Digital Signal Processing & Its Applications
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Lecture 20 A
IIR Filter Design

So, warm welcome to the twentieth lecture on the subject of Digital Signal Processing and Its Applications. We have been discussing the synthesis of Discrete Time Filters, and we should recapitulate for a couple of minutes what we have done so far on that theme.

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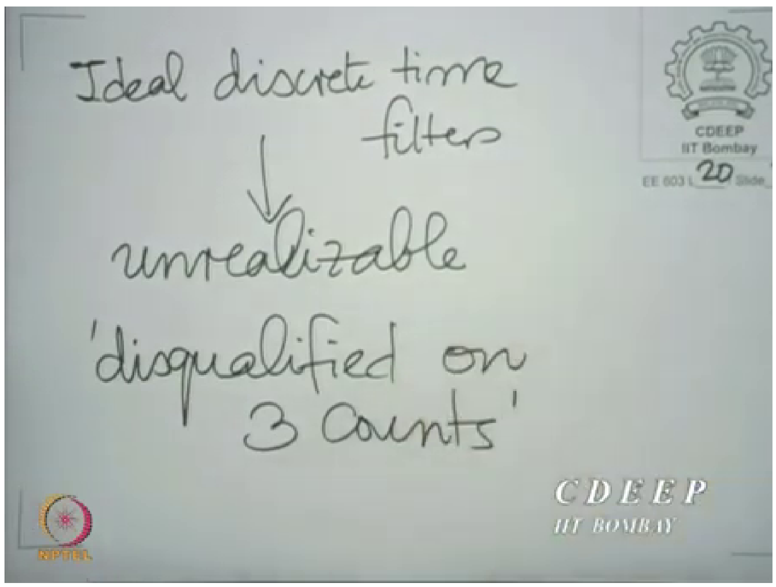

We have been looking at the ideal filter first, the Ideal Discrete Time Filters. And we realized they are unrealizable; they are disqualified so to speak on many counts, on three counts to be more specific.

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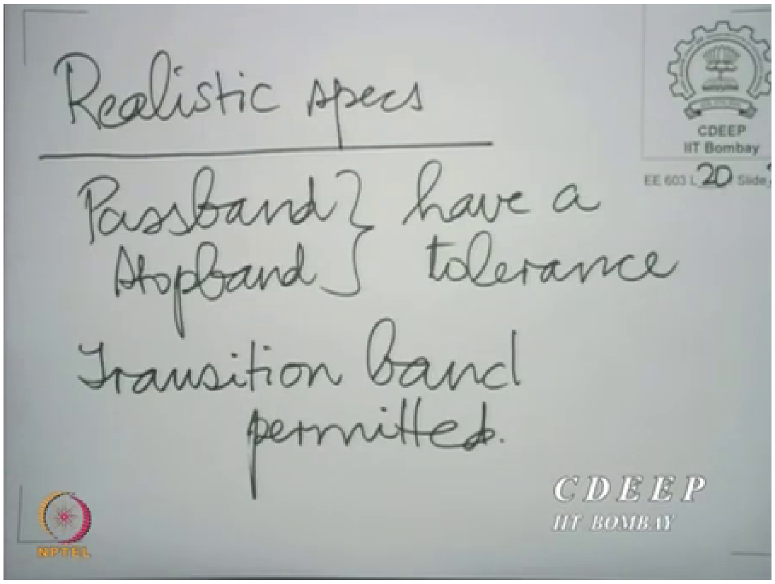

We then move down to putting down a realistic set of specifications. And in the realistic set of specifications, the Pass band, and the Stop band have a tolerance, and there is a Transition band. These are the two things that need to be taken care of. Now, given that you have allowed for a non-zero tolerance in the Pass band and the Stop band or Pass bands and Stop bands, as the case may be and that you have allowed the Transition band, the resultant specifications are always realized, we noted that.

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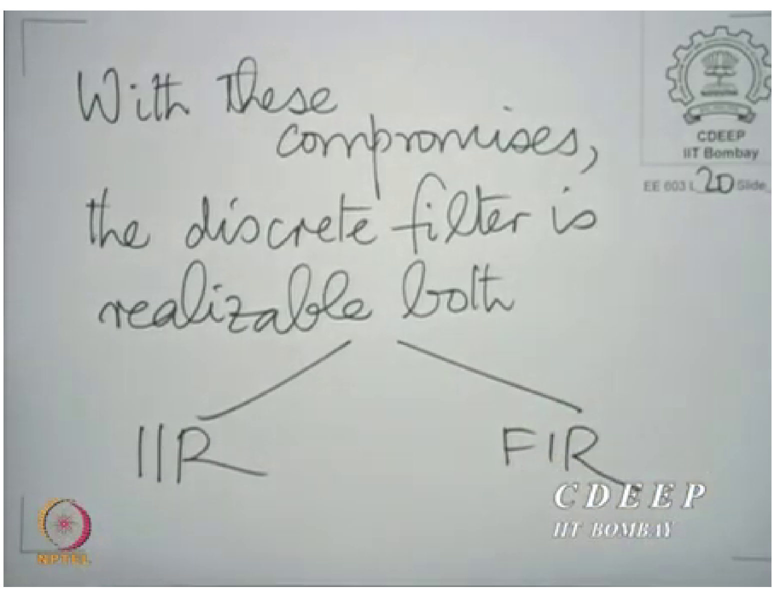

So with these compromises, the discrete filter is realizable. Now, in fact, the good thing is it is realizable both as IIR and FIR. That is, you could realize it by using a Finite Impulse Response system or an Infinite Impulse Response system. You have the choice between the two. Now, obviously the Finite Impulse Response system has several advantages to its credit.

We shall see them, but even right now we can see that the Finite Impulse Response system is stable unconditionally. Even in the presence of numerical errors, it would be stable. That is not the case for Infinite Impulse Response System, they could become unstable. Moreover, we shall see later that it is only the Finite Impulse Response system that can give us the phase response that we desire.

The phase is unavoidable, we noticed that last time. We said the specifications are on the magnitude, not on the phase. The phase is a necessary evil; it normally follows as a consequence of the magnitude response that we are trying to meet, right? But there again the FIR filter scores over the IIR filter, we will see that later.

Now, in all these circumstances why at all do we choose the IIR filter for design then, and we noted that we noted why that was the case. We said that you know, in spite of all these, the IIR filter can offer advantages in the amount of computation, the complexity of the system, or the computational or the hardware-software resource demand and therefore we still consider the prosperity of IIR systems.

And there is another reason IIR systems can be designed by taking inspiration from analog systems. So, there is a history of analog filter design which, of which we can take advantage. We can take advantage of the methods available for analog filter design and use a universal process to convert from analog to discrete time and that is a huge advantage in the design of IIR filters. So for all these reasons we agreed that we would begin with IIR filter design.

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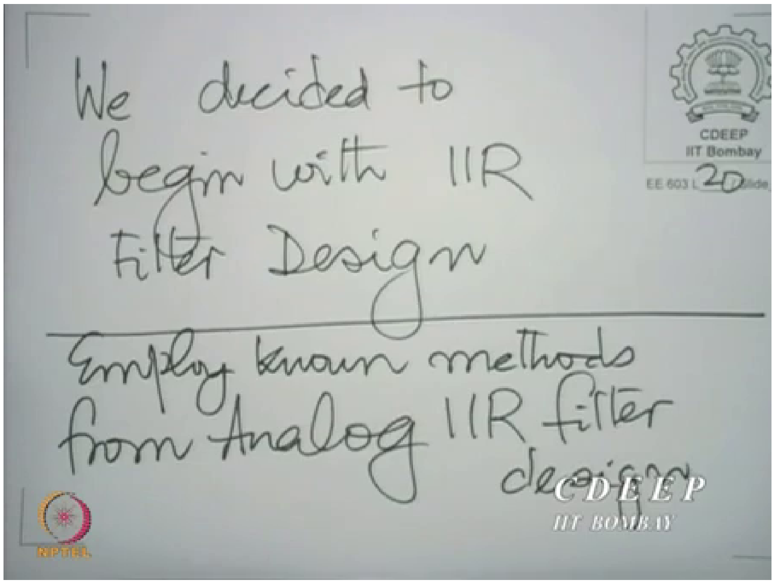

We decided to begin with IIR filter design. And the basic philosophy that we would follow in IIR filter design is to employ known methods from analog IIR filter design. In fact, we also made a remark; we noted last time that when we consider analog filters, we have no choice. There is no finite, there is no nontrivial finite impulse response analog filter.

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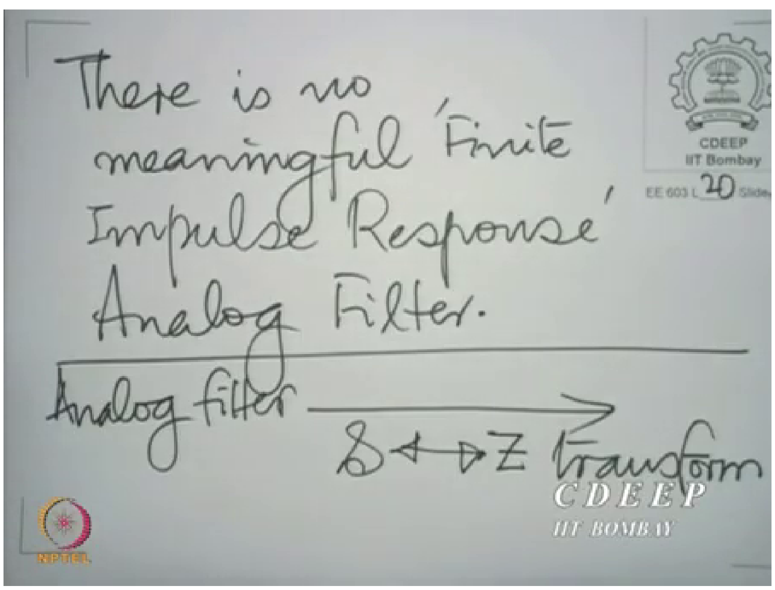

In fact that is one of the reasons why we go to discrete time. There is no Finite Impulse Response Analog Filter, and therefore it is only by IIR filter design that we can take advantage of the knowledge of Analog Filter design. Now after designing the Analog Filter, what do you do next? The Analog Filter is subjected to what we called an s to z transformation.

Essentially we said that the Analog Filter is described in terms of the Laplace variable. So we looked at the system function of the analog filter which would essentially be the function of the Laplace variable s. We are also given an interpretation to the Laplace variable the last time. We noted that the Laplace variable is essentially representative of the derivative operation. And you know we could spend just a minute on justifying that again..

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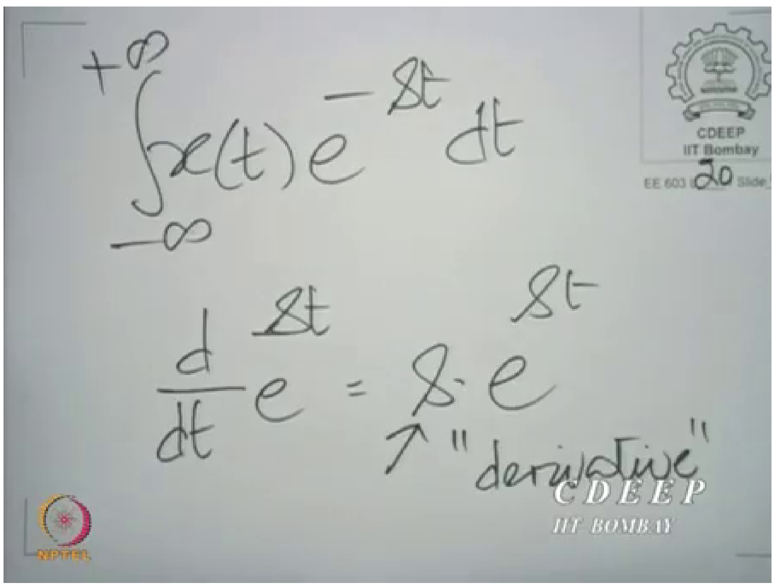

Actually, the Laplace transform is an effort to express any function in terms of e^{st}, complex exponential. And it is quite clear the $\frac{d}{dt}e^{st} = se^{st}$. So multiplication by s is like a derivative operator on e^{st}.

And if you think of any function as comprised of several e^{st}'s, then multiplication by s individually on each of these e^{st}'s amounts to taking a derivative. So in that sense multiplication by s or the operator s or the variable s is representative of a derivative. Now, we note this because this is what will inspire us to come up with an s to z transformation.

In contrast, the variable z or the complex variable z in the discrete domain relates to shifts, to movements. When we multiply a z transform by z inverse, we are shifting the sequence by one step forward and when we multiply it by z; we are shifting it by one step backwards. So z corresponds to shift. Is that right? Let us make a note of that.

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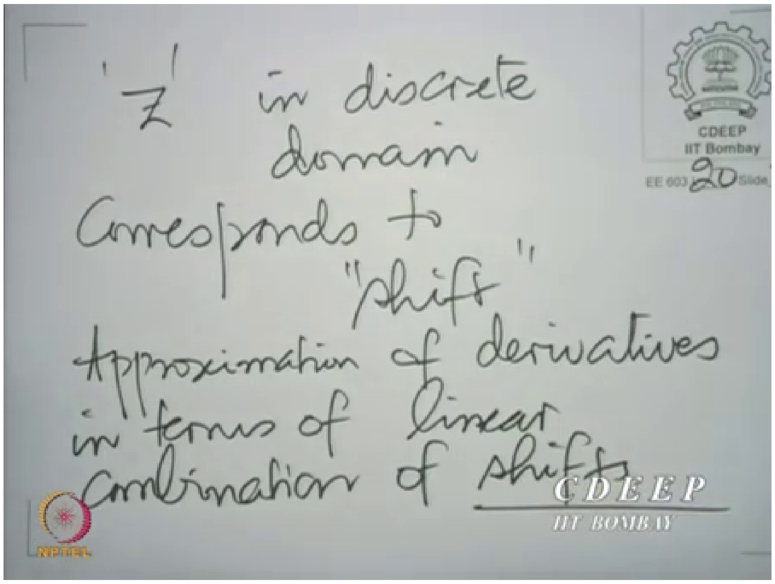

z in the discrete domain corresponds to shift. So what we need is an approximation of derivatives in terms of shifts, in terms of in fact linear combination of shifts. The better we can make the linear, the better we can approximate a derivative in terms of shifts, the better we will do in going from analog to discrete time by replacing s by, of course, a rational function of z. Now, we will also put down certain criteria on that rational function of z.