Digital Signal Processing & Its Applications
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Lecture No. 03 c
Minimum Sampling Frequency, Filters and Discrete Systems

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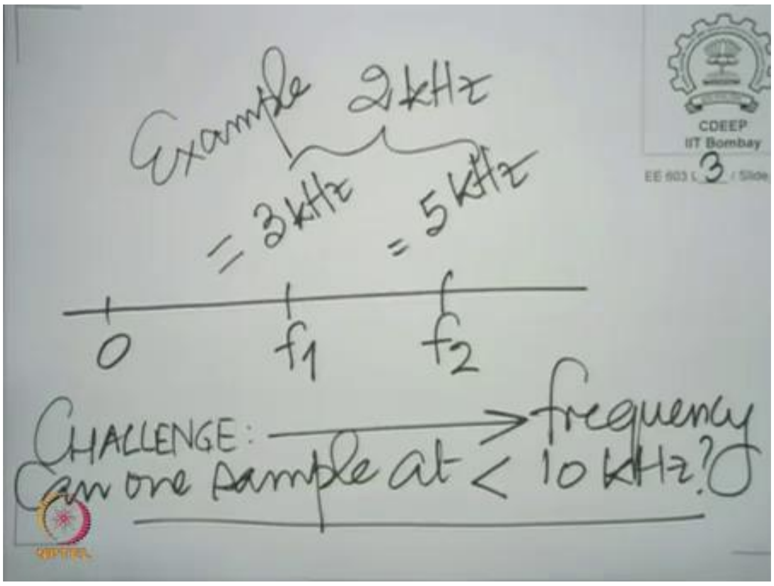

But can I do better is the question, that is the challenge. So challenge, can one sample in this case at less than 10 kilohertz and still reconstruct the signal in this particular case. And of course, if you answer the question for this particular case, you will come up with a more general answer as well.

I mean, pay attention to the question. In general, if all the frequencies from 0 to 5 kilohertz are occupied, then I have no choice at all. I need to sample at more than 10 kilohertz. But here I know the signal occupies the band only between 3 kilohertz and 5 kilohertz. So, actually it is really occupying a band of 2 kilohertz on the frequency axis.

Do I really need to sample at more than 10 kilohertz with this knowledge? Or can I do with less? And if I can do with less? What are those smaller sampling frequencies that I can use? And if I do use such smaller sampling frequencies, how will I reconstruct the original signal from these sampled versions? This is the question before you as a challenge.

I told you I love throwing challenges to the class. And you will find that this happens in subsequent lectures as well. But anyway, let us not get carried away too far by this challenge. Let us now come down to business if you want to call it that. We have agreed that we understand what it takes to sample a signal adequately. And we also agreed that if you do not sample a signal adequately, then you are going to run into the problem of ALIASING.

So, we want to avoid ALIASING, we have sampled the signal at least, in fact more than twice the maximum frequency component present in the original signal. And we are agreeing to reconstruct the original signal by cutting off all frequencies beyond the highest frequency component in the original signal.

And how do you do that? Put what is called a low pass filter, which cuts off after FM on the frequency axis. Incidentally, we are going to do filters in great detail, discrete-time filters, we will talk more about filters. But for the moment, we understand what a filter does. A filter retains some part of the frequency axis on a signal and throws out the rest. That is how we understand the filter for the moment.

In other words, it modifies the amplitudes and phases of the Sine waves present in any signal that is given to it in a certain way. What I mean by that is, it does not matter what amplitudes and phases, the signal that is given to it has, what matters is how it modifies them.

So, whatever be the amplitudes and phases of the Sinewave up to frequency FM if you put a low pass filter with a cut off of FM, it retains all the amplitudes and phases up to FM as they are and beyond the frequency FM, it makes all the amplitude 0. This is how we should understand the low pass filter.

Similarly, if you had a high pass filter with the cut-off of FM, it would mean that after the frequency FM, this filter would retain the amplitudes and phases as they are and before FM it would make all the amplitude 0. This is irrespective of what amplitudes and phases they are at after FM or before FM, it does the same thing to all signals irrespective of what those amplitudes and phases are.

Now, this is the kind of system that we are going to try and design in this course. It is very easy to describe the system. In fact, it is so simple, in a few sentences I have told you what a filter is. But you will realize as you go along the course, that you can never do this exactly. We will understand slowly, why?

You can only approximately do this. And the whole art or the science of discrete-time signal processing is how well you can do this, how closely you can do this thing that which you ideally want to do. You remember in the first lecture, I had talked about the problem of separation of male and female voices through an audio recording. Now, if you speak the language of frequency axis, what would that mean? That would mean that you have a cut-off point.

In fact, let me give you numbers. Typically, speech waveforms, for example, do not go beyond 4 kilohertz on the frequency axis, audio waveforms tell them go beyond 15 kilohertz, and definitely not beyond 20 kilohertz. So, for an audio signal, if you sample at more than 20 kilohertz, you are doing a good job.

For speech signals, if you sample at more than twice of 4, which is 8 kilohertz, you are doing a good job. Now, there again, between 0 and 4, you may reasonably assume that frequencies above 2 kilohertz would have a predominance of female component. And frequencies below 2 kilohertz may have a predominance of male component.

So, if you want to separate the male and female components, you may wish to break up the signal into its bands between 0 and 2 kilohertz, on one side, and between 2 kilohertz and 4 kilohertz on the other side. So, if you pose the problem in that language, then you may ask, can I exactly put the cut-off at 2 kilohertz or can I exactly break the signal between 0 and 2 kilohertz on one side and exactly between 2 kilohertz and 4 kilohertz on the other?

And unfortunately, the answer is no. You can never do this exactly. You can only do it approximately. And if you ask me to summarize, why we need a whole semester to design filters, this is essentially the reason, that though the task is easy to specify, it is not so easy to do.

What are the hurdles that we encounter when performing this task? The hurdles are first to describe a general class of systems which will do this, do what? Do exactly, the same thing to all signals, irrespective of what original amplitudes and phases they had, at different frequencies.

So, the system should not be partial, it should not look at how much of male component or female component there is in the audio waveform and then decide what it will do. It should be impartial, between 0 and 2, I am going to keep between 2 and 4 I am going to cut or between 2 and 4 I am going to keep between 0 and 2 I am going to cut, it should be impartial. Achieving this impartiality means that the system need to have several properties. And we are now going to understand what those properties are. In fact, we will see soon that.

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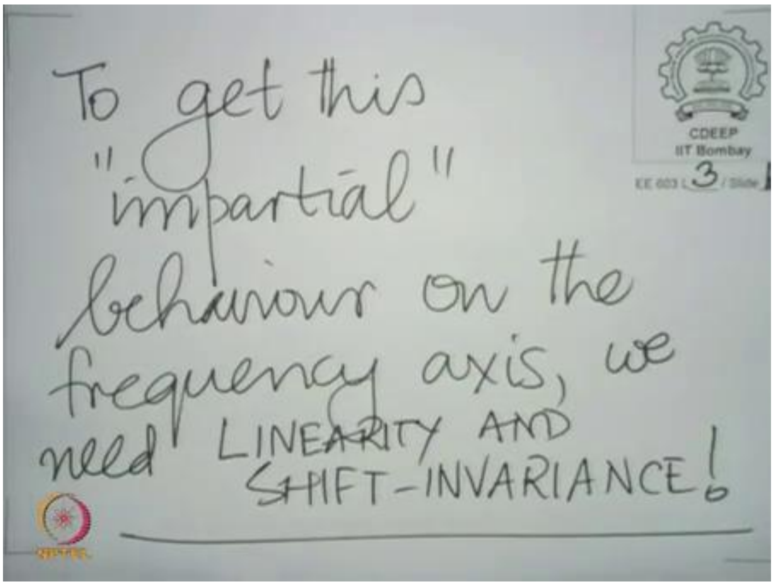

To get this impartial behaviour, let me write it down, to get this so-called impartial behaviour on the frequency axis, we need linearity and shift-invariance. In many discussions on systems and signals, we encounter these terms, we encounter the term linearity, we encounter the term shift-invariance.

And we have possibly had a lot of discussions in our past curriculum or in our past degrees on linearity, on shift-invariance and their consequences. It is useful at this point to reflect, if necessary, with the benefit of hindsight, why we started talking about linearity and shift-invariance in the first place? And the answer is this.

See the whole picture of what you want to do, you want to get this impartial system which does the same thing on the frequency access to all the signals that are given to it. And to get this impartial nature, you need a linear shift-invariance system. I am stating this at the moment, but in the subsequent lectures, we are going to prove this.

Now, this is true, whether you are talking about continuous time, or the independent variable being continuous, or the independent variable being discrete. Perhaps some of us may have been exposed to this idea when we talked about continuous independent variable systems. Now, we are going to describe and then prove these results in the context of discrete independent variable systems. But before that, let us put down what we mean by a discrete system.

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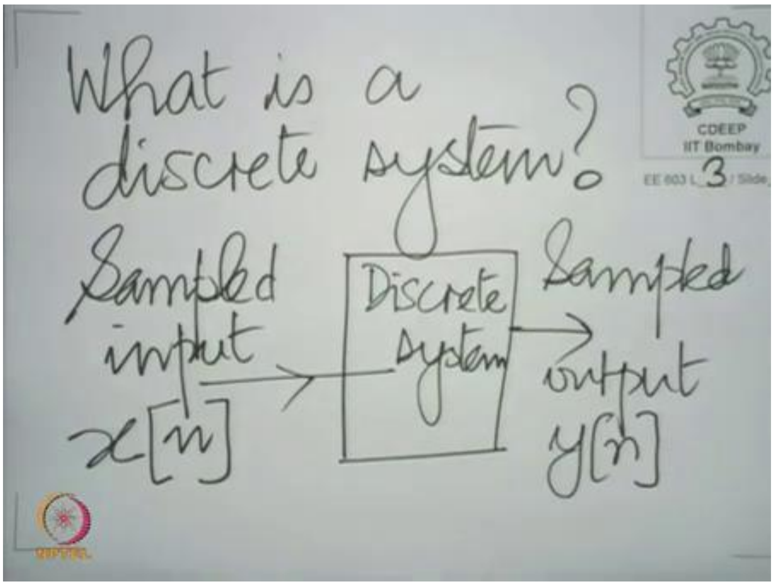

So, next question that we need to answer is, what is a discrete system? A discrete system has a sampled input and produces a sampled output. And if we accept the notation that we have introduced some time ago, we shall use X[N], you remember, when I started with the discussion of a Sinusoid being sampled, I said that if you take samples at all multiples of T, then you could essentially substitute t by NT. And you could call X [NT] as X[N]. So, we will use that notation in future. We have a sampled input, we have a discrete system, which gives you a sampled output. And we will use the standard notation, y[n]to denote the sampled output here.

Now, you have the sampled input, we will agree now. Henceforth, that the sampled input has been sampled according to the Nyquist principle, or the Nyquist Shannon which take a theorem, you have made sure that you have ascertained, that you have taken samples at more than twice the maximum frequency component present in the original signal. That means you know how to reconstruct the original signal from its samples, just put it through a low pass filter, which cuts off at the maximum frequency component present in the original signal.

Now, of course, you could put that X[N] into a discrete system, do what you want with the samples, and then the output can also be put through the same low pass filter. So, if you were to take an analog system, which had the original continuous-time signal as the input with maximum frequency component FM if you were to do some operations on the frequency axis with that analog signal, and if you were to look at the output. Now if you sample the input and sample the output at the same instance, you would get what you are calling X [N] and y[n] in this case.

That is what I mean by prevalence. You have sampled the input. You have done something with those samples, you have generated an output. The input X [N] has generated the output y[n], X [N] are essentially samples of the input at the Nth instance, I mean, N refers to the instant number N equal to 0 means the point T equal to 0, N equal to 1 means the point T equal to, I mean t equal to T, N equal to 2 means the point t equal to 2 times T and you can do this N equal to -1 means the point t equal to -T, and so on, so forth.

So, N is essentially the sample instant to the sample number, you have the sampling instance for the input and you have the sampling instance for the output, you have a relation between them, there is a discrete system, which creates that relationship. And we are assuming that the discrete system does exactly what the original analog system would want to do.

That you take again the example of male and female voice separation. If you have this up to 4-kilohertz signal, which is a conglomerate of several male voices and several female voices, you would have an analog separator which would take the frequencies from 0 to 2 kilohertz and put it on one side, and take the frequencies from 2 kilohertz to 4 kilohertz and put it on the other and you would have a corresponding discrete-time system.

Which does exactly the same thing that means, it would sample the original audio signal or speech signal, it would put essentially output a stream of samples and if you reconstructed the output signal, as you did, as you would reconstruct the input signal by the same process that I described, then you would get after reconstruction of the output the male voices in the female voices in principle in different baskets. (())(16:30) So, then for discrete systems, this is what a discrete system is, it takes a discrete input gives you a discrete output. Now, remember a discrete system. And let me note this down before we conclude the lecture today.

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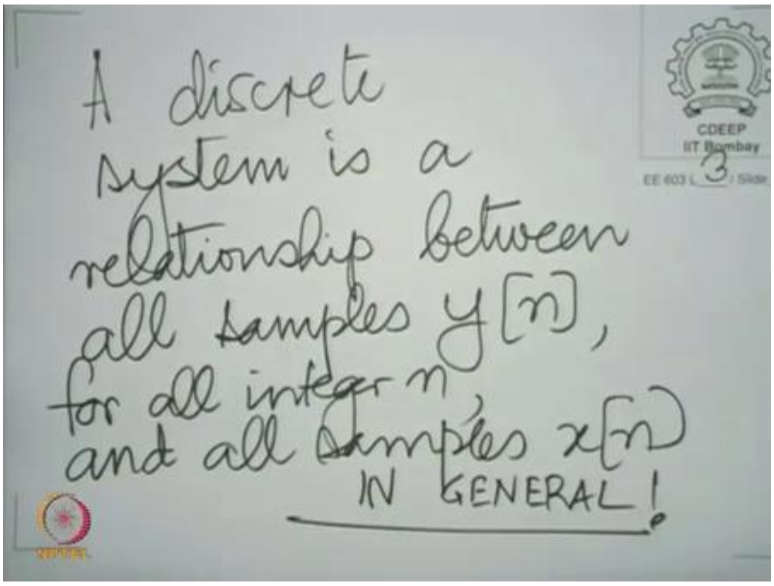

A discrete system is a relationship between all the samples x[n], for all integer n I mean, and all samples x[n] in general. So, you must not think of it as a point-by-point relationship in general. y[10] could be related to x[10], x [9], x [8], x [7] all the way, and also x [11], x [12], x of in principle, yes, I mean, that could happen. Or maybe at least y [10] might be related to a few samples, x[10], 9, 8, and maybe 11, 12.

In fact, there could be a relation between a group of samples of y and a group of samples of x. So, you must remember, a system is a relation between streams of samples, 2 full streams of samples, that makes a system so much richer in nature. And now we need to start studying systems by going step by step. We cannot deal with this entire reality all at once. So, we shall do that, starting from the next onwards. Thank you.