot.

For example, if we know the system is causal, obviously, the impulse response must be either finite length or right sided, it cannot be left sided. So, you know, the context does tell us a lot about what to expect of the region of convergence. So very often, that region of convergence is not specified. But then, technically, a system function is a Z transform, and therefore has an expression and a region of convergence. Now with that observation, let us take an example of an x n being given to this LSI system to produce y n.

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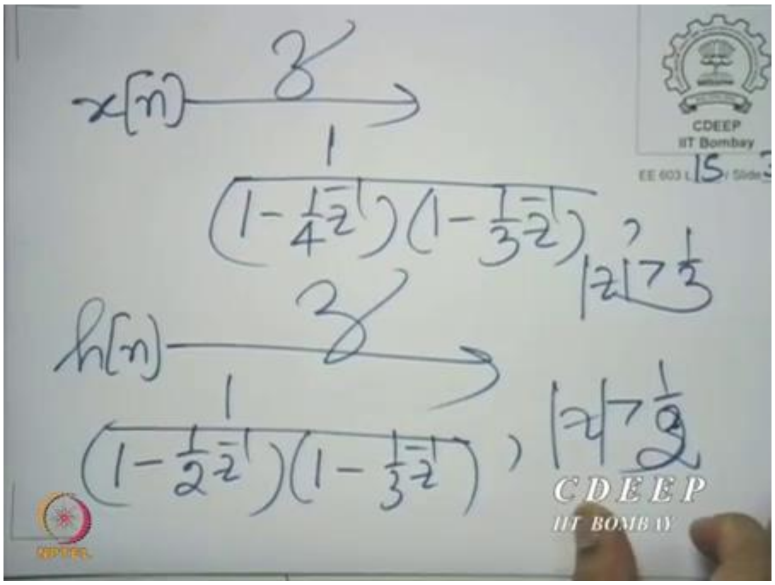

And let us take a specific example of x n and h n. So, we have taken accent, to have the Z transform that straightaway work in the Z domain. To have the Z transform 1 by 1 minus 1 fourth Z inverse 1 minus one third Z inverse. And h n to have the Z transform. Of course, I must specify the region of convergence.

So, I have different, I have 3 possible regions, I shall take mod Z greater than one third as the region of convergence. And let h n have the Z transform, 1 minus half Z inverse times 1 minus one third Z inverse. Again, with mod Z greater than one third as the region of convergence, greater than half, has to be greater than the greater of the 2.

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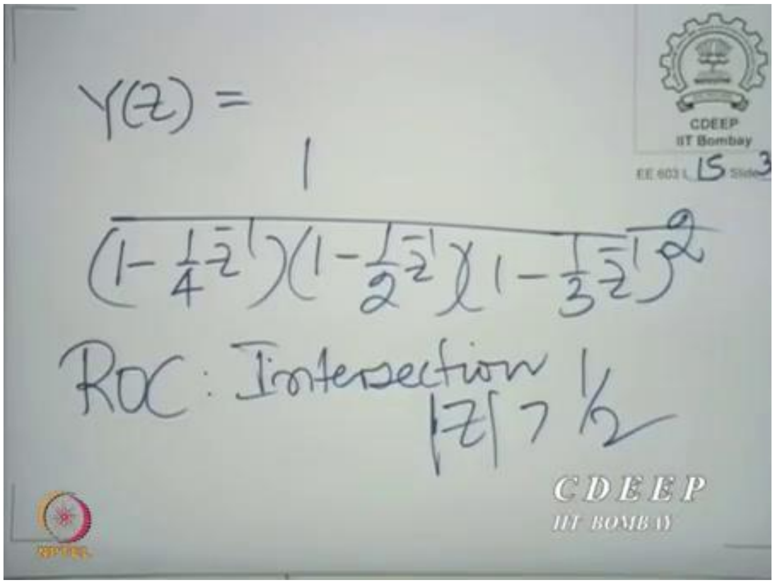

Now, what would, of course, the Z transform of the output is going to be the product to the Z transforms to the input and the impulse response, so we have Y Z is 1 by 1 minus one fourth Z inverse 1 minus half Z inverse, 1 minus one third Z inverse. And of course, here, 1 minus one third Z inverse the whole squared. The note notice that you have now multiple poles at Z equal to one third introduced by virtue of convolution or by virtue of the input being subjected to the impulse response.

Now of course, you would carry out a partial fraction expansion, you need to know the region of convergence, the region of convergence here is the intersection. Here we could take it to the intersection, the intersection, you of course happens to be mod Z greater than half. So, it is greater than one third, greater than one fourth and greater than half as well. You can invert this we can invert, that is very easy to do.

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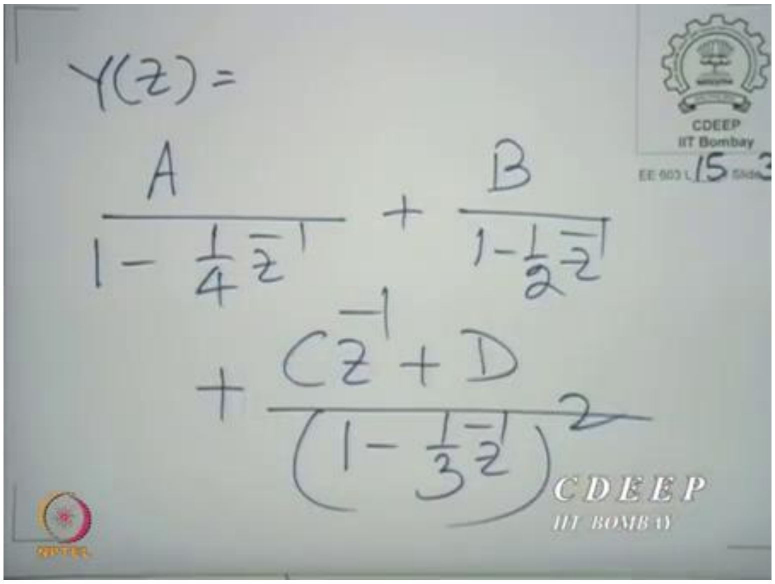

So, Y Z can be decomposed, it can be decomposed in the form A by 1 minus one fourth Z inverse plus B by 1 minus half Z inverse. Now, please note plus CZ inverse plus D divided by 1 minus one third Z inverse the whole squared. And you know how to decompose each of these; you know how to invert each of these terms. But what is to be noted here is that, by virtue of the input and the impulse response coming together or by virtue of the input being applied to the system, even though the input and the impulse response have simple poles that Z equal to one third the output has a double pole.

That means, although you had just a one third to the power n kind of term, an exponential with exponential factor one third present in the input and impulse response separately, when they came together, they gave rise to a new term of the form n times one third raise to the power of n, you know, whatever u n and so on that is beside the point, but an exponential multiplied by a polynomial. So, a new term is created by virtue of the same pole being present in the input and the system.

This would give us the clue how to deal with situations in which the system and the input have similar kinds of terms, the system by system I mean the impulse response of the system. When they have similar kind of terms, there is what is called resonance. In fact, we will look at this in slightly more depth when we talk about linear constant coefficient difference equations. Systems which resonate with the input produce new terms of this kind.

Anyway, what we intended to do today was to illustrate how we could deal with rational Z transforms in much more depth with much more variety. And we also intended to put down some principles of inversion. Let me conclude this lecture with a couple of remarks before we move on to the next lecture, which would deal further with systems, which have rational system functions.

The first point I wanted to mention here was that the word system function and transfer function are interchangeable in discrete time signal processing, we do not clearly distinguish between them. In networks, in electrical networks, people use driving point function and transfer function, there is a slight difference. If the input and output are at the same place, we call it a driving point function, like the impedance.

But if there are different places, we call it a transfer function. The notion of having the input and output different at the same location, different inputs and outputs at the same location does not happen in discrete time systems, we do not have the notion of a driving point function at all, we only have transfer functions or system functions as we understand them.

Secondly, what we have noticed is that inverting a Z transform is an art. It is like, you know, if you if you remember, writing differentiation is a process, we know how to differentiate functions, I mean find the derivatives of functions. Integration of functions is an art. It involves knowing derivatives, recognizing derivatives and inverting them the same is true for the Z transform.

Obtaining the Z transform is often a process you know, it could, it could be done by using standard approaches of summation of series. But inverting a Z transform is an art, it involves recognizing forms and inverting them. There is another way to formally invert a Z transform, if you go back to the interpretation that we gave of the Z transform in the beginning. Namely, the discrete time Fourier transform of a suitably exponentially weighted sequence.

Now, I put it as a challenge before you, come out with a formal way to invert the Z transform based on that interpretation, can you treat the inverse Z transform as the inverse discrete time Fourier transform of some properly chosen sequence and then get back the original sequence from that properly chosen sequence, that is the challenge. With that challenge then, we come to the end of this lecture and we proceed to look further at rational systems in the next lecture. Thank you.