Digital Signal Processing & Its Application
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Lecture 18c
Four Ideal Piecewise Constant Filters

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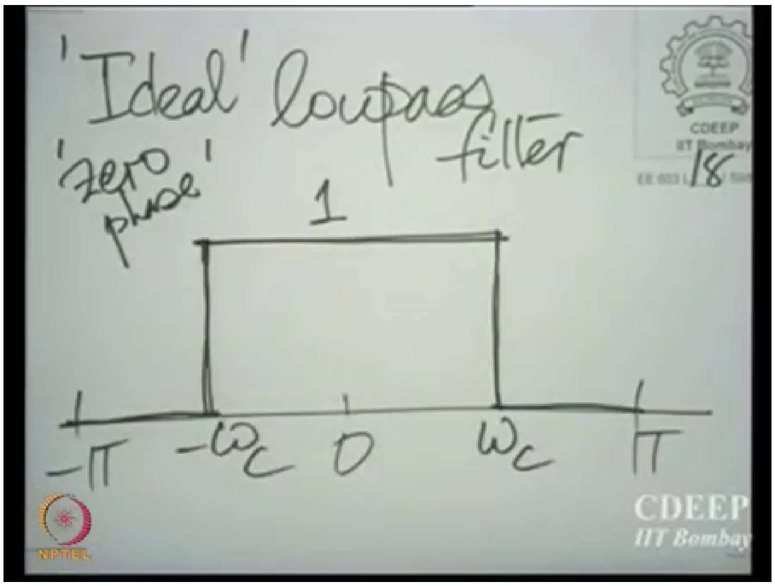

Now, the first of the four ideal piecewise constant filters is the ideal low pass filter. And the ideal low pass filter would have a frequency response, which has zero phase. In fact, all the filters ideally would have a zero phase response. And a magnitude response that looks like this, one between minus omega c and plus omega c and zero else.

Now this is very clearly an illustration of why we need to talk about defined almost everywhere. If you look at this response, it is precisely specified for all points between minus pi and pi except at the points omega c and minus omega c. At the points omega c and minus omega c the responses are discontinuous, the response is discontinuous.

So, it cannot be specified the right limit and the left limit are not equal. So, at that point, it is not really correct to talk about the value of the magnitude or the value of the response. Therefore, the frequency response exists at all points except at omega c and minus omega c. And therefore, we say that for this ideal filter the frequency response.

I mean by whatever it is, I mean, there is an impulse response that would possibly correspond to this frequency response, the discrete time Fourier transform of that impulse response is defined almost everywhere, except for the point omega c and minus omega c. At those two points it does not converge. That is why we needed to put the almost everywhere.

Anyway, so this is the ideal low pass filter. Correspondingly, we can have now you know, we this also tells us why we are calling it piecewise constant, the ideal response is constant between minus omega c and plus omega c, constant between omega c and pi, constant between minus pi and minus omega c. Here it is zero ideally, here it is zero ideally.

And here it is one, ideally, in magnitude. From on pieces, you can divide the frequency axis between minus pi and pi into a finite number of pieces. And on each of these pieces, the response is a constant, that is why we are calling it piecewise constant. So, the ideal responses that we are trying to meet are all piecewise constant. Let us look at the other piecewise constant responses that we would try to design.

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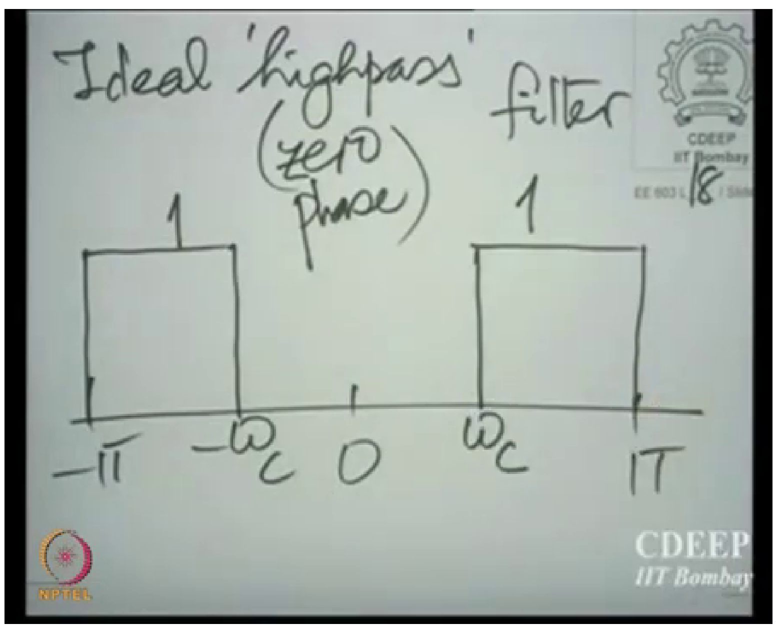

The ideal high pass filter. High and low remember, is only between 0 and pi. So here the, the ideal high pass filter would ideally take a magnitude of one from some omega c to pi and from minus pi to minus omega c and zero else. So this is the ideal high pass filter, of course, it is zero phase and this is the magnitude.

Here again, you notice it is piecewise constant, its constant on the three pieces minus pi to minus omega c minus omega c to plus omega c and plus omega c to plus pi.

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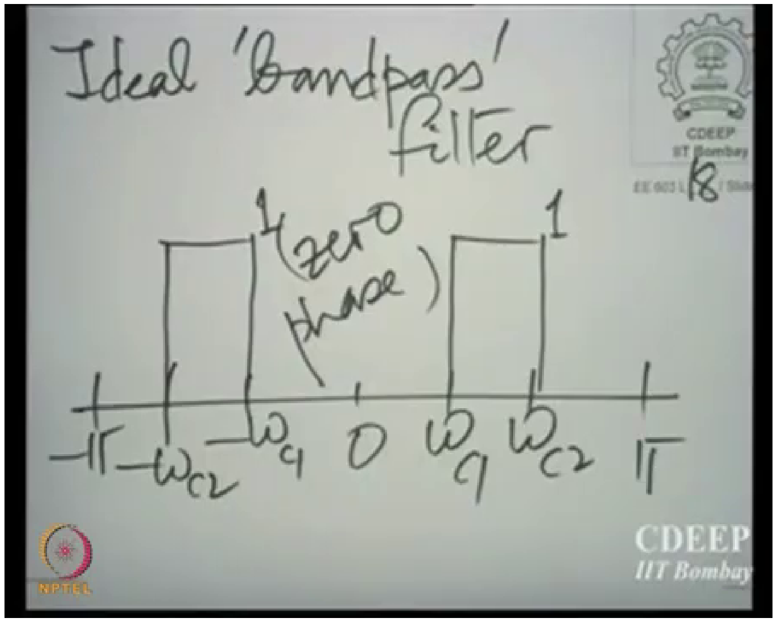

We then have the ideal bandpass filter and that is easy again. There are two points omega c1 and omega c2, between those two points, the response is a 1 and otherwise it is 0. And of course, once again it is zero phase. Here again, it is piecewise constant. There are exactly 1, 2, 3, 4, 5, 5 pieces on which the response is the constant.

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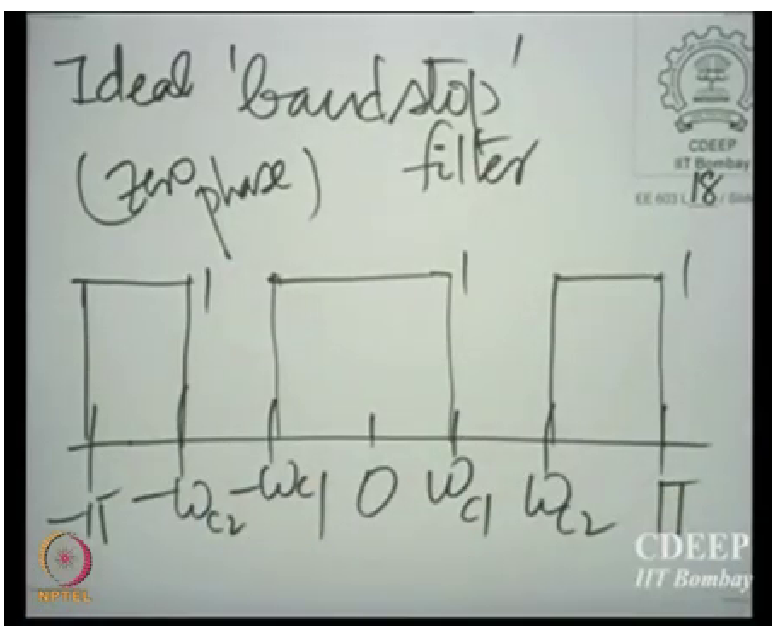

And finally, we have an ideal bandstop filter. As the name suggests this filter would stop a band and pass the rest, it would pass the rest and stop a band to stop the band between omega c1 and omega c2 and pass the rest. And here again, this is zero phase and is piecewise constant on 5 pieces, this one, this one, this one, this one and this one.

It is 1 on these three pieces and 0 on these two pieces. Now, we should also note that it is possible to obtain the ideal impulse response corresponding to each of these ideal filters. And we shall illustrate that by starting with the low pass filter. In fact, we have already done that job; we do not need to repeat it.

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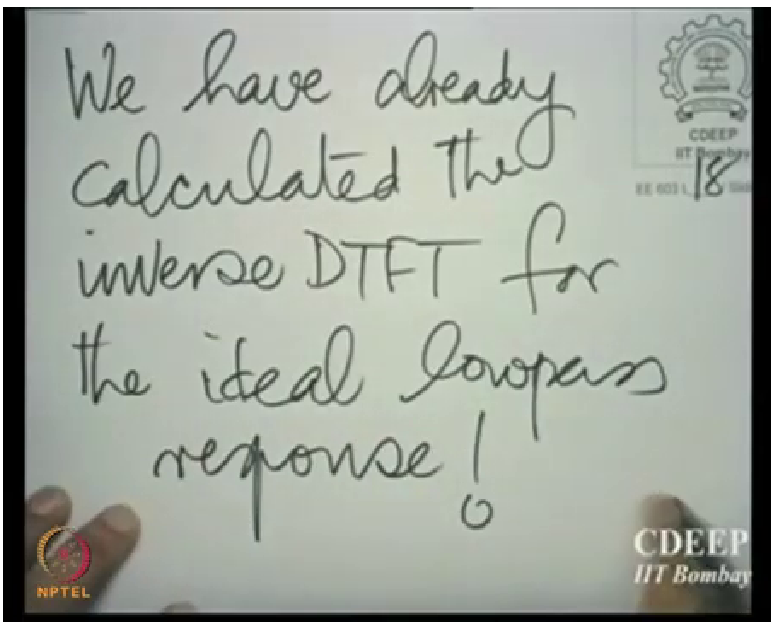

We have already calculated the ideal impulse response of the low pass filter. We have already calculated the inverse DTFT of this, for the ideal low pass response and we can straight away write that down.

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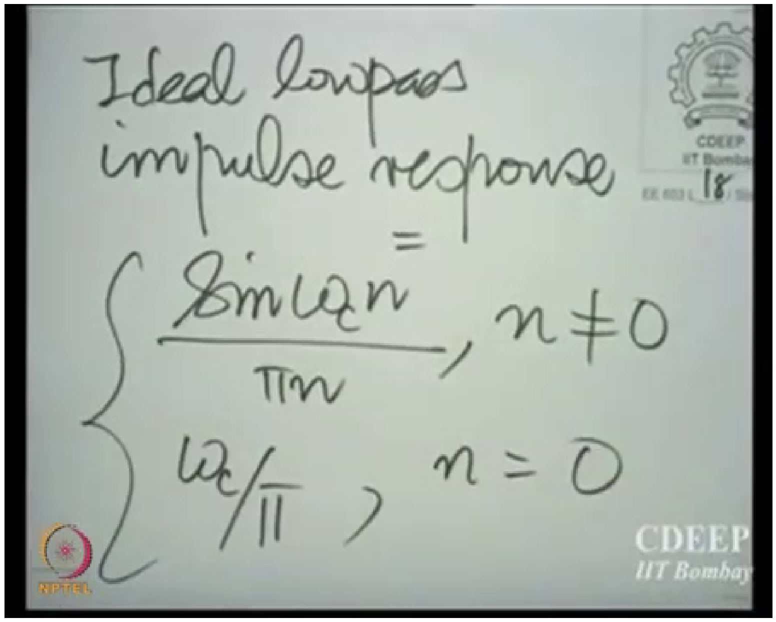

And the ideal low pass impulse response is equal to sine omega cn divided by pi n whenever n is not equal to 0 and omega c by pi when n is equal to 0. We have already calculated that, correct.