Digital Signal Processing & Its Application

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Lecture 16c
Introduction to Resonance in Rational Systems

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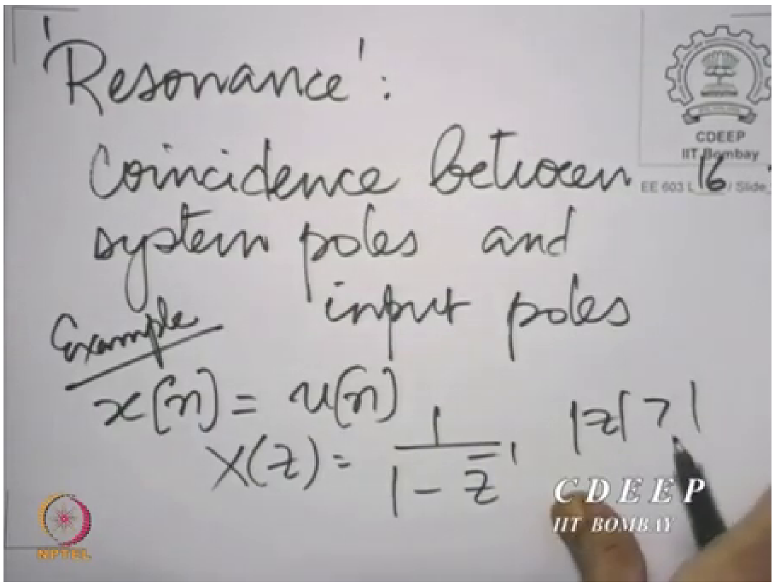

1 remark about resonance, we briefly alluded to resonance in the previous lecture, we said resonance occurs when there is a coincidence between system and input poles. For example, we could take x n equal to u n, what is the Z transform, the Z transform is 1 by 1 minus Z inverse with mod z greater than 1. Let us take h n to be the same sequence.

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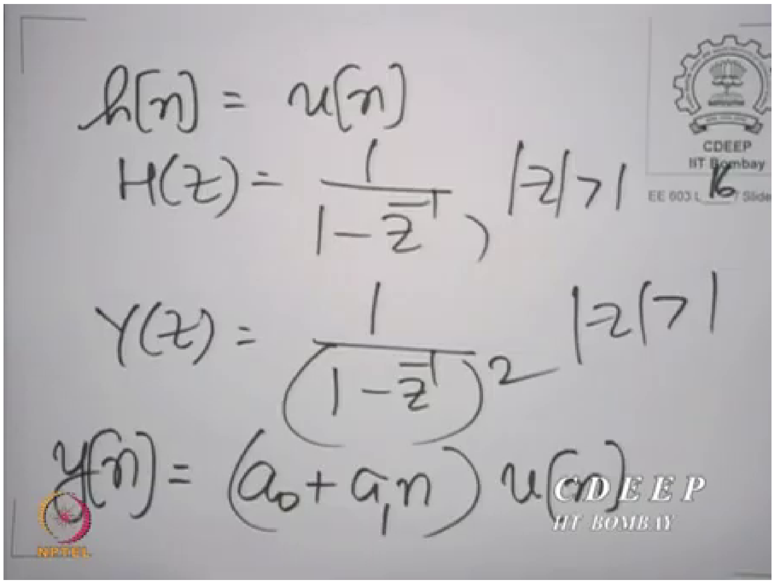

And whereupon, you have H Z is also 1 by 1 minus Z inverse with mod Z greater than 1. Now, Y z of course, would be 1 by 1 minus Z inverse the whole squared with mod Z greater than 1. And we know the form of Y Z, the form of y n, y n would be of the form some a0 plus a1 n times u n. Now it is very interesting, here is a system where the input, of course, has a pole located, co located with the impulse response, it is the same place.

If this were to be treated as an LSI system with this impulse response, that LSI system is unstable. Because the input is the, impulse response is not absolutely sum of them. So, it is an unstable system. But what we see is where this instability is manifested, the instability is manifested when we irritate the system by giving it the same input as the impulse response otherwise, that instability is not manifested in the system.

Here you irritated the system by giving the same input as the impulse response to it otherwise, the system is, you know, unstable, but, you know, it is a, it is an unstable system in a very hidden way, it is a you know as they say latent, latent you know, it is not easy to identify the instability in the system. It is that proverbial case, where you need to find a cause to irritate the system, that cause is giving the input which is the same as the impulse response. If you give it any other kind of input, which is absolutely summable.

You know, for example, if you were to give it some other complex exponential sequence, not just constant, you would find that the output is bounded. If you give it a sine wave as an input, starting from some point, you would find that the output is bounded. But it is for this troublesome case that the output becomes unbound. Of course, this output is unbounded. Now, some people like to call such systems marginally stable, of course that marginally stable is only an informal term.

Now you understand why marginally because, you know, it is it kind of instable in most cases, but then it has these troublesome cases where it is, it manifests its instability. Now, this idea of resonance is an idea which is used very frequently in analog service. Resonance happens when the input and the impulse response match, you know.

And the consequence of resonance is that although the input and impulse response by themselves might be bounded, the response is unbounded. In fact, this used to happen when soldiers mass marched across a bridge, this is a common example given. Typically, a bridge is constructed so that it remains stable with all its mechanical elements.

But there are certain unfortunate frequencies of resonance in a bridge, when a bridge is constructed, the mechanical elements could have certain resonant frequencies. And you know, when people randomly walk across the bridge, there is no problem. None of those frequencies are given in a significant way.

But when you have these soldiers marching in strict rhythm across the bridge, if that rhythm happens to resonate with the system frequencies, bad luck, the bridge begins to oscillate, and the output of the bridge oscillations can grow without bound, and the bridge can collapse. This was observed many times when soldiers marched across the bridge.

And people were puzzled, why should it happen? When people walk randomly, nothing happens. But when people march in rhythm, suddenly you have resonance. Yes, there is a quest. The question is, if you have and if the system allows the output to grow without bound, where is the question of conservation of energy?

Now, you know, if you look at it that way, an unstable system is only a concept. So his question is, you know, where is the conservation of energy law valid? Well, actually, yes, I mean, therefore, you know, an unstable system can only be approximated. For example, if you have a bank account, you know, you will never put your money into a bank where the bank account is a stable system.

You would like your money to grow without bound in principle, but of course, money cannot grow without bound, there is a finite limit to the economy of the country. So, an unstable system is a concept you know, it operates unstably up to a point. For example, when a transistor starts getting heated, you know, and heated inordinately that happens up to a point where the transistor remains a transistor after some time it burns and that system disappears.

So instable, instability is a concept and valid only for a certain time. Anyway, with that observation of resonance, we conclude this lecture. And we shall in the next lecture go deeper into the question of when rational systems are stable and causal. Thank you.