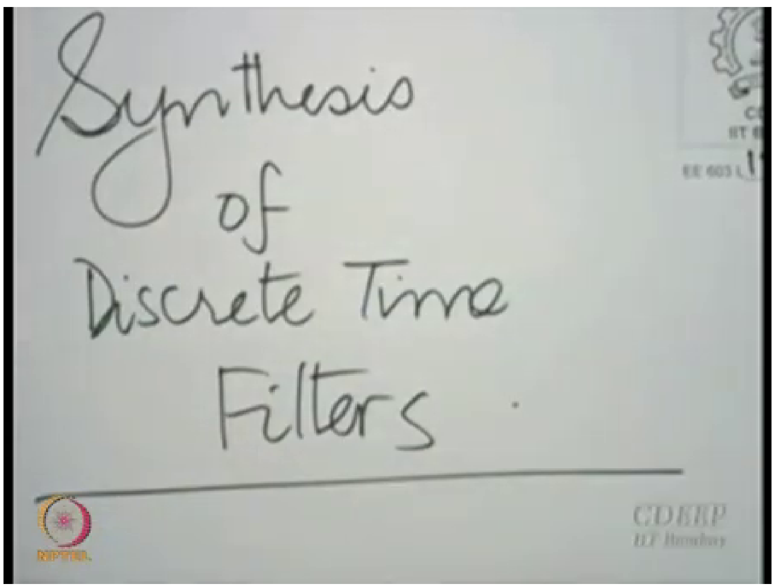
Digital Signal Processing and Its Application
Professor Vikram M. Gadre
Department of Electrical Engineering
Indian Institute of Technology, Bombay
Lecture 19a
Magnitude Response Specification for Realizability

A warm welcome to the 19th lecture on the subject of Digital Signal Processing and Its Applications, let us take a few minutes to recapitulate what we did in the previous lecture. In the previous lecture, we had begun discussing the synthesis of discrete time systems.

(Refer Slide Time: 0:34)



In fact, the synthesis of digital filters, we had looked at the ideal filters, the four ideal filters that we most commonly encounter namely the ideal low pass filter, the ideal high pass filter, bandpass filter and band stop filter. I had taken the specific instance of the ideal low pass filter. And we had looked at the impulse response of the ideal low pass filter.

From the impulse response, we had drawn three, in fact, from the frequency response the ideal frequency response, and from the impulse response, we had drawn three conclusions. The first one was that the filter was, of course, non-causal. But unfortunately, it could not even be made causal if we allow to delay, a finite delay.

So, by introducing a finite delay, we could not make the filter causal because the number of non *0* samples on the negative side of *n* was infinite. So, you see, if it were finite, if you have a finite number of negative samples or samples on negative *n*, then you can take the farthest back samples.

So, for example, suppose you have samples which are non *0* for negative *n*, all the way from *n* going from *0* to *n* going to -20 , you can introduce a delay of 20 samples and make the filter positive. If you have a finite number of non 0 samples on the negative side, it is always possible to make the filter causal even if it is non causal to begin with. Unfortunately, that is not the case with the ideal filter.

Secondly, we noted that the ideal filter was unstable, we took the specific example of $\omega_c = \frac{\pi}{2}$, but I encourage you to take other examples of ω_c as well. So, you could look at $\omega = \frac{\pi}{4}$ if you like, or $\omega = \frac{3\pi}{4}$ or some other values and convince yourself that in each of these cases, the filter would become unstable.

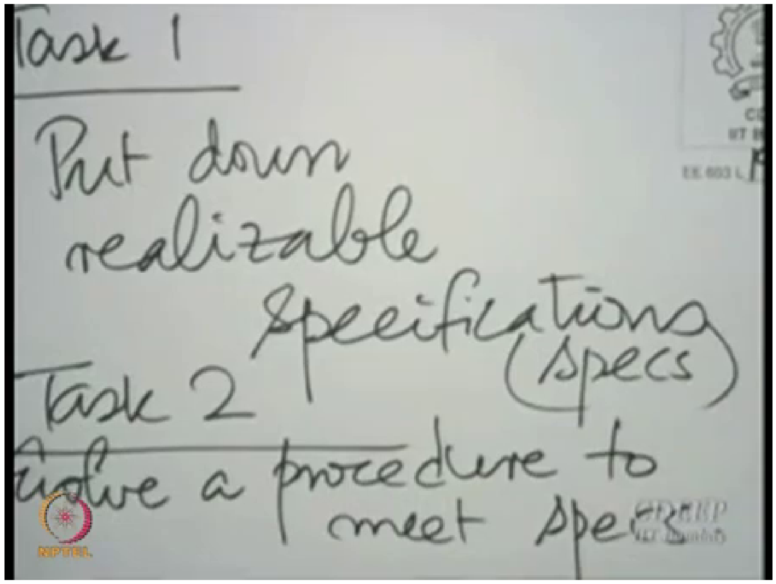
In fact, I put it before you as a challenge to show that for general ω_c , this filter is unstable. That is a bit of a more demanding challenge, we had shown it for the specific case of $\omega_c = \frac{\pi}{2}$, but show it more generally for any ω_c between 0 and π and the filter is unstable. Essentially, you will need to show that the filter impulse response is non summable, non-absolutely summable.

Thirdly, we noted that the ideal filter was irrational and that was because we could not have a continuum of zeros in the frequency response if the filter were rational. Now, of course, some people like to talk about what is called the Paley Wiener criterion. You know, possibly at this stage, I think it is adequate to note that for a rational function, you cannot have a continuum of zeros as we do in the ideal filter.

And in fact, because the ideal filter is irrational, we are now left with the trouble that it is unrealizable. So, on all these counts, the filter is describable, but not realized, describable means you could put down an impulse response for it, you could describe what it does, but you can never realize it.

What we wish to do in the process of filter design is to realize meaningful specifications. We said that yesterday. If we put down the specifications as we did yesterday, for example, for the ideal low pass filter or, for that matter for the other kinds of filters as well. We are not in a position to ever meet those specifications with any rational system.

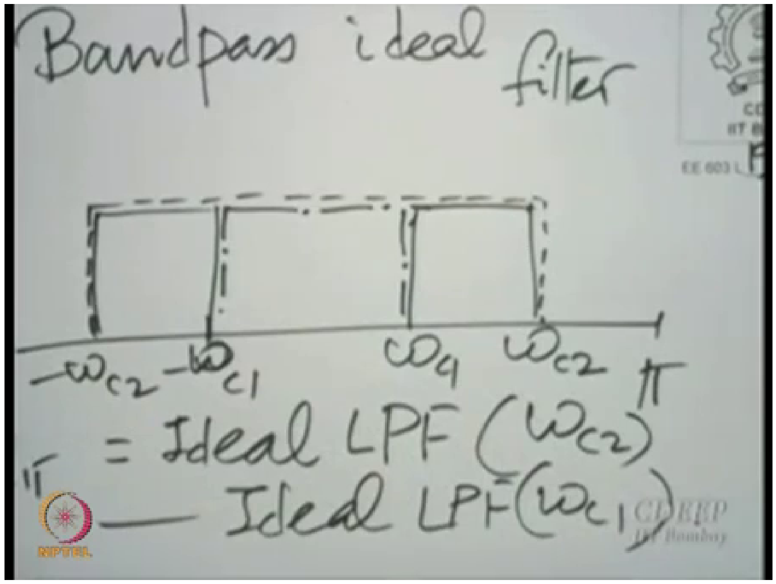
(Refer Slide Time: 5:13)



So, one task that we have before us today is to put down realizable specifications.And the second task not just today but onwards from now is to evolve a procedure to meet the specs. We shall in future abbreviate specifications by specs so we should be would need to evolve a procedure to meet the specifications.

Before we go on to discussing meaningful specifications we must complete one little detail and that is we saw the ideal impulse response of the low pass filter but we did not really look at the ideal impulse response and the other three kinds of filters. Let us spend a minute in writing down a process which will take us from the low pass to the other kinds of filters.

(Refer Slide Time: 6:11)

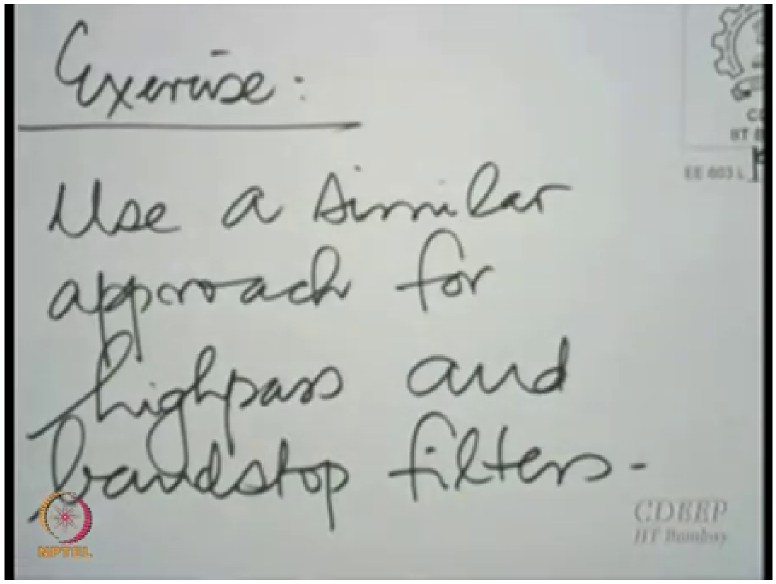


So, in fact that is very easy if you look for example at the bandpass filter, its ideal response is between ω_{c1} and ω_{c2} and of course its mirror image on the other side $-\omega_{c1}$ to $-\omega_{c2}$. It is very easy to see that this can be construed to be a filter which emerges from three low pass filters, 1 with a cut off at ω_{c2} and the other with a cut off at ω_{c1}.

So, essentially a bandpass filter is an ideal low pass filter with a cut off at ω_{c2} minus an ideal low pass filter with a cut off at ω_{c1}. Now, the discrete time Fourier transform and the inverse discrete time Fourier transform are both linear operators and therefore if we have this relationship between the frequency responses the same relationship would carry over to the inverse discrete time Fourier transform.

So, therefore one can compute the ideal impulse response of the bandpass filter by using a combination of 2 impulse responses.

(Refer Slide Time: 7:49)

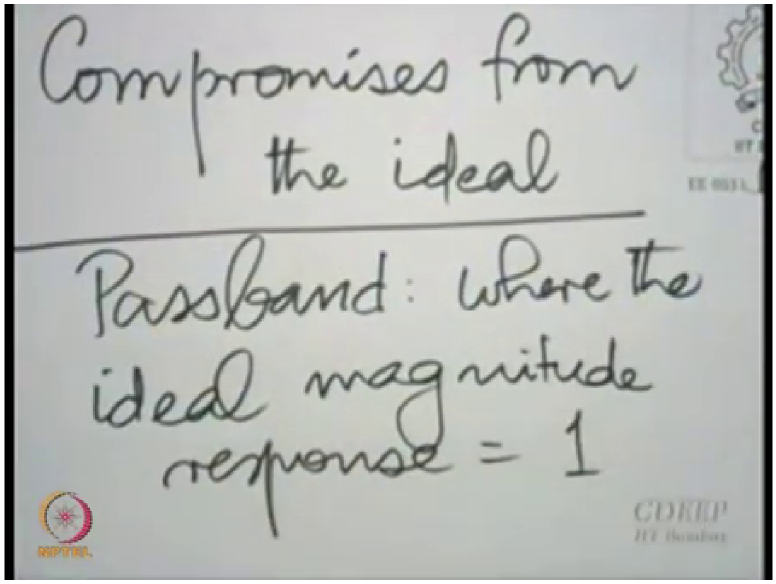


Now, I leave it to you as an exercise to do the same for the other 2 ideal filters. Use a similar approach for the high passband stopband filters essentially the approach involves invoking the linearity of the discrete time Fourier transform and the inverse discrete time Fourier transform. So, we talked and we are in a position to compute all the ideal responses but as we have noted n1 of them is going to be realized.

Now, of course, I have not shown it for each case but I leave it to you as an exercise to generalize this for the other cases. In fact, you could take a bandpass filter, for example, with a cut off between $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ if you like and then see what happens to it is it stable or not stable.

And in fact, you will find that all of them are unstable and unrealizable anyway. Now, let us get down to business by putting down specifications that we can actually realize. So, we need to understand what we need to compromise from the idea there are three things that we need to compromise from the idea.

(Refer Slide Time: 9:13)

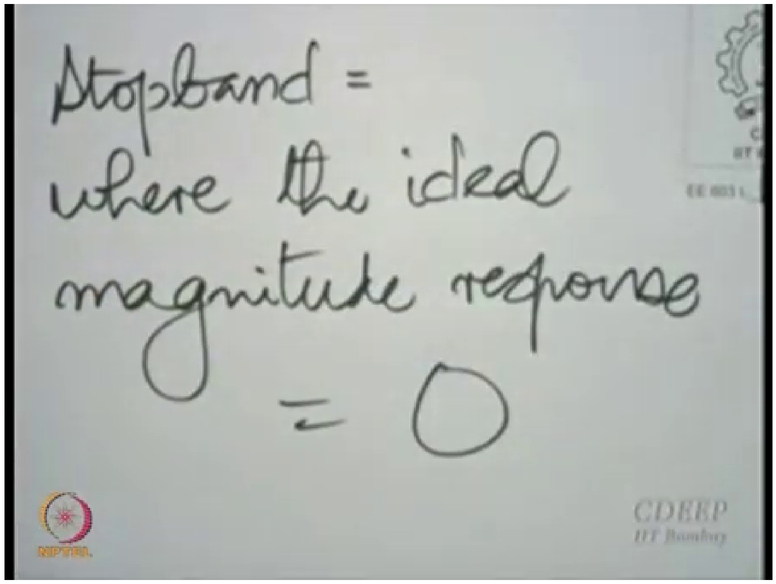


You see that part, now you notice in all the ideal responses that we have taken here, the responses, the frequency responses are piecewise constant. In fact, we explain what piecewise constant means but it is more than just piecewise constant. In fact, you can classify them as some regions which you want to pass and some regions that you want to stop.

So, it is more specific than piecewise constant. You can also have piecewise constant responses where different parts of the frequency axis have different non zero magnitudes ,that is also possible that is more general, but we are not looking at that case here. We are looking at a case where each band either carries a 1 response rate or a 0 response on it.

So, therefore, we have the notion of pass bands were the ideal magnitude is 1, so in the pass band you are trying to make the ideal response, ideal magnitude response equal to 1.

(Refer Slide Time: 10:38)



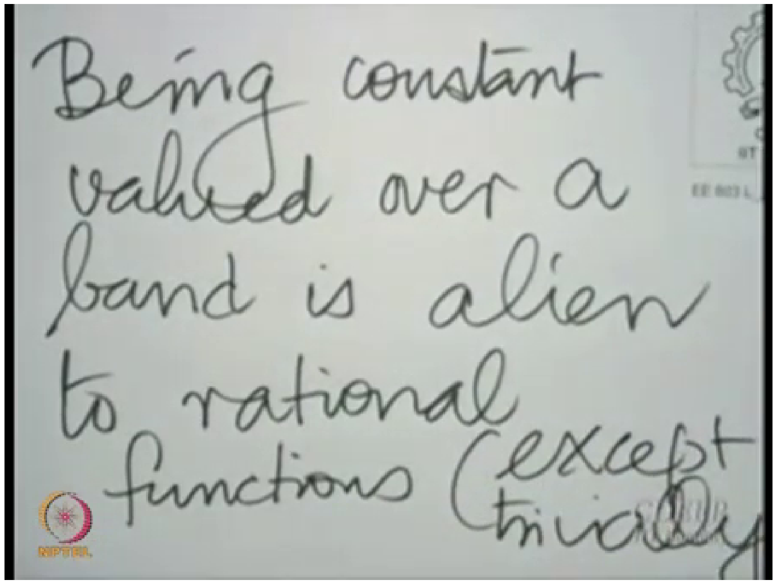
And in the stopband you are trying to make the ideal magnitude response equal to 0 and these are the only 2 kinds of bands that we have in the filters that you have seen. So, the first compromise that we need to effect is that we cannot have the magnitude response 1 or for that matter constant either in the passband or in the stopband if we wish that the filter be rational. Now, again a minute's thought will convince us why this is so.

Suppose indeed the response for constant in the pass band, it is very easy to see that if you have a rational system which gives you a constant value all over a continuum of the independent variable, there if you subtract 1 from that, let that constant value be 1 without any loss of generality, it could be any constant does not matter.

If you subtract that constant from the rational function the resultant function must be rational. A constant is the rational function. The difference of 2 rational functions is always rational. So, you have here a rational function which then becomes 0 over a continuum and you run the same problem you run into a contradiction.

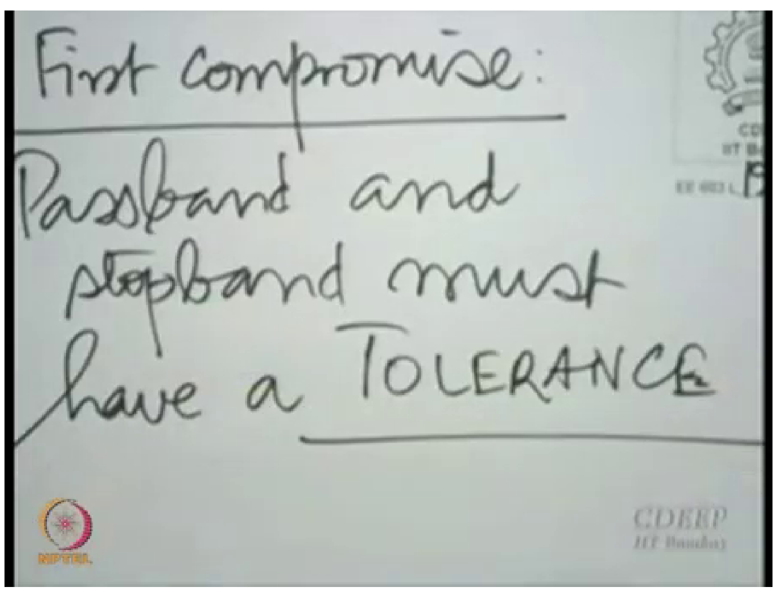
So, either the rational function is trivially 0, which of course, is totally useless or there is a contradiction you could not have had a rational function in the first place which is constant all over a band. So, the very idea of being constant over a band is alien to rational functions and therefore the first compromise that we must live with.

(Refer Slide Time: 12:37)



Being constant valued over a band is alien to rational functions except trivially, by trivially I mean the rational function itself is a constant, then of course it is of no use at all. Is that correct? So, therefore we have to compromise that, we cannot insist that the rational function be constant over a band.

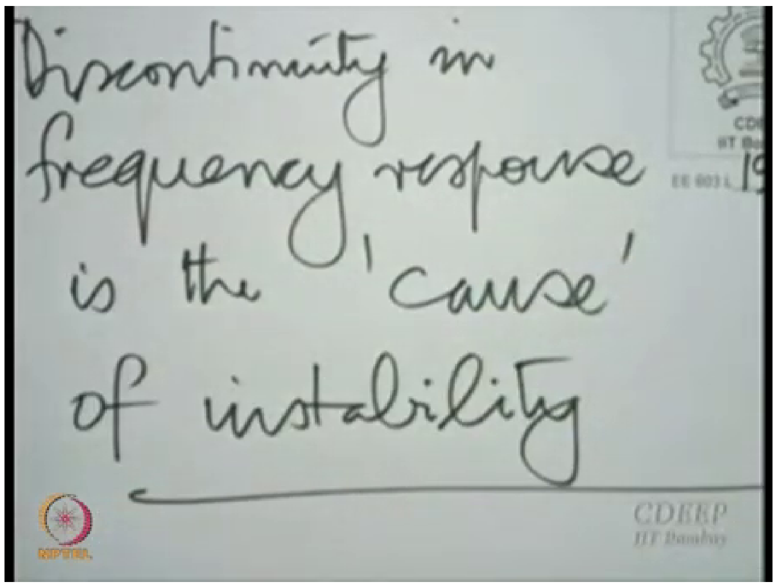
(Refer Slide Time: 13:26)



And therefore our first compromise is that passband and stopband must have a tolerance, by tolerance we mean that the magnitude response is allowed to vary in a certain region, we cannot insist that it be a constant. Now, the other thing that disqualifies the ideal filter is a discontinuity. In fact, I briefly remarked about this in the previous lecture.

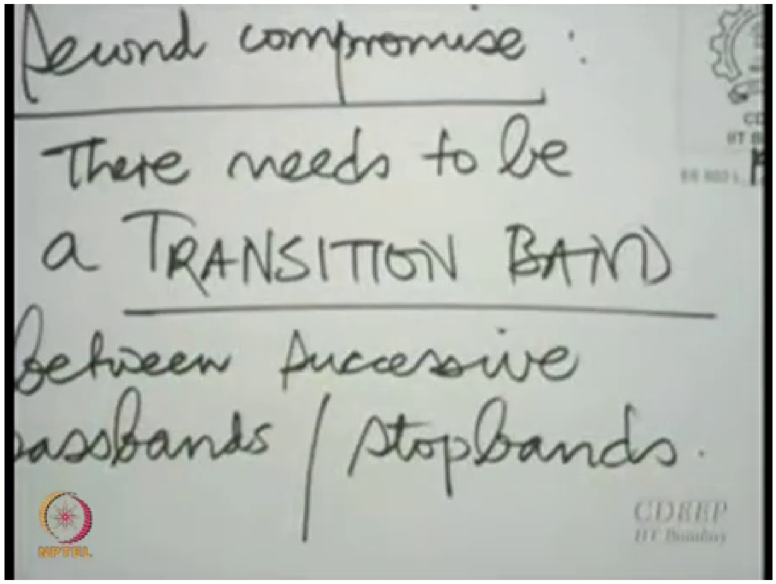
I put the challenge before you to show that the fact that you have a discontinuity is the cause of instability. It is a challenging problem, not at all simple to solve. Anyway the discontinuity is the trouble in the ideal field, therefore the next compromise that we need to make is not to have a discontinuity.

(Refer Slide Time: 14:43)



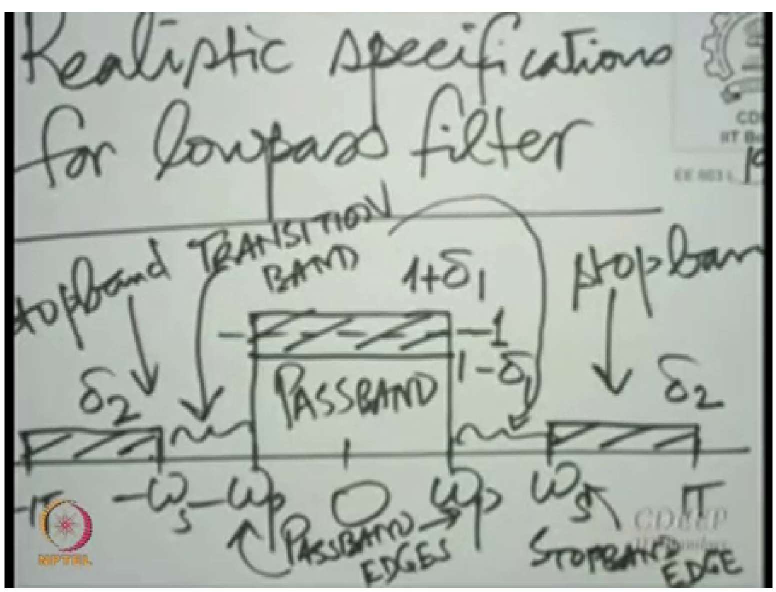
Discontinuity in frequency response is the cause of instability and therefore the second compromise is, we must insist on a continuous frequency response, that is the ideal towards wish to strive all the specifications that we wish to meet must allow the frequency response to become continuous. In fact, for a rational function it needs to be much more, it needs to be analytic, anyway so we put that compromise.

(Refer Slide Time: 15:26)



Second compromise: There needs to be a transition band between successive bands, you cannot have a passband touching the stopband, there must be a band of transition during which you may allow the response to move gradually from the passband to the stopband or from the stopband to the passband. So, let us take therefore the example of the low pass filter. What kind of specifications can we put which can be met?

(Refer Slide Time: 16:13)



So, in the low pass filter realistic specifications would look like this. You would of course, have a passband edge which we call ω_p. The passband itself would have a tolerance, of course, the ideal response that you want is a 1, but you must allow the response to vary from $1 + \square_1$ to $1 - \square_1$. You also need to have a stopband edge here.

And again in the stopband you cannot have the response go to 0 all over you must have a tolerance and therefore you must allow the response to vary in the shaded regions, in the passband and in the stopband. So, this is a stopband and so is this and this is the passband and this is the transition band and so is this.

So, if you look at it, there are three actually there are two basic compromises and three compromises in all you have a passband tolerance here you have a stopband tolerance in these two and you have a transition band, ω_p is called the passband edge. These are called the passband edges and this is called the stopband edge, so is this. And obviously the passband and stopband edge cannot coincide, that is what a transition band is all about.

Now, we have good news. No matter how small the tolerance in the pass band is as long as it is non-zero and how small the stopband tolerances and how small the transition band is, the filter is realizable, that is the good news. So although we started with the bad news that the ideal filter is unrealizable, we now have the good news that the moment we make these two basic compromises the filter becomes realizable.

And realizable either with an infinite impulse response system or with a finite impulse responses that is the beauty of it. Let us make a note of that. Yes, there is a question.

Student: Why we need passband tolerance?

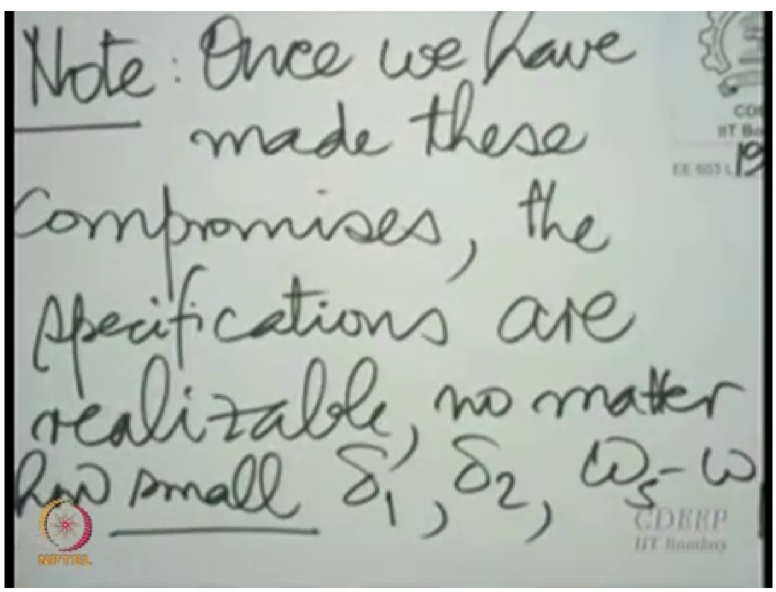
Professor: Okay, so the question is why do we need a passband tolerance? Now, you see suppose you did not allow a passband tolerance, that means you insisted that the response be constant in the passband, in fact the same thing holds to stopband, suppose the response is constant in the passband.

Now, let that constant be c and let us assume that you could indeed get a rational function which meets that constant response c in the passband. When you subtract c that constant from the rational function the resultant function is also rational. You see c, constant is a rational function, 1 rational function minus another rational function is rational.

So, the difference is the rational function, the difference is 0 all over the passband and you run into the same trouble that you did when you want a continuum of zeros for a rational function. Is that correct? Is that clear? Is it clear now why you cannot have a constant passband or stopband response? Yes, because that leads to contradiction to rationality. In fact, now that this question has been raised let me put 1 more challenge before you.

It is not only constancy that is forbidden by a rational function. My contention is other things are forbidden too, what are those other things, can, what are the other kinds of responses that cannot be for a rational function that is left to you to think. Anyway it is not relevant to what we are doing right now. So, I would not like to, but I put it as a challenge before you.

(Refer Slide Time: 21:25)

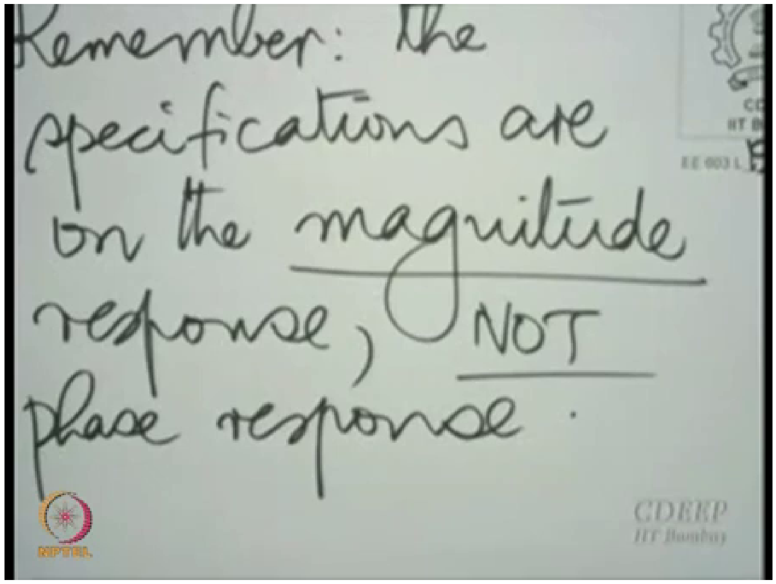


So now coming back to this discussion let us make a note of this we will note, therefore that once we have made this compromise specifications are realized or realizable no matter how small are δ_1, δ_2 and $\omega_s - \omega_p$, no matter how small all of them are. Yes, there is a question.

Student: What about the causality condition?

Professor: The question is what about the causality condition. Well, the beauty is that you can realize these with causal filters, but you will have to allow a phase. Now, what we are talking about here is the magnitude response. We have put specifications, so please note, let us make, I think that is a good question, so the specifications that we are putting are on the magnitude response and the phase response comes as a necessary evil.

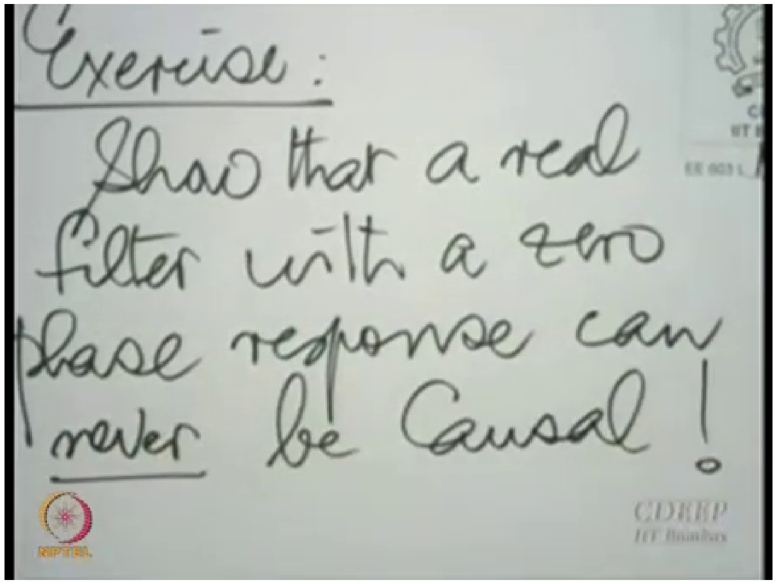
(Refer Slide Time: 22:37)



So remember the specifications are on the magnitude response not the phase response. In fact, we cannot put a specification on the phase response that is the problem. If we also insist on putting specifications in the phase response then we become restricted, not that we cannot realize it, but what we cannot realize is 0 phase response that is not possible.

And now that also answers why phase response is a necessary evil. Phase response is essentially a consequence of causality. If you had 0 phase response you could never get causal systems, in fact, I put this as an exercise for you to reason.

(Refer Slide Time: 23:44)



Show that a real filter with a 0 phase response can never be causal. Yes, there is a question.

Student: How can the phase decide the causality of a system?

Professors: So, the question is how can the phase decide the causality of a system? Well, what I am saying is that the phase is a consequence of the causality of the system, it does not decide, if you want the system to be causal you have no choice but to allow phase response. But just because the system has a face response does not mean it is causal. So, you can have non causal systems also with a phase response.

But if you want causality, phase response is a must. And if you want causality, there is a certain kind of phase response that you need to have. What it means is? You say, what is phase response after all? A phase response shifts each sine wave in time, by a pre specified angle. Is that correct? What does a magnitude response do? It multiplies each sinusoid in the input by what the magnitude response specifies.

What does the phase response do? It adds that phase to each sine wave in the input, depending on what the phase response is at that frequency. When you add a phase to a sine wave, what are you doing 2 sine waves, essentially, you are moving the sine wave. So, what you need to do is to move all sine waves adequately to make the system causal that is what the phase response must satisfy if you want to ensure causality.

If you want to answer the question in a broad sense, then the phase response must be such that sine waves are all shifted in a manner that causality is ensured that you are not asking for future inputs to come before you can deal with the present output. So, essentially, you are asking, you have to wait for some time, that is what it means, waiting for some time for an output to emerge is a consequence of causality.

The effect of the current sample is not going to be felt only now, it is going to be felt for some time from now as well. That is another way to understand causality. And in a way you have to wait for that time. So, waiting time that is a consequence of the system being causal is reflected in the phase response or another word and in other words, the phase response is necessary if you want the system to be causal.

Unfortunately, when you have put down magnitude response specifications, then phase response specifications cannot be put as well and then we cannot insist that they be met too; that is not possible. So, if you meet the magnitude response specifications, whether it be with an infinite impulse response system or with a finite impulse response system, there is only a certain class of phase responses that you can then meet.

You cannot ask for an arbitrary phase response and have that independently met as well. That is what it is all about. The phase response is not really in your, not too much in your control after you have met the magnitude response specifications.