Digital Signal Processing & Its Applications

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Lecture 21 a

Basic IIR Filter Design Steps

A warm welcome to the 21st lecture on the subject of Digital Signal Processing and Its Applications. We continue in this lecture to discuss the subject of filter design and specifically Infinite Impulse Response filter design. We need to recapitulate a few ideas before we proceed to a specific kind of design. We have agreed that for IIR filter design, we shall follow the following process.

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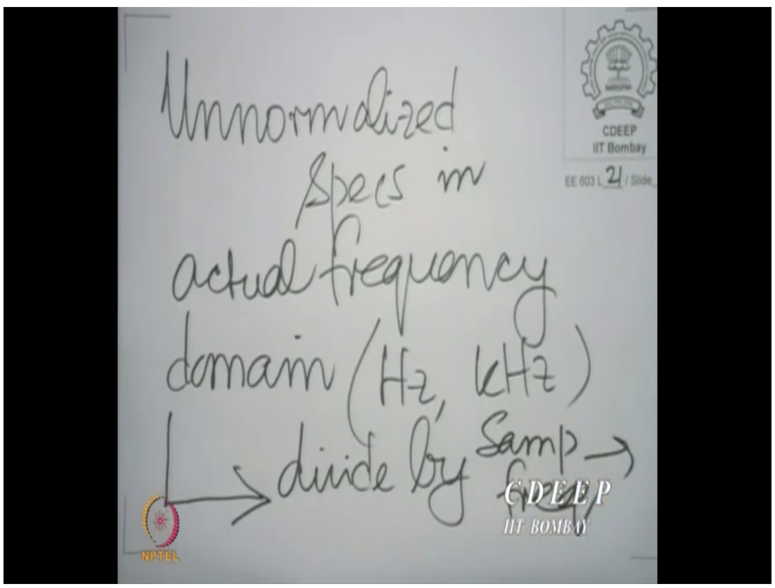

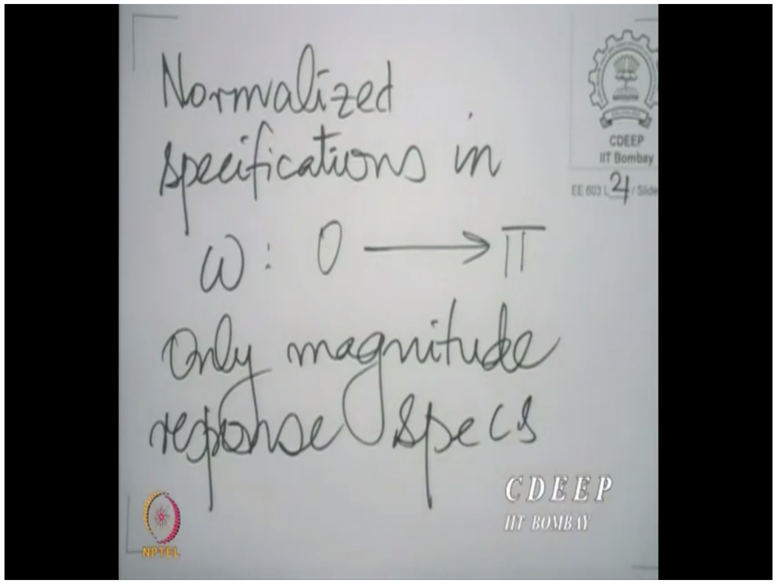

We would first identify what are called the unmoralized specifcations; those would be given to us. These would be in the actual frequency domain that is in terms of Hertz, Kilo-Hertz, whatever, So, we divide these by the sampling frequency and we get what are called the normalized specifications.

And the normal specifications are in the normalized angular frequency, which of course would be between 0 and π because you are of course going to design a real filter and therefore what you have between 0 and π is also going to be mirrored between -π and 0.

Further, what we have as specifications are only the magnitude specifications. Is that right? So, only magnitude specifcations. Phase response is just going to be a consequence of the design. We have very little control on it. We will just accept what phase response we get.

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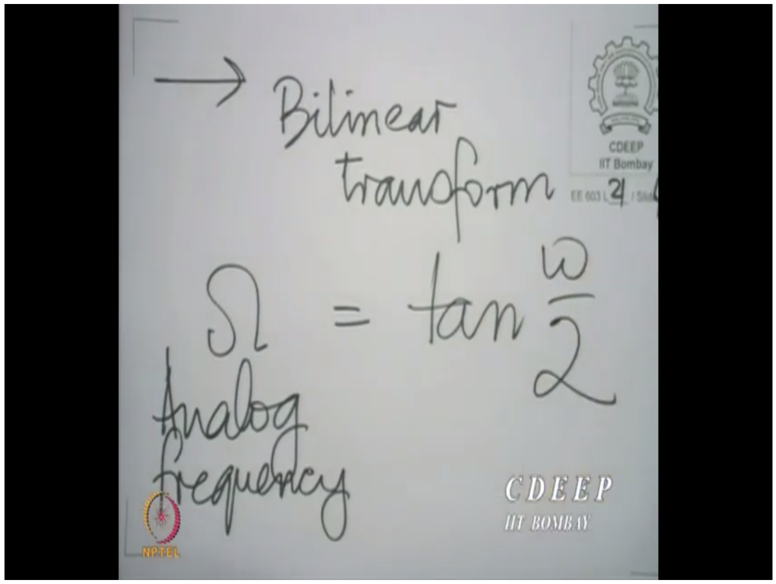

Now, having got the normalized angular frequency specifications, the next step is to use the bilinear transform. So we have already looked at the bilinear transform. So, and then we have the analog frequency, Ω, given by tan of the discrete frequency by two and therefore, a range of 0 to π maps into a range of 0 to ∞ in this process of bilinear transformation.

Now one must remember that the so called analog filter frequency that we are talking about here is only an intermediate design step. We would design that analog filter. We would need the frequency, the the response specifications of the analog domain, but we are not going to directly lose that analog filter at all.

And moreover, the analog frequencies that we get are not the frequencies that we will see anywhere in the operation of a discrete filter. They are only an intermediate calculation that we have employed for the purpose of design. Is that right? So we have this analog frequency which we have got. We have got frequency specifications in the analog domain and remember we have already seen the properties of this function tangent.

It is a monotonically strictly increasing function between 0 and π, and therefore whatever be the nature of the filter in the discrete domain, that nature is preserved in the analog domain. If it happens to be a low-pass filter, it is transcreated into an analog low-pass filter. If it happens to be a high-pass filter, it is transcreated into an analog high-pass filter. It happens to be a band pass filter, analog band pass, and so too for band stop.

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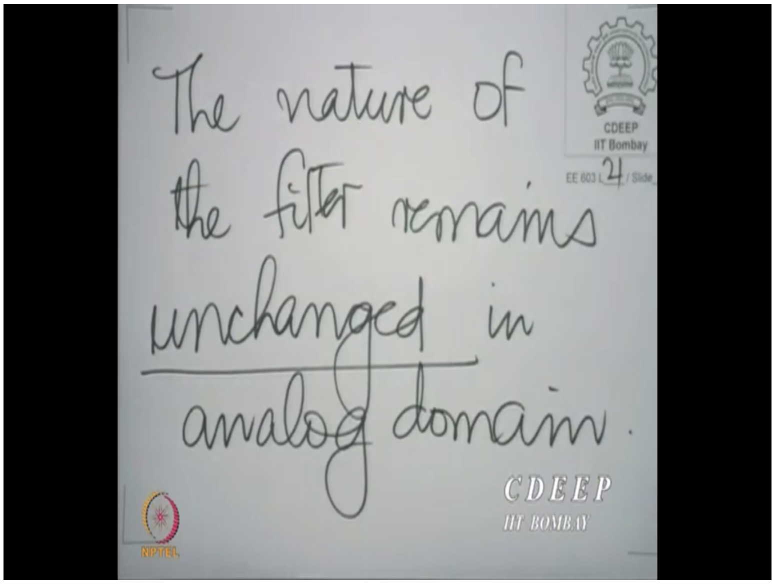

So therefore the nature of the filter remains unchanged. You must make a note of this or emphasize this. The nature of the filter remains unchanged in the analog domain. In fact, we must make a remark here. You see, when we talk about nature, there are two, in fact three parts in the nature. One is the actual specifications on the dependent variable.

That means how much is the passband tolerance, how much is the top band tolerance or if there are multiple passband than tolerances, and similarly stop bands. What kind of filter that is the second. And the third is the nature of the passband and the stop band. That is something that we have not yet talked about and we shall do so very shortly. So in the passband, there are two possibilities, the magnitude may be monotonic, or the magnitude may be non-monotonic.

So you know, even though there is a tolerance, nothing says that you must have a monotonic change of magnitude from one end to the other in the passband. The magnitude may be non-monotonic. So we have a choice between monotonic and non-monotonic magnitudes in the passband and the stop band, and that nature is also carried.

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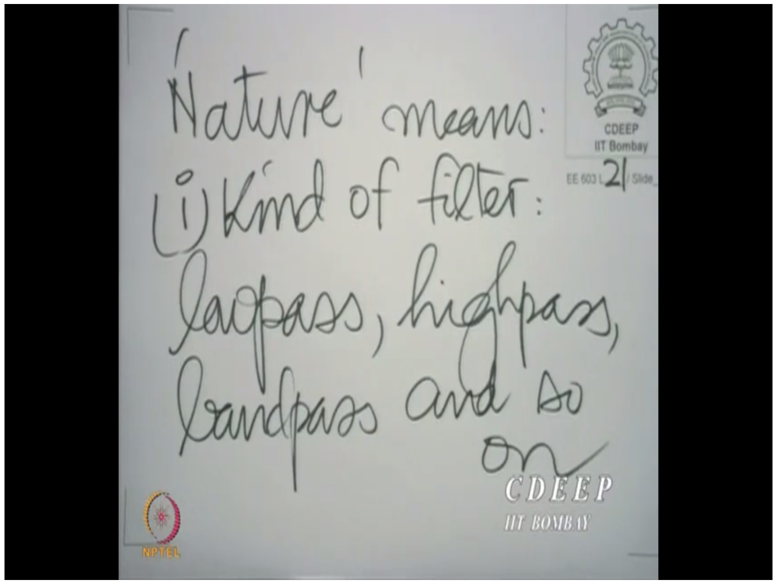

So all the three aspects of nature, that is, let us write them down, nature means, one, kind of filter, low-pass, high-pass, etc. That is the first aspect of the nature of the filter.

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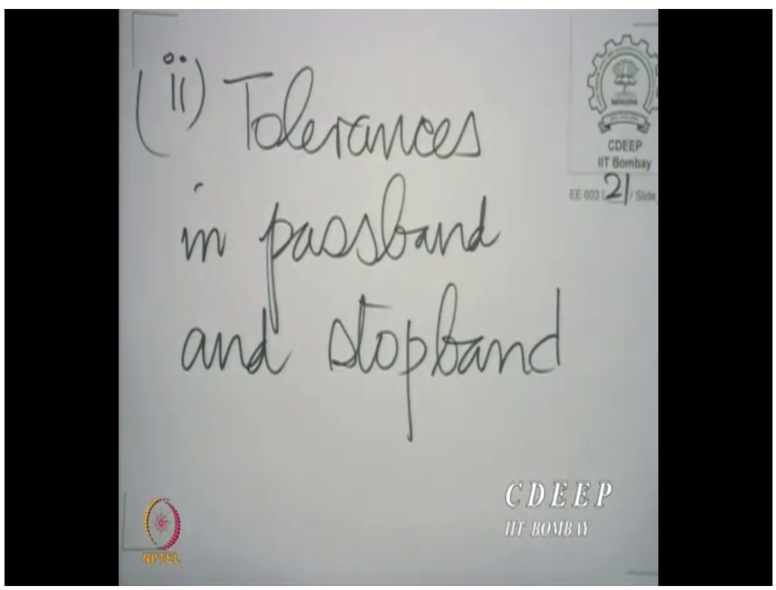

The second aspect of the nature is the tolerances, tolerances in the passband and stop band. That is also carried over because what we have distorted is the independent variable, not the dependent variable. The dependent variable is carried with the independent variable. There is no change of tolerances.

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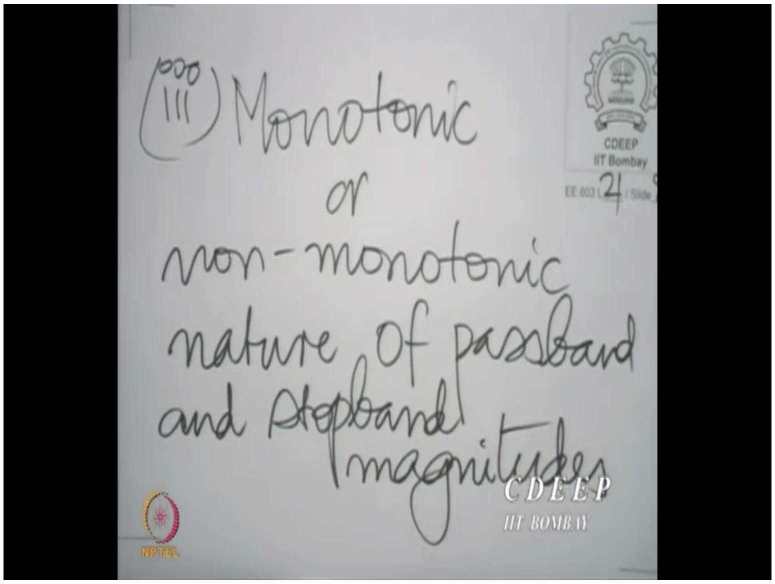

And the third is the monotonic, or non-monotonic nature of passband and stop band magnitudes. So in the passband, the magnitude may vary monotonically from one end of the passband to the other, or it may be non-monotonic. It may increase, and decrease, and so on, and both have their merits.

It is not that any one of them is better than the other. Each has its merits. We shall see an example of both, but anyway, let us now come to an observation that once we have done this, we have carried the nature as it is to the analog domain.

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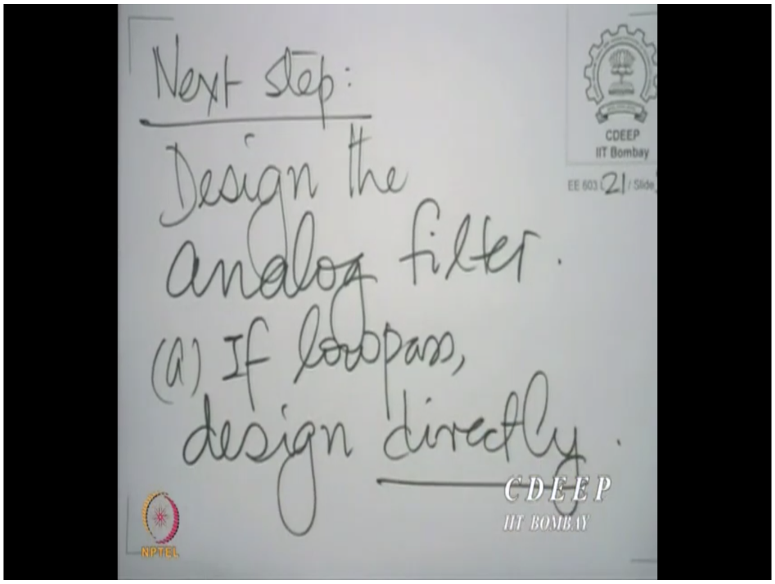

So, next step is of course, design. Next step, design the analog filter. Now here again, there are two parts to this step. The first part is, if it is low-pass, design directly, and we are going to see how we can design analog low-pass filters directly from the specifications.

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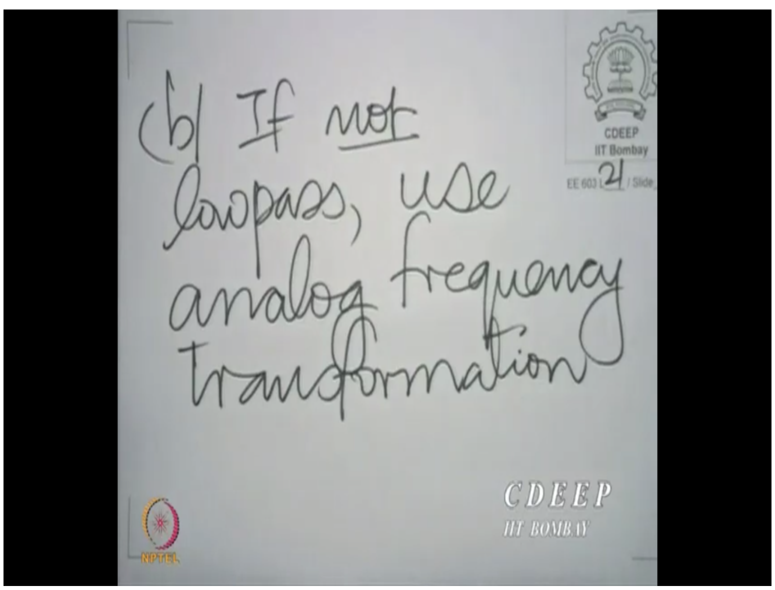

On the other hand, if the filter is not low-pass, use what is called an analog frequency transformation. So in here, we are going to study later mechanisms by which you can convert a filter other than low-pass into a low-pass filter for the purpose of design, or why one shouldn’t really say convert filters, convert specifications on filters other than low-pass into equivalent specifications on a low-pass filter.

Design the low-pass filter and then run back to the kind of filter that you wish to realize. Now that would be the subject of a subsequent lecture, but at this point it is recognized that if you are not designing a low-pass filter then we will have to perform one more step here.

You will have to make a transformation in the analog domain itself, which we will study later, but for the moment, let us first look at the possibility of designing low-pass filter because if you are able to design low-pass filters then this additional one step would take us to any other kind of filter that we wish to design. Anyway, so once we have designed the analog filter, having designed the analog filter.

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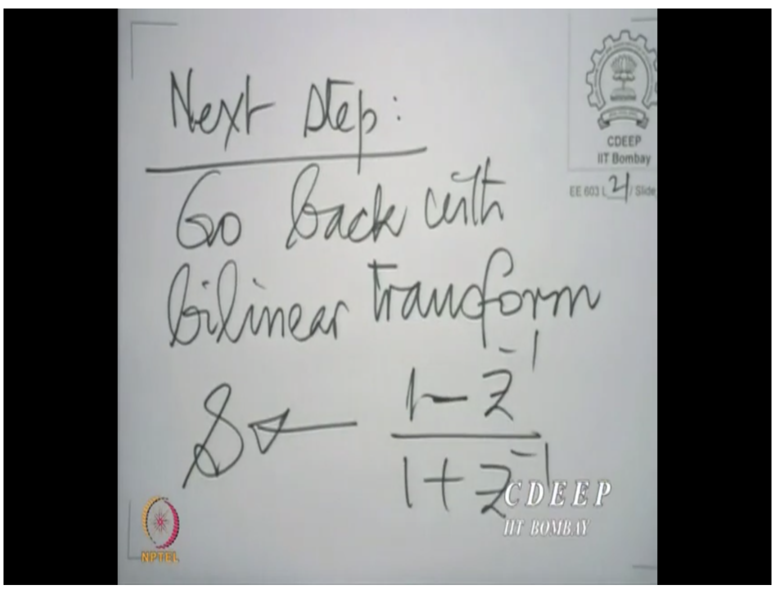

Next step, go back with the bilinear transform, and how do you go back with the bilinear transform in the analog domain? The variable $s \leftarrow (1 - z^{-1})/(1 + z^{-1})$. And therefore, rational analog filter as designed is converted into a rational discrete time filter; the specifications are carried as they are.

And of course, once we have done this, once we have written down a rational system function, the design is done, but then we have other steps later, we want to realize the filter. Realize means convert that rational system function into a hardware-software structure.

That of course, we shall study in detail later as well. So realization step itself is a big step, over which we have spent quite some time. But for the moment, let us be satisfied with this step. Once we have come here, we have the rational discrete time filter ready before us, and for the moment our job is done.

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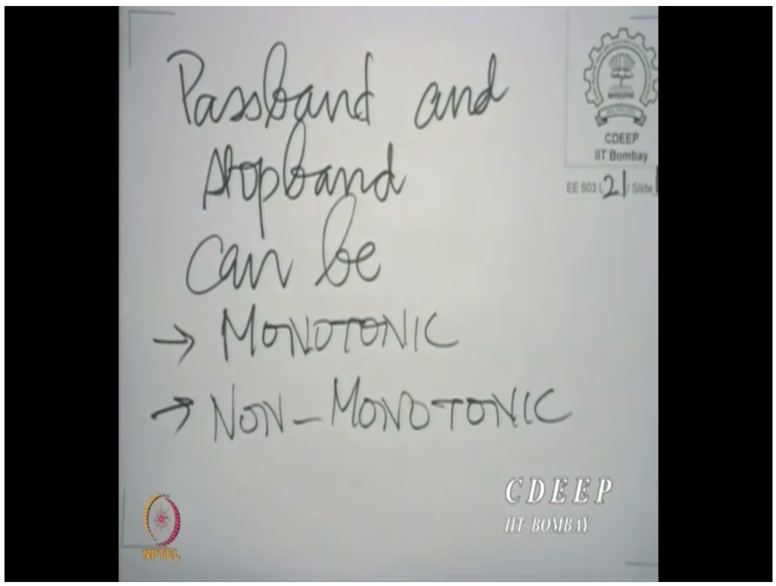

So, now let us look at how we can design low-pass filters. Now again, we have agreed that there are two possibilities for each passband and stop band. The passband and stop band can be monotonic or non-monotonic. By monotonic, we mean that the magnitude either increases monotonically from one end of that band to the other end, or decreases monotonically. It depends on the kind of band that we are talking about.

Non-monotonic means this is not the case. So it may go through a point of maximum or a point of minimum within the passband or the stop band. Now it turns out that if we look at the possibilities of non-monotonic design, the best choice of a non-monotonic response is what is called an equiripple response. Let us write that down.

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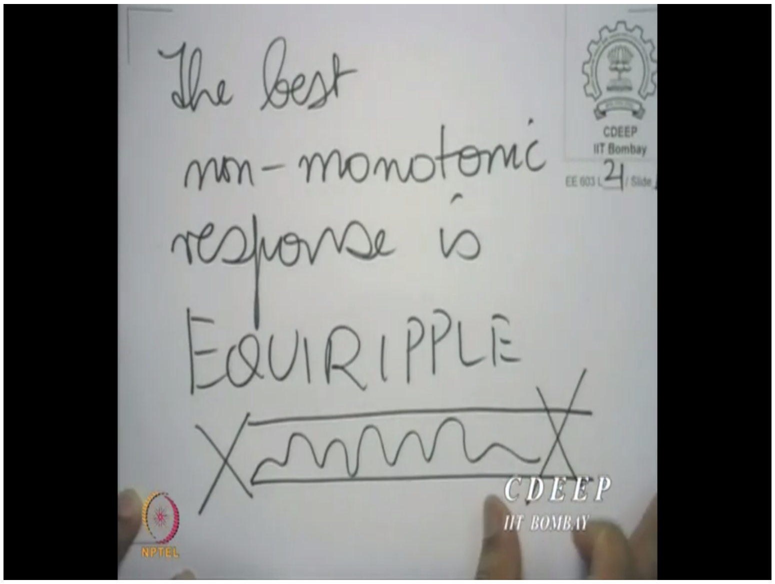

So, the best non-monotonic response is equiripple. Equiripple means all the oscillations, all the changes of magnitude are of the same intensity. They are all, you know, if there, so for example, you can visualize a band here, this would have been, you know this would have been a non-monotonic response, but this is not permitted. Here, they are not all of the same, the maxima and minima are not all of the same height. This is not equiripple, this is not accepted, and rather what is accepted is a response something like this.

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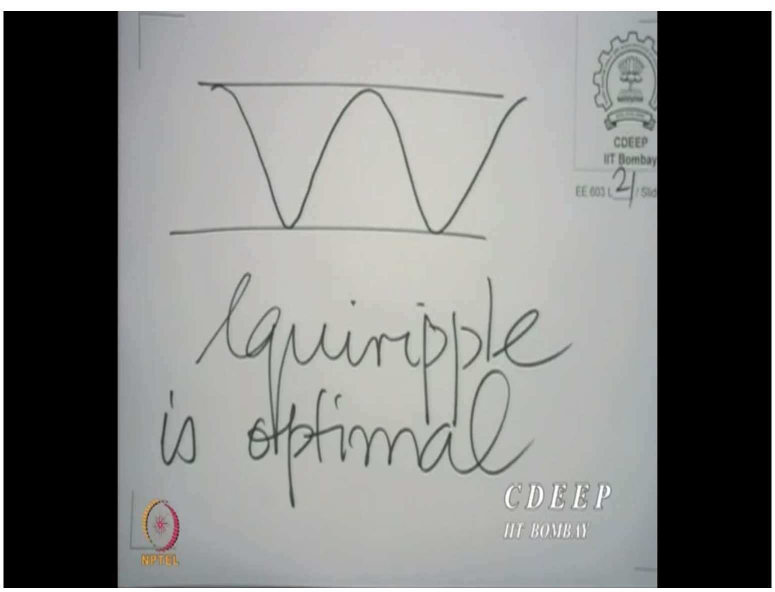

Equiripple is optimal, that is what we employ in our design process, and if you are going for a non-monotonic response, the best thing to do is use an equiripple one. Why I am not going to that discussion here? For those of you who are interested, you might want to look up rational approximations, a mathematics textbook on rational approximations, but that is not of prime interest here.

Here we will not concern ourselves with that so much at the moment. But anyway, let us accept that we either have a monotonic response, or any equiripple response, and again, we have four possibilities. We have the passband and the stop band, each of them being monotonic or equiripple.

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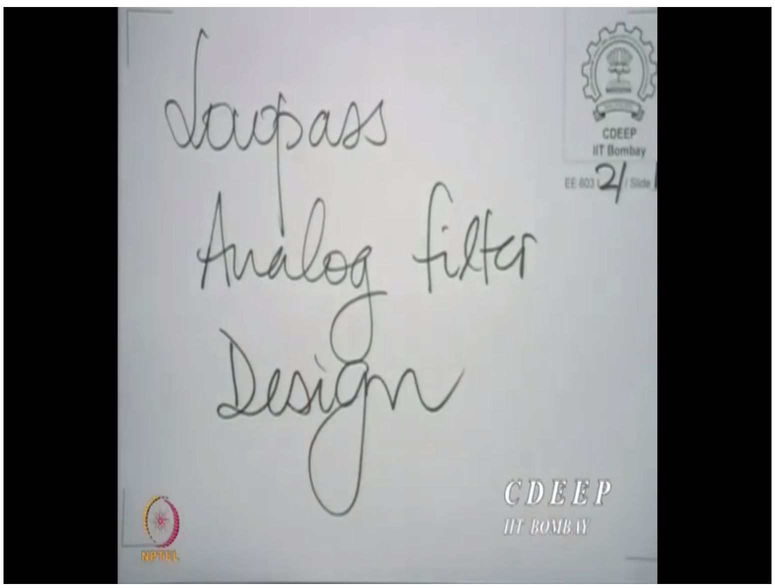

So, let us consider low-pass analog filter design now.

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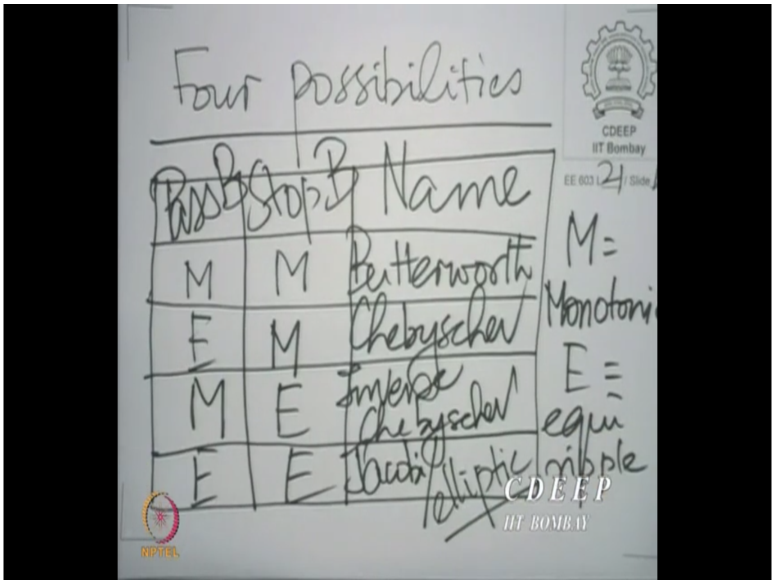

And as I said, we have four possibilities, and we gave a name to each of those four possibilities. Passband, stop band, and name. M stands for monotonic, and E for equiripple. So passband and stop band can both be monotonic and this called the Butterworth’s form of design. Incidently, these names are all after scientists who have built these concepts, this kind of design, so Butterworth is the name of a scientist, a researcher.

The passband could be equiripple, and the stop band monotonic, which kind of design is called as the Chebyshev design, again after the mathematician who introduced the whole class of Chebyshev functions. You have a monotonic passband, and equiripple stop band which is called as the inverse Chebyshev for obvious reasons.

And finally, you could have both the passband and stop band to be equiripple in which ways in in which possibility we call it the Jacobi possibility, again after the name of a person Jacobi, or they are also called elliptic filters because they are based on what are called the elliptic functions.

We shall only look at the Butterworth and the Chebyshev forms of approximation in this course, and we shall give a hint how to build the inverse Chebyshev design. It’s not too difficult to go from the Chebyshev to the inverse Chebyshev, and we shall leave it as a task for you to do on your own, given some steps of process. Is that right? So these are four possibilities.