Digital Signal Processing & Its Application
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Lecture 16b
Solving Linear constant coefficient difference equations which are valid over a finite range of time

Now, we share a very important problem or challenge you may look at it in either sense, which nature poses, whether it is in discrete time or continuous time. The ideal systems that we want to realize are irrational, we shall see that very soon. The ideal system, when we put down ideal specifications for what we want to do, the corresponding system turns out to be irrational.

And the whole game that we are going to play for several lectures all over the subject of filter design is to approximate this irrational ideal by rational system. The rational systems see nature, in engineering this is probably a frequent occurrence. What we want is what we cannot realize, what we can realize often not what we want, but we have to draw a compromise between the 2.

That is often the case in engineering and technology and discrete time processing is no exception. Anyway, let us look a little more at rational systems. And specifically, linear constant coefficient difference equations. Now, you see, we have a linear constant coefficient difference equation.

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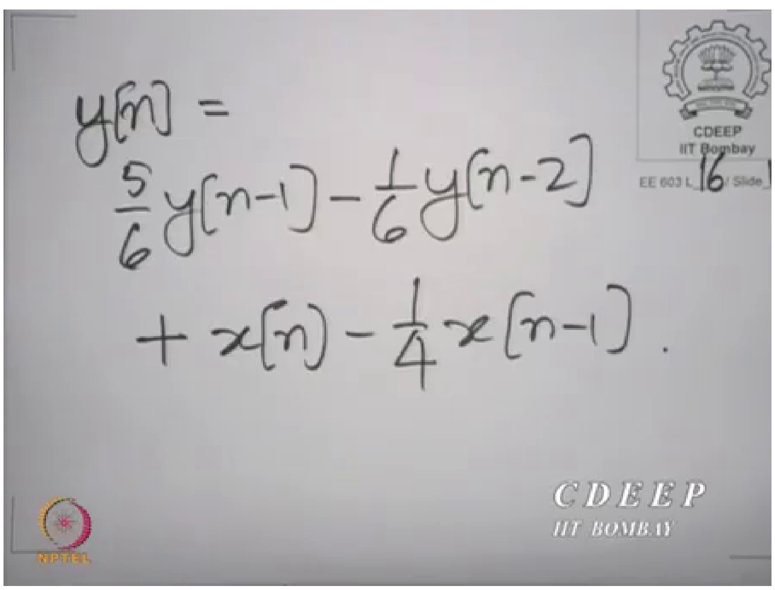

Let us take for example, the linear constant coefficient difference equation that we had a minute ago y n is 5 by 6 y n minus 1 minus 1 by 6 y n minus 2 plus x n minus 1 by 4 x n minus 1. Now, I want to make a few remarks about solving such difference equations. By what I mean by solving is you are provided with the value of x n over a certain interval of time.

So, let us take an example. Let us assume, assume that x n is known, known for n equal to let us say capital N1 to n equal to capital N2, in particular capital N1 might be 0 and capital N2 might be say 20 or 100. So, what we are saying is in a sense is that you know the input over a certain interval of the natural domain.

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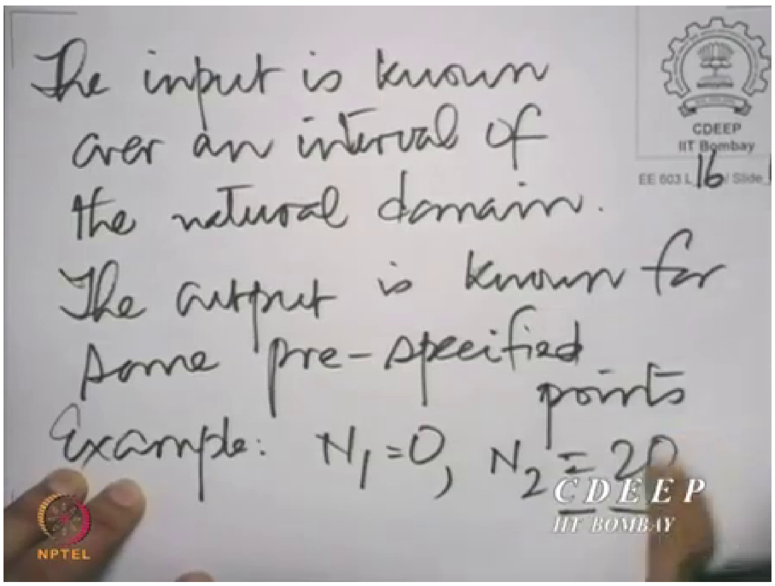

So, the input is known over an interval of the natural domain. The output is known for some pre specified points. Example, suppose N1 is equal to 0, N2 is equal to 20 we continue on this.

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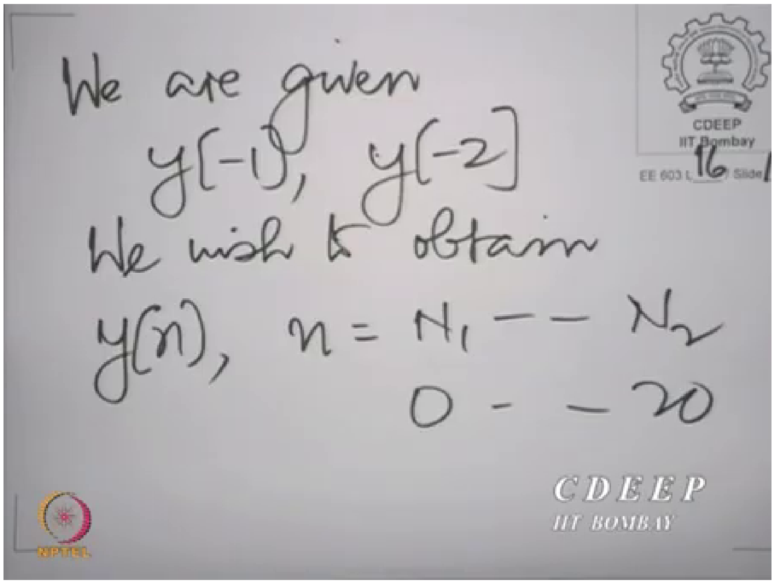

We are given y of minus 1 and y of minus 2. So, you know the, what we call the initial values. And we wish to obtain y of n, for n equal to N1 to N2, namely 0 to 20. How would we proceed in this case? Well, the approach to proceeding is to for a moment, assume that this difference equation lasted forever.

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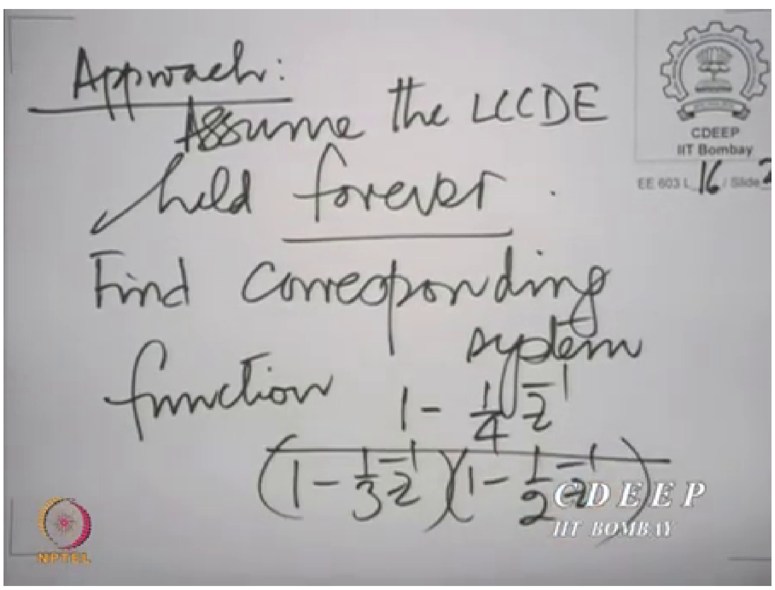

So, the approach is assume the difference equation lasted forever, assume the LCCDE held forever. Find the corresponding system function, what do you mean by the corresponding system function the ratio of y z to x z. And what is that corresponding system function here, you know it, it is 1 minus one fourth Z inverse divided by we began with 1 minus one third Z inverse into 1 minus half Z inverse.

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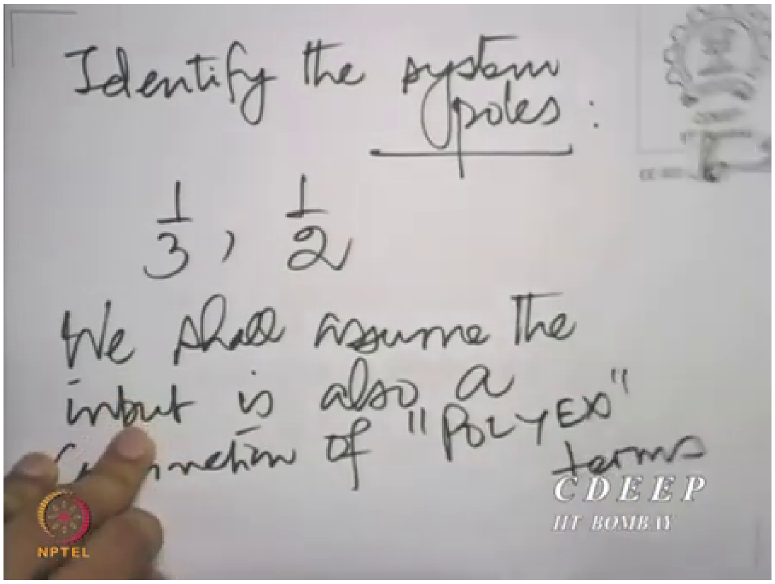

Identify the poles, identify the poles. Identify what are called the system poles now, where are the system poles, here they are at one third and half. Now, of course, we are only going to deal with the solution of such equations when the input also has a rational or you know the input is also a sum of what are called poly externs. So, we shall assume, the input is also a combination of what are called polyex terms. I will explain what are polyex terms, polyex refers to polynomial multiplied by exponential.

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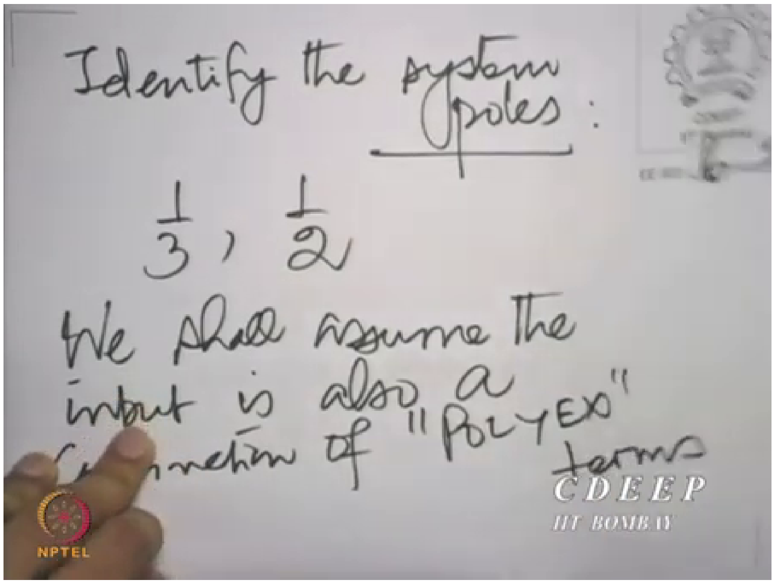

So, for example, you could take the polyex term, example of a polyex term, 2n squared plus 5n plus 1 multiplied by the one fifth raise to the power of n, this is a polyex term, this is the exponential part of the term and this is the polynomial, polynomial in the natural domain variable. Poly comes from here and ex comes from here.

A polynomial in the natural variable multiplied by an exponential is a polyex term, now of course you can see a pure exponential is also a polyex term with the polynomial of degree 0. A pure polynomial is also a polyex term with the exponential factor equal to 1. A constant is a very special case of a polyex term with the degree of the polynomial equal to 0 and the exponential factor equal to 1 and so on.

So all these are covered in the class of polyex terms. So, when you have a polyex input as we call it that means an input which comprises of a sum of polyex terms we can solve this linear constant coefficient difference equation very easily. What we do is to identify what we call the input poles.

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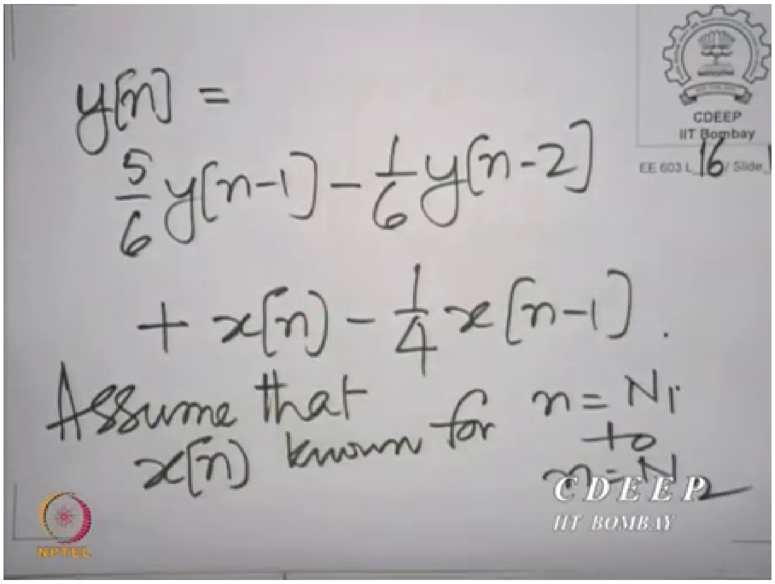

Now, now here for example, the pole corresponding, so the pole corresponding to this polyex term is one fifth but with a multiplicity of 3, that means one fifth taken 3 times. Why is it taken 3 times because there are 3 coefficients in the polynomial, polynomial as of degree 2, so it has 3 degrees of freedom, the constant part of the polynomial, the first degree part and the second degree part.

So as many as are the degrees of freedom in the polynomials, so many times is that pole repeated. If the polynomial is of degree 0, of course, that pole occurs with multiplicity 1 that means, it occurs only once. If the polynomial is of degree 1 that pole occurs twice, if the polynomial is of degree 2 as we see here, that pole occurs 3 times, and so on so forth.

Now, of course, you must remember, even if you have just an n squared term and a 0 degree term, you must still assume that you have a repetition 3 times, the pole is repeated 3 times, it is the highest power of n that matters. So the eventually the multiplicity of the pole is 1 more than the highest degree of highest power of n in the polynomial.

So, suppose you apply, now let us take the simple case, suppose you apply this polyex term as the input x n, of course, valid only in that region 0 to 20. Now, remark about how large or how small that region can be. Large is no problem, it can go all over the integer axis, small is a problem, you cannot make it smaller, than the degree of the difference equation, that means the highest past sample that you need to deal with.

In fact, here that you need to deal with two past samples, you cannot just assume that this, this difference equation holds for 3 samples and then solve it by the approach that you are using. If the length over which this difference equation holds is large enough to cover the span of the difference, span means how many samples you're involving at a time, then it is valid to use the procedure that we are describing.

Otherwise, there is no solution but to operate the difference equations step by step. So, of course, you know, y n in terms of y n minus 1, y n minus 2, you know, let us go back to that difference equation. So, if you look at it, you can always sit down and solve it term by term, you know, let us go back to this equation here. So, here you have y n is 5 by 6 y n minus 1 minus 1 by 6 and so on.

So, you know, if you know y of minus 1 and if you know y of minus 2, and you know x of 0 and you know x of minus 1, then of course, I can find y of 1 or y of 0. Once you know y of 0, you can go one step further, you can find y of 1. So, you can keep solving this difference equation step by step and obtain the output that you can do, when the difference equation is valid only for two small interval.

But that is not a practical way of doing it when it is valid for a very long time. And what we are describing is a process for solving it when it is valid for a very long time. If it is valid for a very small time, it is not worth doing all that we are talking about. Anyway, so long enough means, long time means, long enough for all the samples to be covered, samples that are involved.

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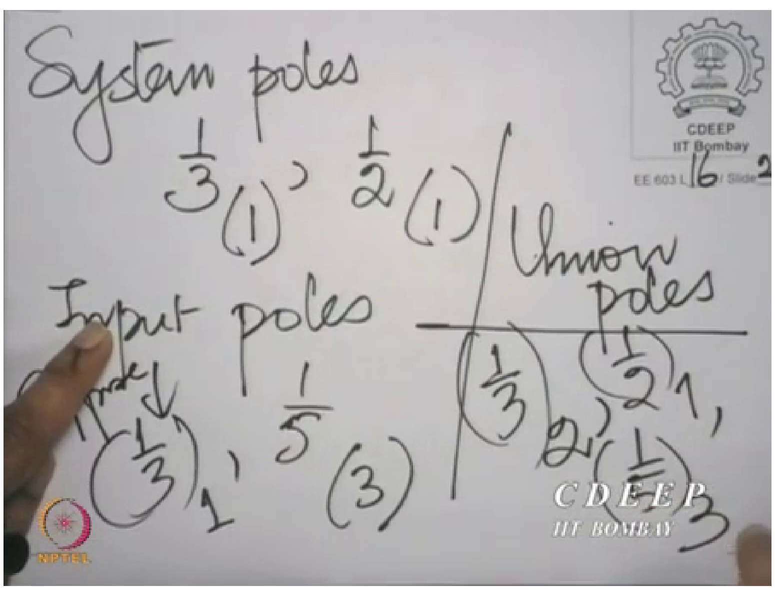

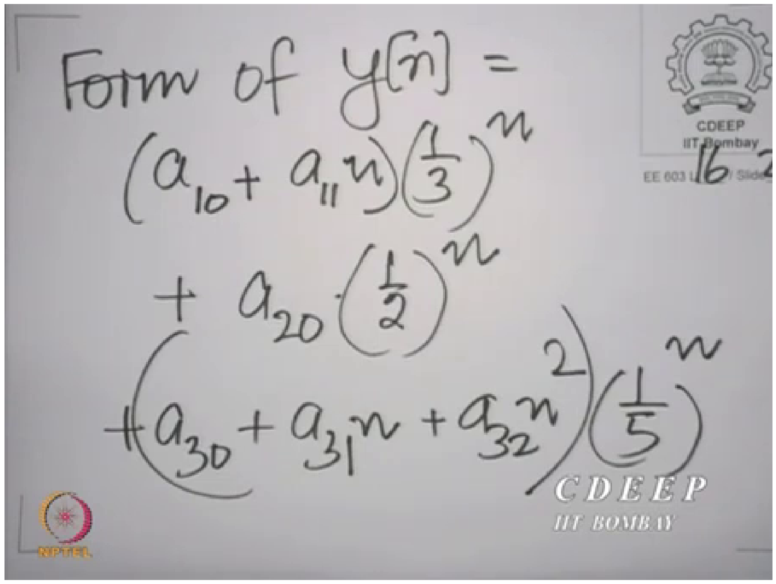

Now, in this case, we say the system poles are at one third and half. The input poles, and you know with each pole you write its multiplicity, the input poles are at one fifth and with a multiplicity of 3. Now we write down what are called the union poles, we write down what are called the Union poles. Now what we mean by the union poles are, the poles of the system union with the poles of the input. And now how do you take, now I said union, not union means when you have a common pole, you must bring them together.

Now of course in this case, there is no common pole. But suppose, we also had one mod, we had a one third with a multiplicity of 1 here, also in the input, we introduce that too. That means the input is a combination of two polyex term, a one fifth to the power of n multiplied by a polynomial of degree 2 plus a one third to the power of n multiplied by a polynomial of degree 0.

Suppose it is the input is a combination of 2 polyex terms, what would be the union poles? Now look for the common poles, which are the common poles, one third, one third is common. Now, one third occurs with multiplicity 1 here and multiplicity 1 here. So, in the union it will occur with multiplicity 2, half occurs with multiplicity 1, one fifth occurs with multiplicity 3, this is the set of union. Now, once you have the union poles, you know the form of the output immediately, you know the form of the output immediately.

What is the form of the output, the form of y n, now take each pole in turn. One third occurring with a multiplicity of 2 contributes a10 plus a11 n times one third to the power n half with a multiplicity of 1 contributes a20 into half raise to the power of n. And one fifth with a multiplicity of 3 contributes a30 plus a31n plus a32n squared into one fifth raise to the power of n, this is the form of y.

Any questions on this so far, this is a very important point. If we understand this half the job is done. Now, all these a's, a10, a11, a20, a30, a31, a32 all of them are unknown constant, we have to determine the constants. How do we determine the constants, we determine the constants by imposing the constraints that are given to us.

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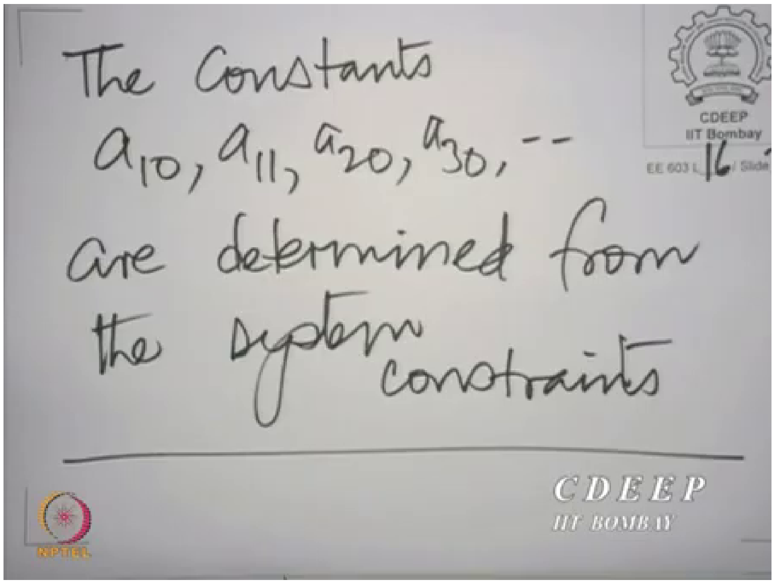

So, the constants are determined, are determined from the system constraints. Now, in fact, what do you mean by the system constraints, one constraint is the LCCDE itself. So, put this y n back into the LCCDE and some constants will emerge on their own, then take into account the conditions on why that are given to you, some more of these constants will be constrained.

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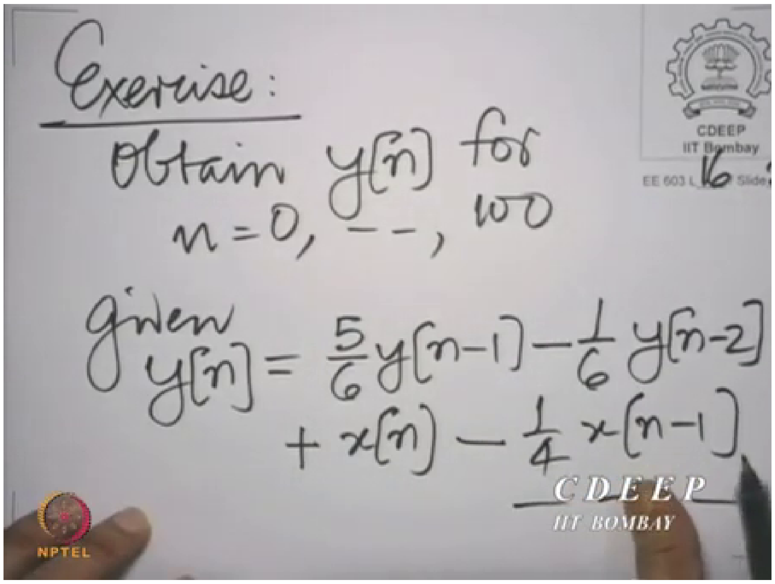

So, I give you a little exercise to help you understand this concept better, the exercises is as follows. Obtain y n for n equal to 0 to 100 given that y n minus 5 by 6 y n minus 1 or you know we will do the same thing 5 by y n is equal to 5 by 6 y n minus 1 minus 1 by 6 y n minus 2 plus x n minus 1 by 4 x n minus 1.

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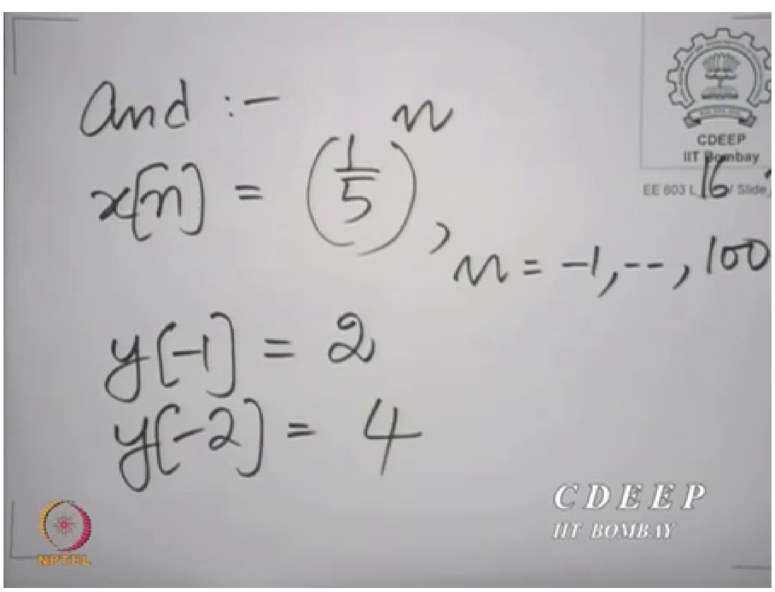

And x n is equal to 1 by 5 raise to the power of n for n equal to minus 1 up to 100. Y of minus 1 is equal to 2, and y of minus 2 is equal to 4. So, I leave this as an exercise for you to work out, using the process that we have just described. Now, one remark about resonance.